## Cost of new residential property

## Introduction

The subject of my project is to analyze the cost of new residential property in Luxembourg. There appears to be a strong and widely recognized dependency of the prices to the distance from Luxembourg City. This dependency is the main motivation of this study which attempts to identify other factors and explanatory variables and to propose a multiple linear regression model of the property prices.

## Data

The data are taken from a periodic publication of the prices of the new residential properties available for sale and grouped by region (Center, North, South, West and East) and includes:

- Name of the location
- Type of property (apartment or house)
- Number of bedrooms
- Area of the property (in $\mathrm{m}^{2}$ )
- Price of the property (in Euros)

The distances from a given location to Luxembourg City are published by the government. ${ }^{1}$
In total, prices for 425 different properties were published in a given publication in 2011, grouped by region as detailed in the table below. 87 properties ( $20 \%$ of the total) are located in Luxembourg City.

Table 1 Data Count

| Region | Incl. L-City | Excl. L-City |
| :--- | :---: | :---: |
| Center | 154 | 67 |
| South | 79 | 79 |
| North | 109 | 109 |
| West | 32 | 32 |
| East | 51 | 51 |
| Total | $\mathbf{4 2 5}$ | $\mathbf{3 3 8}$ |

Full data excluding Luxembourg City is listed in Appendix.

## Analysis

As the prices vary greatly between small and large properties, the common practice is to measure the prices in Euros per square meter. We define the response variable $\mathbf{Y}$ as the prices in Euros per square meter. We start with the first explanatory variable $\mathbf{X}_{\mathbf{1}}$ for the distance from Luxembourg City, in km.

As seen from Chart 1, the property prices in Luxembourg City (at 0 km distance) are considerably higher than in the rest of the country and they have larger variation. This may be explained by a larger heterogeneity of various factors affecting the price, such as: higher variation of the price of the land within the city, city district, infrastructure, higher variation in cost of construction itself, higher demand (in particular for higher class property), etc. Based on these considerations, the Luxembourg City is excluded from further analysis.

[^0]Chart 1 Prices in EUR/m2 as a function of distance from Luxembourg City (all data)


The price of the property has two components: the price of the land and of the construction itself. While one can expect that the land becomes cheaper as the distance from the capital city increases (but still would have a certain limit), the prices of the construction should rather remain consistent throughout the country. Therefore, one would not expect a linear relation between prices per meter squared and the distance, and one can indeed discern a convexity in the chart.

According to Tukey and Mosteller's bulging rule, this particular non-linearity can be corrected by power transformation of either $\mathbf{X}$ or $\mathbf{Y}$ or both in the direction of decrease of the power. The Chart 2 shows the plot of $\ln (\mathbf{Y})$ as a function of $\mathbf{X}^{1 / 2}$ :

Chart 2 Power transformation $Y$ to $\ln Y$ and $X$ to $\sqrt{ } X$


For simplicity of further analysis, the designations $\mathbf{X}_{1}$ and $\mathbf{Y}$ are kept for the transformed variables.
The following table provides the summary of the analysis of variance (ANOVA) for this initial singlevariable model.

Table 2 ANOVA for the single-variable model

| Regression Statistics |  |
| :--- | ---: |
| Multiple R | 0.67353 |
| R Square | 0.45365 |
| Adjusted R Square | 0.45202 |
| Standard Error | 0.15841 |
| Observations | 338 |

ANOVA

|  | df |  | SS | MS | F |
| :--- | ---: | ---: | :--- | ---: | ---: |
| Regression | 1 | 7.00091 | 7.00091 | 278.98743 | Significance $F$ |
| Residual | 336 | 8.43159 | $0.025093 \mathrm{E}-46$ |  |  |
| Total | 337 | 15.43250 |  |  |  |


|  | Coefficients | Standard Error | $t$ Stat | P-value | Lower 95\% | Upper 95\% |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Intercept | 8.76120 | 0.03160 | 277.25710 | 0 | 8.69904 | 8.82336 |
| X Variable 1 | -0.10942 | 0.00655 | -16.70292 | $5.05853 \mathrm{E}-46$ | -0.12231 | -0.09654 |

The $\mathrm{R}^{2}$ of the regression is equal to $45.365 \%$, and the $t$ statistic of -16.7 (or the F-test of 279) with the pvalue of $5 \times 10^{-46}$ show that the null hypothesis (the slope $\mathrm{B}=0$ ) can be rejected. Therefore, this simple regression is highly significant and confirms the commonly recognized decrease of prices as the distance from Luxembourg City increases (the slope B is negative).

Nevertheless, there is significant variation not explained by the distance alone. Based on the available data, it would be possible to consider the number of bedrooms as an additional explanatory variable $\mathbf{X}_{2}$, as shown in the chart below.

Chart 3 Two-variable model : Distance and number of bedrooms


The correlation between the two variables (distance and number of bedrooms) is only $-3 \%$, which means that variables are highly independent.

The table below shows the ANOVA results of the two-variable model.

Table 3 ANOVA for the two-variable model

| Regression Statistics |  |
| :--- | ---: |
| Multiple R | 0.68300 |
| R Square | 0.46649 |
| Adjusted R Square | 0.46330 |
| Standard Error | 0.15677 |
| Observations | 338 |


| ANOVA |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | df | SS | MS | F | Significance $F$ |  |
| Regression | 2 | 7.19910 | 3.59955 | 146.45826 | 1.97895E-46 |  |
| Residual | 335 | 8.23340 | 0.02458 |  |  |  |
| Total | 337 | 15.43250 |  |  |  |  |
|  |  |  |  |  |  |  |
|  | Coefficients | Standard Error | t Stat | $P$-value | Lower 95\% | Upper 95\% |
| Intercept | 8.83120 | 0.03982 | 221.77900 | 0 | 8.75287 | 8.90953 |
| X Variable 1 | -0.10994 | 0.00649 | -16.95020 | 5.6804E-47 | -0.12270 | -0.09718 |
| X Variable 2 | -0.02708 | 0.00954 | -2.83969 | 0.00479 | -0.04584 | -0.00832 |

As expected, the $\mathrm{R}^{2}$ increased (from $45.37 \%$ to $46.65 \%$ ) with the addition of the second explanatory variable. The p -value of the variable $\mathrm{X}_{2}$ is much smaller than 1 , and the slope coefficient is non-zero with at least $95 \%$ confidence. Therefore, the number of bedrooms variable appears to be significant for the model.

As the data shows a strong correlation between the number of bedrooms and the type of property (apartment or house), with the correlation coefficient of $57 \%$, it is interesting to study the model where the dummy variable $\mathbf{D}_{\mathbf{1}}$ for the type of property ( 1 for house, 0 for apartment) is used instead of number of bedrooms. The ANOVA summary for this model is given in the table below:

Table 4 ANOVA for the model with one quantitative and one dummy variable.

| Regression Statistics |  |
| :--- | ---: |
| Multiple R | 0.67356 |
| R Square | 0.45368 |
| Adjusted R Square | 0.45042 |
| Standard Error | 0.15864 |
| Observations | 338 |


| ANOVA | df |  | SS | MS | F |
| :--- | ---: | ---: | :--- | :--- | ---: |
|  | 2 | 7.00143 | 3.50072 | 139.09757 | Significance $F$ |
| Regression | 335 | 8.43106 | 0.02517 |  |  |
| Residual | 337 | 15.43250 |  |  |  |
| Total |  |  |  |  |  |


|  | Coefficients | Standard Error | $t$ Stat | P-value | Lower 95\% | Upper 95\% |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Intercept | 8.76208 | 0.03223 | 271.86373 | 0 | 8.69868 | 8.82548 |
| X1 | -0.10943 | 0.00656 | -16.67918 | $6.783 \mathrm{E}-46$ | -0.12234 | -0.09653 |
| D1 | -0.00268 | 0.01860 | -0.14415 | 0.88547 | -0.03927 | 0.03391 |

One can see that the addition of the dummy variable does not improve the $\mathrm{R}^{2}$ of the regression and that this variable is in fact insignificant: its p -value is 0.885 , and the confidence interval of the slope value includes zero ( $-0.039 ; 0.034$ ).

The same conclusion about the insignificance of the type of property dummy variable can be drawn from the analysis of the model with both $\mathbf{X}_{2}$ and $\mathbf{D}_{\mathbf{1}}$. Indeed, as seen from the table below, the p-value of $\mathbf{D}_{\mathbf{1}}$ is 0.071 , and the null hypothesis is not rejected based on the $95 \%$ confidence interval, since it includes the value of zero.

Table 5 ANOVA for the model with two quantitative variables and one dummy variable.
ANOVA

|  | $d f$ | SS | $M S$ | $F$ | Significance $F$ |
| :--- | ---: | ---: | :--- | ---: | ---: |
| Regression | 3 | 7.27916 | 2.42639 | 99.39648 | $5.33608 \mathrm{E}-46$ |
| Residual | 334 | 8.15334 | 0.02441 |  |  |
| Total | 337 | 15.43250 |  |  |  |


|  | Coefficients | Standard Error | $t$ Stat | P-value | Lower 95\% | Upper 95\% |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Intercept | 8.84898 | 0.04088 | 216.45563 | 0 | 8.76856 | 8.92939 |
| X1 | -0.11004 | 0.00646 | -17.02251 | $3.17539 \mathrm{E}-47$ | -0.12275 | -0.09732 |
| X2 | -0.03910 | 0.01159 | -3.37297 | 0.00083 | -0.06190 | -0.01630 |
| D1 | 0.04046 | 0.02234 | 1.81098 | 0.07104 | -0.00349 | 0.08441 |

Therefore, the dummy variable $\mathbf{D}_{\mathbf{1}}$ for the type of property is not selected for the model.
Finally, it would be interesting to study the differences between regions. First, a set of dummy variables as shown in the table below can be introduced.

Table 6 Dummy variables for the region

| Region | D2 | D3 | D4 | D5 |
| :--- | :---: | :---: | :---: | :---: |
| Center | 0 | 0 | 0 | 0 |
| North | 1 | 0 | 0 | 0 |
| South | 0 | 1 | 0 | 0 |
| West | 0 | 0 | 1 | 0 |
| East | 0 | 0 | 0 | 1 |

In this model, we are effectively only varying the intercept, while keeping the slope of each regression line (plane) the same.

The following ANOVA summary is obtained for the model with two quantitative variables $\left(\mathbf{X}_{1}, \mathbf{X}_{2}\right)$ and four dummy variables $\left(\mathbf{D}_{2}, \mathbf{D}_{3}, \mathbf{D}_{4}, \mathbf{D}_{5}\right)$ :

Table 7 ANOVA for the model with 2 quantitative variables and 4 dummy variables

| Regression Statistics |  |
| :--- | ---: |
| Multiple R | 0.72397 |
| R Square | 0.52413 |
| Adjusted R Square | 0.51550 |
| Standard Error | 0.14895 |
| Observations | 338 |

ANOVA

|  | $d f$ |  | SS | MS | $F$ |
| :--- | ---: | ---: | :--- | ---: | ---: |
| Significance $F$ |  |  |  |  |  |
| Regression | 6 | 8.08862 | 1.34810 | 60.76107 | $1.63118 \mathrm{E}-50$ |
| Residual | 331 | 7.34388 | 0.02219 |  |  |
| Total | 337 | 15.43250 |  |  |  |


|  | Coefficients | Standard Error | $t$ Stat | P-value | Lower 95\% | Upper 95\% |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Intercept | 8.97864 | 0.04838 | 185.60361 | 0 | 8.88347 | 9.07380 |
| X1 | -0.13968 | 0.01104 | -12.65190 | $3.38585 \mathrm{E}-30$ | -0.16139 | -0.11796 |
| X2 | -0.03542 | 0.00932 | -3.80108 | 0.00017 | -0.05374 | -0.01709 |
| D2 | 0.07514 | 0.03874 | 1.93946 | 0.05330 | -0.00107 | 0.15136 |
| D3 | -0.07528 | 0.02639 | -2.85281 | 0.00461 | -0.12719 | -0.02337 |
| D4 | -0.06487 | 0.03296 | -1.96843 | 0.04985 | -0.12970 | $-4.18455 \mathrm{E}-05$ |
| D5 | 0.07223 | 0.03160 | 2.28604 | 0.02288 | 0.01008 | 0.13438 |

Although one can note the improvement of $\mathrm{R}^{2}$ to $52.41 \%$, the significance of dummy variable $\mathbf{D}_{2}$ can be questioned, given the $p$-value of 0.0533 . Furthermore, this model assumes the constant slope coefficient across all regions and only varies the intercept point. However, as seen below, the analysis of the pricedistance plots by regions suggests that different regions have different slope coefficients.

## Chart 4 Conditional plots by regions




It would therefore seem reasonable to introduce interactions between the distance variable $\left(\mathbf{X}_{1}\right)$ and each of the dummy variables ( $\mathbf{D}_{2}$ through $\mathbf{D}_{\mathbf{5}}$ ). The results of regression of the model with two quantitative variables ( $\mathbf{X}_{1}, \mathbf{X}_{2}$ ), four dummy variables ( $\mathbf{D}_{2}, \mathbf{D}_{3}, \mathbf{D}_{4}, \mathbf{D}_{5}$ ) and four interactions ( $\mathbf{X}_{1} \mathbf{D}_{2}, \mathbf{X}_{1} \mathbf{D}_{3}, \mathbf{X}_{1} \mathbf{D}_{4}, \mathbf{X}_{1} \mathbf{D}_{5}$ ) are given in the following table:

Table 8 ANOVA for the model with 2 quantitative variables, 4 dummy variables and 4 interactions

| Regression Statistics |  |
| :--- | ---: |
| Multiple R | 0.74484 |
| R Square | 0.55478 |
| Adjusted R Square | 0.54117 |
| Standard Error | 0.14495 |
| Observations | 338 |


| ANOVA |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $d f$ | SS | MS | $F$ | Significance $F$ |  |
| Regression | 10 | 8.56168 | 0.85617 | 40.74721 | 1.06362E-51 |  |
| Residual | 327 | 6.87082 | 0.02101 |  |  |  |
| Total | 337 | 15.43250 |  |  |  |  |
|  | Coefficients | Standard Error | t Stat | $P$-value | Lower 95\% | Upper 95\% |
| Intercept | 8.80674 | 0.07582 | 116.16059 | 0.00000 | 8.65760 | 8.95589 |
| X1 | -0.08253 | 0.02152 | -3.83477 | 0.00015 | -0.12488 | -0.04019 |
| X2 | -0.04144 | 0.00916 | -4.52421 | 0.00001 | -0.05946 | -0.02342 |
| D2 | 0.37406 | 0.12280 | 3.04618 | 0.00251 | 0.13249 | 0.61562 |
| D3 | 0.64206 | 0.17843 | 3.59834 | 0.00037 | 0.29104 | 0.99309 |
| D4 | 0.20549 | 0.14217 | 1.44541 | 0.14930 | -0.07419 | 0.48517 |
| D5 | -0.02701 | 0.17824 | -0.15151 | 0.87966 | -0.37765 | 0.32364 |
| X1D2 | -0.07547 | 0.02685 | -2.81073 | 0.00524 | -0.12829 | -0.02265 |
| X1D3 | -0.19037 | 0.04605 | -4.13400 | 0.00005 | -0.28096 | -0.09978 |
| X1D4 | -0.07761 | 0.03676 | -2.11144 | 0.03549 | -0.14993 | -0.00530 |
| X1D5 | 0.00422 | 0.04067 | 0.10381 | 0.91738 | -0.07579 | 0.08423 |

The $\mathrm{R}^{2}$ of this model is $55.48 \%$, and the significance is high $(\mathrm{F}=40.74)$. However, as seen in the plots above, the East data exhibits considerable variation, and has a slope and intercept comparable to those of the Center region. The ANOVA results are in line with these observations, showing high $p$-values and small slope coefficients for $\mathbf{D}_{5}$ and $\mathbf{X}_{1} \mathbf{D}_{5}$. Based on these considerations, the model is refined by combining East with Center, i.e. removing the $\mathbf{D}_{5}$ and $\mathbf{X}_{\mathbf{1}} \mathbf{D}_{\mathbf{5}}$ variables. The regression results of this model are as follows:

Table 9 ANOVA for the model with 2 quantitative variables, 3 dummy variables and $\mathbf{3}$ interactions

| Regression Statistics |  |
| :--- | ---: |
| Multiple R | 0.74477 |
| R Square | 0.55469 |
| Adjusted R Square | 0.54386 |
| Standard Error | 0.14453 |
| Observations | 338 |

ANOVA

|  | $d f$ |  | SS | MS | $F$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Regression | 8 | 8.56023 | 1.07003 | 51.22610 | $2.13679 \mathrm{E}-53$ |
| Residual | 329 | 6.87227 | 0.02089 |  |  |
| Total | 337 | 15.43250 |  |  |  |


|  | Coefficients | Standard Error | t Stat | P-value | Lower 95\% | Upper 95\% |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Intercept | 8.81030 | 0.05811 | 151.61465 | 0.00000 | 8.69599 | 8.92462 |
| X1 | -0.08437 | 0.01330 | -6.34591 | 0.00000 | -0.11052 | -0.05822 |
| X2 | -0.04127 | 0.00911 | -4.53210 | 0.00001 | -0.05919 | -0.02336 |
| D2 | 0.37000 | 0.11190 | 3.30661 | 0.00105 | 0.14987 | 0.59012 |
| D3 | 0.63781 | 0.17068 | 3.73701 | 0.00022 | 0.30206 | 0.97357 |
| D4 | 0.20147 | 0.13279 | 1.51717 | 0.13018 | -0.05976 | 0.46270 |
| X1D2 | -0.07362 | 0.02077 | -3.54449 | 0.00045 | -0.11447 | -0.03276 |
| X1D3 | -0.18845 | 0.04261 | -4.42279 | 0.00001 | -0.27227 | -0.10463 |
| X1D4 | -0.07577 | 0.03254 | -2.32835 | 0.02050 | -0.13979 | -0.01175 |

Note that while $\mathrm{R}^{2}$ has slightly decreased compared with previous model, the adjusted $\mathrm{R}^{2}$ is higher than previous ( $54.39 \%$ vs $54.12 \%$ ) and the $F$ value has improved from 41 to 51 . The only remaining questionable significance is that of $\mathbf{D}_{4}$ and to a lesser extent of $\mathbf{X}_{1} \mathbf{D}_{4}$. The poor p-values for these variables can be explained by a small amount of data in the West region (32 data points). Moreover, the
standard error of all dummy variables is relatively high, since these variables represent the addition to the intercept value, and there is a leverage effect of the slope error on the intercept error.

## Conclusion

The final model selected is determined by the following equation with coefficients from Table 9:
$\mathbf{Y}=8.8103-0.08437 \mathbf{X}_{\mathbf{1}}-0.04127 \mathbf{X}_{\mathbf{2}}+0.37 \mathbf{D}_{\mathbf{2}}+0.63781 \mathbf{D}_{3}+0.20147 \mathbf{D}_{\mathbf{4}}-0.07362 \mathbf{X}_{\mathbf{1}} \mathbf{D}_{\mathbf{2}}-0.18845 \mathbf{X}_{\mathbf{1}} \mathbf{D}_{\mathbf{3}}$ $-0.07577 \mathbf{X}_{1} \mathbf{D}_{\mathbf{4}}$,
where Y is the natural logarithm of the price in Euros per square meter, and $\mathrm{X}_{1}$ is the square root of the distance from Luxembourg City in km .

Substituting the corresponding values of dummy variables for each region, the following equations are obtained:

Center / East: $\quad \mathbf{Y}=8.81030-0.08437 \mathbf{X}_{1}-0.04127 \mathbf{X}_{2}$
North : $\quad \mathbf{Y}=9.18030-0.15799 \mathbf{X}_{\mathbf{1}}-0.04127 \mathbf{X}_{\mathbf{2}}$
South : $\quad \mathbf{Y}=9.44811-0.27282 \mathbf{X}_{\mathbf{1}}-0.04127 \mathbf{X}_{\mathbf{2}}$
West : $\quad \mathbf{Y}=9.01177-0.16014 \mathbf{X}_{\mathbf{1}}-0.04127 \mathbf{X}_{\mathbf{2}}$
Note that intercept and slope coefficients for $\mathbf{X}_{1}$ compare well with those of the single variable regressions in Chart 4.

Appendix - Original data, excl. Luxembourg City, sorted by distance

| Price, EUR | Area, m2 | Bedrooms | Type | Distance, km | Region | Price, EUR | $\begin{aligned} & \text { Area, } \\ & \mathrm{m} 2 \end{aligned}$ | Bedrooms | Type | $\begin{gathered} \text { Distance, } \\ \text { km } \end{gathered}$ | Region | Price, EUR | $\begin{gathered} \text { Area, } \\ \mathrm{m} 2 \end{gathered}$ | Bedrooms | Type | $\begin{gathered} \text { Distance, } \\ \text { km } \end{gathered}$ | Region |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 524,713 | 128 | 2 | A | 4 | Center | 610,470 | 139 |  | H | 16 | South | 536,952 | 130 | 4 | H | 26 | North |
| 592,430 | 157 | 3 | A | 4 | Center | 251,339 | 49 | 1 | A | 16 | South | 449,813 | 146 | 4 | H | 26 | North |
| 588,608 | 168 | 4 | A | 4 | Center | 387,569 | 81 | 2 | A | 16 | South | 433,803 | 94 | 2 | A | 26 | North |
| 495,000 | 105 | 2 | A | 4 | Center | 488,680 | 104 | 3 | A | 16 | South | 388,500 | 87 | 2 | A | 26 | North |
| 612,900 | 92 | 1 | A | 4 | Center | 476,890 | 114 | 3 | A | 16 | South | 571,842 | 113 | 3 | A | 26 | North |
| 457,950 | 82 | 2 | A | 4 | Center | 248,146 | 54 |  | A | 16 | South | 408,952 | 87 |  | A | 26 | North |
| 575,900 | 104 | 3 | A | 4 | Center | 390,303 | 85 | 2 | A | 16 | South | 618,281 | 125 | 3 | A | 26 | North |
| 971,006 | 173 | 4 | H | 5 | Center | 558,400 | 116 | 3 | A | 16 | South | 466,000 | 139 | 3 | H | 26 | North |
| 1,017,500 | 180 | 4 | H | 5 | Center | 187,831 | 45 |  | A | 16 | South | 286,501 | 61 | 1 | A | 26 | North |
| 272,500 | 53 | 1 | A | 6 | Center | 316,450 | 83 | 2 | A | 16 | South | 738,295 | 144 |  | H | 27 | East |
| 377,500 | 75 | 2 | A | 6 | Center | 385,511 | 96 | 3 | A | 16 | South | 584,096 | 133 | 3 | H | 27 | East |
| 487,500 | 94 |  | A | 6 | Center | 598,905 | 143 | 4 | A | 16 | South | 490,119 | 90 | 1 | A | 27 | East |
| 551,240 | 121 | 2 | A | 6 | Center | 284,604 | 59 | 1 | A | 16 | South | 309,365 | 76 | 2 | A | 27 | East |
| 338,236 | 63 | 1 | A | 6 | Center | 434,293 | 79 | 2 | A | 16 | South | 385,450 | 107 |  | A | 27 | East |
| 611,961 | 128 | 3 | A | 6 | Center | 214,900 | 46 | 1 | A | 16 | South | 312,922 | 106 | 2 | A | 27 | East |
| 515,873 | 123 | 2 | A | 6 | Center | 349,250 | 79 | 2 | A | 16 | South | 191,400 | 55 | 1 | A | 27 | East |
| 520,966 | 124 | 3 | A | 6 | Center | 444,100 | 105 | 3 | A | 16 | South | 149,800 | 44 | 1 | A | 27 | East |
| 393,592 | 89 | 2 | A | 6 | Center | 364,068 | 101 | 2 | A | 16 | South | 160,389 | 44 | 2 | A | 27 | East |
| 481,454 | 111 | 3 | A | 6 | Center | 349,808 | 97 | 3 | A | 16 | South | 401,175 | 112 | 3 | A | 27 | East |
| 312,642 | 61 | 1 | A | 6 | Center | 459,146 | 134 | 3 | A | 16 | South | 235,939 | 59 | 1 | A | 27 | West |
| 565,642 | 117 | 3 | A | 6 | Center | 377,282 | 127 | 2 | A | 16 | South | 424,230 | 126 | 3 | A | 27 | West |
| 853,450 | 173 | 3 | H | 7 | Center | 365,608 | 85 | 2 | A | 16 | South | 425,000 | 129 | 3 | A | 28 | North |
| 461,900 | 90 | 2 | A | 7 | South | 745,000 | 234 | 4 | H | 16 | East | 431,430 | 165 | 4 | H | 28 | North |
| 539,500 | 114 | 3 | A | 7 | South | 598,988 | 170 | 3 | H | 16 | East | 673,116 | 174 |  | H | 28 | North |
| 730,100 | 137 | 4 | A | 7 | South | 673,042 | 186 | 4 | H | 16 | East | 330,637 | 90 | 2 | A | 28 | North |
| 476,000 | 93 | 2 | A | 8 | West | 530,000 | 140 | 3 | H | 17 | East | 406,509 | 118 |  | A | 28 | North |
| 791,343 | 225 | 3 | H | 8 | West | 471,203 | 160 | 3 | H | 17 | West | 590,000 | 130 | 3 | H | 28 | North |
| 795,375 | 174 | 3 | H | 8 | West | 380,100 | 88 | 2 | A | 17 | West | 495,000 | 162 | 4 | H | 29 | North |
| 529,400 | 105 | 2 | H | 8 | West | 485,400 | 114 | 3 | A | 17 | West | 516,600 | 123 | 3 | A | 29 | North |
| 556,000 | 124 | 3 | A | 8 | West | 846,042 | 221 | 3 | H | 17 | West | 404,754 | 100 | 1 | A | 29 | North |
| 437,694 | 83 | 2 | A | 9 | Center | 211,520 | 53 | 1 | A | 17 | West | 434,861 | 107 | 2 | A | 29 | North |
| 476,650 | 96 | 3 | A | 9 | Center | 263,600 | 68 | 2 | A | 17 | West | 427,308 | 115 | 3 | A | 29 | North |
| 968,347 | 214 | 3 | H | 9 | Center | 461,805 | 99 | 2 | A | 18 | East | 539,320 | 143 | 3 | A | 29 | North |
| 777,300 | 140 | 3 | H | 9 | Center | 411,427 | 85 | 2 | A | 18 | East | 558,000 | 134 | 3 | H | 29 | East |
| 1,268,673 | 198 | 3 | H | 10 | Center | 1,300,000 | 367 | 3 | A | 18 | East | 596,150 | 141 | 4 | H | 29 | East |
| 979,076 | 198 | 4 | н | 10 | Center | 532,265 | 146 | 1 | A | 18 | East | 603,881 | 176 | 3 | H | 29 | East |
| 709,420 | 150 | 3 | H | 10 | Center | 461,805 | 98 | 2 | A | 18 | East | 320,000 | 91 | 2 | A | 30 | North |
| 562,000 | 114 | 3 | н | 10 | Center | 512,550 | 99 | 3 | A | 18 | East | 827,760 | 142 | 4 | н | 30 | East |
| 272,899 | 63 | 2 | A | 10 | Center | 279,913 | 76 | 2 | A | 18 | West | 255,971 | 58 | 1 | A | 30 | East |
| 427,721 | 99 | 3 | A | 10 | Center | 316,838 | 83 | 2 | A | 18 | West | 336,253 | 106 | 2 | A | 32 | North |
| 887,581 | 180 | 4 | H | 10 | Center | 376,800 | 98 | 3 | A | 18 | West | 414,087 | 141 | 3 | H | 32 | West |
| 423,183 | 92 | 2 | A | 10 | South | 375,000 | 98 | 2 | A | 19 | Center | 631,083 | 165 | 3 | H | 32 | West |
| 609, 196 | 131 | 3 | H | 10 | South | 268,000 | 84 | 2 | A | 19 | South | 554,940 | 150 | 3 | H | 33 | North |
| 840,000 | 157 | 4 | H | 10 | West | 445,270 | 106 | 3 | A | 19 | South | 305,750 | 75 | 2 | A | 33 | North |
| 268,021 | 51 | 1 | A | 10 | West | 306,000 | 95 | 2 | A | 19 | South | 395,000 | 144 | 3 | A | 34 | East |
| 628,025 | 162 | 3 | н | 10 | West | 200,000 | 66 | 1 | A | 19 | South | 672,500 | 175 | 4 | H | 34 | East |
| 1,052,940 | 198 | 3 | H | 10 | West | 215,000 | 65 | 2 | A | 19 | South | 229,188 | 58 | 1 | A | 36 | North |
| 440,166 | 101 | 2 | A | 11 | South | 329,000 | 125 | 3 | A | 19 | South | 319,122 | 91 | 2 | A | 36 | North |
| 611,950 | 151 | 3 | A | 11 | South | 231,156 | 54 | 1 | A | 19 | South | 218,000 | 77 | 1 | A | 36 | North |
| 590,700 | 149 | 3 | A | 11 | South | 256,796 | 56 | 2 | A | 19 | South | 220,000 | 77 | 2 | A | 36 | North |
| 655,495 | 160 |  | н | 11 | West | 175,000 | 56 | 1 | A | 19 | South | 238,000 | 86 | 1 | A | 36 | North |
| 750,000 | 184 | 4 | н | 11 | West | 230,000 | 73 | 2 | A | 19 | South | 302,000 | 101 | 2 | A | 36 | North |
| 469,000 | 108 | 3 | A | 11 | West | 245,000 | 77 | 2 | A | 19 | South | 327,000 | 113 | 3 | A | 36 | North |
| 334,806 | 65 | 1 | A | 12 | Center | 320,200 | 100 | 3 | A | 19 | South | 416,000 | 120 | 3 | H | 36 | North |
| 407,892 | 77 | 2 | A | 12 | Center | 164,164 | 53 | 1 | A | 19 | South | 351,147 | 145 | 4 | H | 36 | North |
| 492,352 | 102 | 3 | A | 12 | Center | 215,000 | 71 | 2 | A | 19 | South | 598,740 | 180 | 3 | H | 36 | North |
| 439,500 | 91 | 2 | A | 12 | Center | 243,984 | 58 | 1 | A | 19 | South | 206,353 | 71 |  | A | 36 | North |
| 1,035,325 | 231 | 3 | H | 12 | Center | 291,225 | 86 | 2 | A | 19 | South | 250,806 | 79 | 3 | A | 36 | North |
| 310,000 | 73 | 2 | A | 14 | South | 244,800 | 61 | 1 | A | 19 | South | 460,000 | 147 | 3 | H | 36 | North |
| 413,080 | 83 | 2 | A | 14 | South | 198,090 | 63 | 1 | A | 19 | South | 272,000 | 71 | 1 | A | 36 | North |
| 474,369 | 104 | 3 | A | 14 | South | 257,108 | 83 | 2 | A | 19 | South | 262,000 | 78 | 2 | A | 36 | North |
| 296,160 | 70 | 1 | A | 14 | South | 259,000 | 51 | 1 | A | 19 | East | 391,000 | 124 | 3 | A | 36 | North |
| 356,952 | 73 | 2 | A | 14 | South | 350,400 | 78 | 2 | A | 19 | East | 453,000 | 130 | 4 | H | 38 | North |
| 419,346 | 104 | 3 | A | 14 | South | 452,300 | 109 | 3 | A | 19 | East | 407,344 | 163 |  | H | 38 | North |
| 227,622 | 56 | 1 | A | 14 | South | 787,139 | 176 | 3 | H | 20 | Center | 463,283 | 135 | 4 | H | 38 | North |
| 328,769 | 78 | 2 | A | 14 | South | 816,956 | 220 | 4 | H | 20 | Center | 273,201 | 76 | 1 | A | 42 | North |
| 256,392 | 71 | 2 | A | 14 | South | 276,000 | 82 | 1 | A | 20 | South | 287,636 | 85 | 2 | A | 42 | North |
| 372,000 | 85 | 2 | A | 15 | Center | 162,180 | 51 | 1 | A | 20 | South | 457,079 | 126 | 3 | A | 42 | North |
| 732,402 | 145 | 3 | H | 15 | Center | 333,020 | 103 | 3 | A | 20 | South | 270,799 | 90 | 2 | A | 42 | North |
| 488,000 | 114 | 3 | H | 15 | Center | 380,000 | 126 | 3 | A | 20 | South | 198,609 | 62 | 1 | A | 42 | North |
| 593,000 | 130 | 3 | H | 15 | Center | 289,000 | 82 | 2 | A | 20 | South | 284,147 | 93 | 2 | A | 42 | North |
| 743,910 | 173 | 3 | н | 15 | Center | 435,000 | 144 | 3 | H | 20 | South | 235,000 | 82 | 1 | A | 42 | North |
| 539,000 | 140 | 4 | H | 15 | Center | 364,902 | 105 | 3 | A | 20 | South | 245,000 | 82 | 2 | A | 42 | North |
| 428,227 | 97 | 2 | A | 15 | Center | 682,645 | 151 | 3 | H | 20 | East | 232,154 | 89 | 2 | A | 42 | North |
| 465,199 | 109 | 2 | A | 15 | Center | 550,960 | 147 | 3 | H | 21 | Center | 180,713 | 70 | 2 | A | 42 | North |
| 460,587 | 106 | 2 | A | 15 | Center | 334,890 | 107 | 3 | A | 21 | East | 268,268 | 103 | 3 | A | 42 | North |
| 395,850 | 93 | 3 | A | 15 | Center | 477,324 | 190 | 3 | H | 21 | West | 248,000 | 67 | 2 | A | 43 | North |
| 501,000 | 150 | 3 | н | 15 | Center | 295,336 | 83 | 1 | A | 23 | Center | 316,000 | 101 | 3 | A | 43 | North |
| 448,000 | 110 | 3 | н | 15 | Center | 368,282 | 92 | 2 | A | 23 | Center | 440,000 | 135 |  | H | 43 | North |
| 595,000 | 114 | 3 | н | 15 | Center | 381,156 | 109 | 3 | A | 23 | Center | 204,618 | 87 | 2 | A | 44 | North |
| 598,000 | 140 | 3 | н | 15 | Center | 495,000 | 127 | 3 | A | 23 | Center | 343,300 | 136 | 3 | A | 44 | North |
| 394,000 | 100 | 3 | H | 15 | Center | 272,991 | 67 | 1 | A | 23 | North | 310,000 | 129 | 3 | H | 44 | North |
| 458,587 | 103 | 2 | A | 15 | Center | 326,745 | 85 | 2 | A | 23 | North | 482,476 | 140 | 3 | H | 45 | North |
| 732,402 | 155 | 3 | H | 15 | Center | 658,005 | 187 | 4 | H | 24 | East | 497,248 | 170 | 3 | H | 45 | North |
| 698,000 | 175 | 4 | H | 15 | Center | 349,000 | 79 | 2 | A | 24 | East | 442,510 | 174 | 4 | H | 45 | North |
| 539,000 | 140 | 4 | H | 15 | Center | 418,000 | 127 | 3 | A | 24 | East | 680,500 | 220 | 4 | H | 49 | North |
| 405,000 | 91 | 2 | H | 15 | South | 660,000 | 130 | 3 | H | 24 | East | 216,000 | 80 | 2 | A | 50 | North |
| 310,000 | 80 | 2 | A | 15 | South | 768,500 | 175 | 4 | H | 24 | East | 246,000 | 94 | 3 | A | 50 | North |
| 638,000 | 125 | 3 | H | 15 | East | 410,000 | 97 | 2 | A | 24 | East | 272,079 | 75 | 1 | A | 50 | North |
| 311,261 | 49 | 1 | A | 15 | East | 610,440 | 227 | 4 | H | 24 | East | 272,710 | 89 | 2 | A | 50 | North |
| 431,562 | 77 | 2 | A | 15 | East | 485,000 | 160 | 3 | H | 24 | East | 346,736 | 116 | 3 | A | 50 | North |
| 549,293 | 103 | 3 | A | 15 | East | 316,800 | 86 | 2 | A | 24 | East | 446,050 | 166 | 3 | H | 50 | North |
| 227,130 | 54 | 1 | н | 15 | East | 411,600 | 115 | 3 | A | 24 | East | 298,400 | 84 | 2 | A | 50 | North |
| 375,970 | 94 | 2 | H | 15 | East | 478,000 | 120 | 3 | H | 24 | West | 445,300 | 142 | 4 | A | 50 | North |
| 425,240 | 108 | 3 | H | 15 | East | 565,742 | 152 | 3 | H | 24 | West | 268,100 | 84 | 2 | A | 50 | North |
| 469,738 | 111 | 3 | A | 15 | East | 628,000 | 120 | 3 | H | 25 | North | 408,100 | 140 |  | A | 50 | North |
| 350,000 | 101 | 3 | A | 15 | East | 667,000 | 155 | 4 | H | 25 | North | 197,060 | 67 | 2 | A | 51 | North |
| 706,017 | 180 | 3 | H | 15 | West | 363,200 | 84 | 2 | A | 25 | North | 275,000 | 90 | 2 | H | 51 | North |
| 570,735 | 157 | 3 | H | 15 | West | 339,600 | 77 | 2 | A | 25 | North | 354,716 | 130 | 3 | H | 51 | North |
| 470,100 | 102 | 3 | H | 16 | Center | 459,000 | 111 | 3 | A | 25 | North | 560,143 | 160 | 3 | H | 51 | North |
| 743,094 | 171 | 4 | H | 16 | Center | 480,775 | 230 | 4 | H | 25 | West | 342,504 | 123 | 4 | A | 51 | North |
| 685,000 | 192 | 4 | H | 16 | South | 402,420 | 160 | 3 | H | 25 | West | 129,049 | 57 | 1 | A | 57 | North |
| 498,000 | 137 | 3 | A | 16 | South | 359,500 | 77 | 1 | A | 26 | North | 208,816 | 97 | 2 | A | 57 | North |
| 237,736 | 57 |  | A | 16 | South | 436,100 | 97 | 2 | A | 26 | North | 214,699 | 99 | 3 | A | 57 | North |
| 393,460 | 93 | 2 | A | 16 | South | 228,000 | 54 | 2 | A | 26 | North | 434,344 | 176 | 4 | H | 57 | North |
| 293,790 | 67 | 1 | A | 16 | South | 685,557 | 187 | 4 | A | 26 | North | 253,104 | 77 | 1 | A | 57 | North |
| 303,240 | 73 | 2 | A | 16 | South | 296,900 | 79 | 2 | A | 26 | North | 274,623 | 82 | 2 | A | 57 | North |
| 349,808 | 114 |  | A | 16 | South | 380,668 | 108 | 3 | A | 26 | North | 111,240 | 37 | 1 | A | 59 | North |
| 320,290 | 91 | 1 | A | 16 | South | 245,000 | 69 | 2 | A | 26 | North | 221,370 | 74 | 2 | A | 59 | North |
| 319,076 | 79 | 2 | A | 16 | South | 498,600 | 130 | 3 | H | 26 | North | 326,490 | 109 | 3 | A | 59 | North |
| 488,593 | 150 | 3 | A | 16 | South | 472,242 | 118 | 3 | H | 26 | North | 180,259 | 59 | 1 | A | 59 | North |
| 285,840 493,530 | 77 124 | 2 | A | 16 16 | South South | 513,820 524,191 | 143 136 | 3 | H | 26 26 | North North | 232,880 | 79 | 2 | A | 59 | North |


[^0]:    ${ }^{1}$ The government lists the distances between all pairs of communities in the country and uses these to determine tax deductions for the cost of commuting between place of residence and work. The distances are calculated along a straight line (bird's flight) between the centers of communities.

