Forecast of the tide level in Venice

Giacomo De Leva

1 Introduction

Beside its immense historical significance and beauty, the city of Venice in Italy has been also known for a phenomenon called in Italian "acqua alta" (high water), namely high tides. Tides in Venice are caused by two distinct components: the astronomical tide, correlated to the motion of the Moon and the Sun, and the meteorological surge due to atmospheric conditions. Under normal conditions, the meteorological contribution is small and the level observed approximately coincides with the astronomical tide. In case of particular weather conditions, typically low pressure and strong sirocco winds, the meteorological contribution becomes important: if it occurs simultaneously with a maximum of astronomical tide, it can produce the phenomenon of high water. Conventionally, in Venice, a sea level higher than 80 cm above the local datum of Punta Salute, is called "acqua alta": at this level a lot of problems about transport and pedestrian use of roads in lowest sides of the town (St. Mark's Square) arise. When the tide exceeds 100 cm, about 5% of the city is affected by flooding. If the tides reaches an height of 110 cm, about 12% of the city is flooded. Finally, a sea level higher than 140 cm causes 59% of the city flooded.

Figure 1: High water in Venice, on November 11, 2012. (Reuters/Manuel Silvestri)

2 Data

In the Appendix we report measures of the tide that had been taken every hour from 2012-11-29 00:00 until 2012-12-31 23:00 at the local datum of Punta Salute (see Figure 2). These data are available at the web page

http://www.comune.venezia.it/flex/cm/pages/ServeBLOB.php/L/IT/IDPagina/25419

of the Tide Centre of the city of Venice. A plot of these data is given in Figure 3. Even if the plot does not show any strong evidence of a trend, the series has a seasonality and does not seem to be stationary. We shall use the data taken from 2012-11-29 00:00 until 2012-12-30 23:00 to build a model to forecast tide level values on 2012-12-31.

Figure 2: Punta Salute.

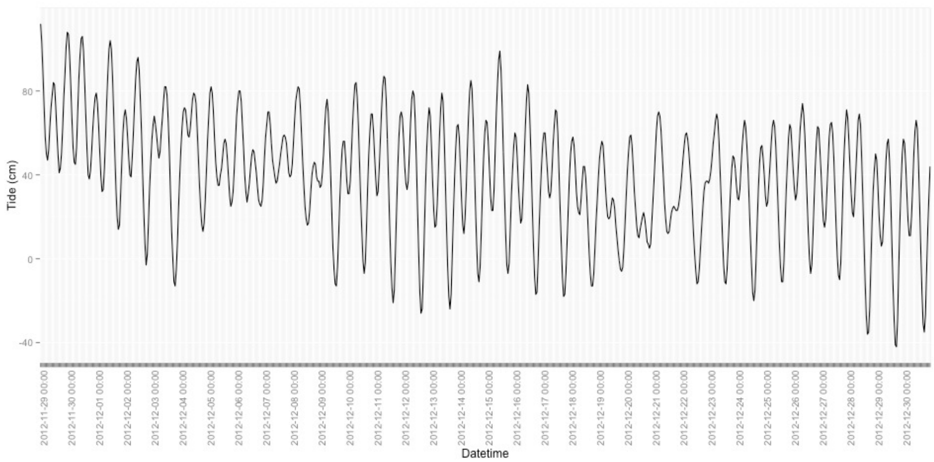

Figure 3: Tide values at Punta Salute from 2012-11-29 until 2012-12-30

3 Model specification

Figure 4 shows the sample of autocorrelation for the series. We notice significant correlation at lag 1, 2, 3, 24, 25, 26 and so on. However, there are also other lags at which there is substantial correlation. Figure 5 shows the time series plot of tide levels after we take the seasonal difference (lag=24).

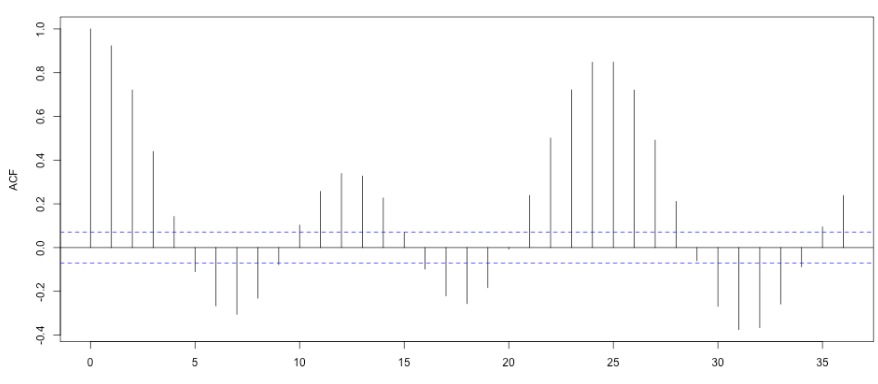

Figure 4: Sample autocorrelation function.

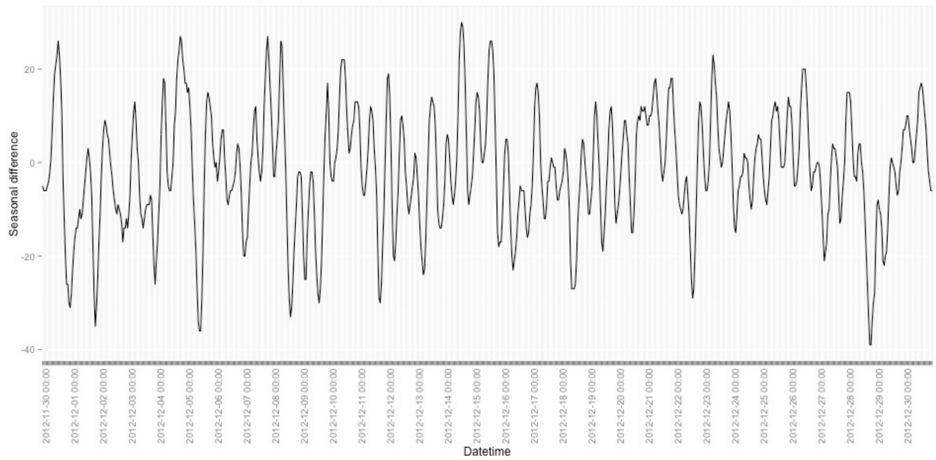

Figure 5: Time series after taking the seasonal difference.

Most of the seasonality has disappeared, but, as we may notice in Figure 6, there is still correlation at many lags.

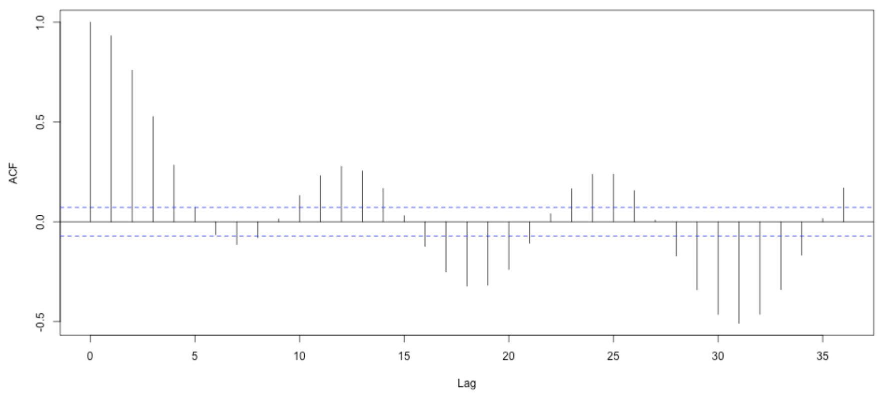

Figure 6: Sample autocorrelation function after taking the seasonal difference.

Hence, we decide to take from our original time series the first and the seasonal difference (Figure 7).

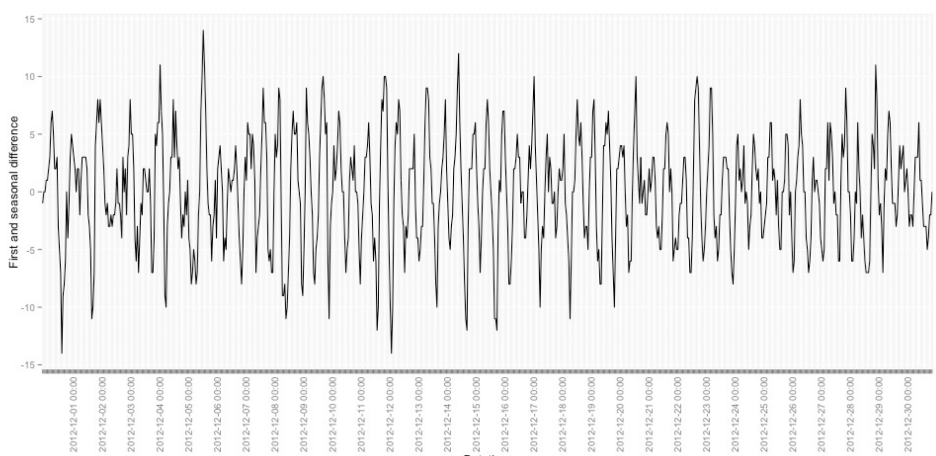

Figure 7: Time series after taking the first and the seasonal difference.

The sample autocorrelation function after taking the first and the seasonal difference shows that the transformed time series could be modeled by an ARMA model. More precisely, if we set $Z_t = \nabla_{24}\nabla Y_t$, where Y_t is the original time series, we shall attempt to model Z_t as an ARMA(2,4) model.

$$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \theta_3 e_{t-3} + \theta_4 e_{t-4}.$$

4 Model Fitting and Diagnostic

We proceed to estimate the parameters of the ARMA(2,4) model by means of the maximum likehood method.

```
> tide_model <- arima(Z,order=c(2,0,4))
```

We find out the following estimates: $\phi_1 = 1.7494$, $\phi_2 = -0.9992$, $\theta_1 = -1.054$, $\theta_2 = 0.075$, $\theta_3 = 0.212$, $\theta_4 = 0.248$. To check these estimations we plot the standardized residuals.

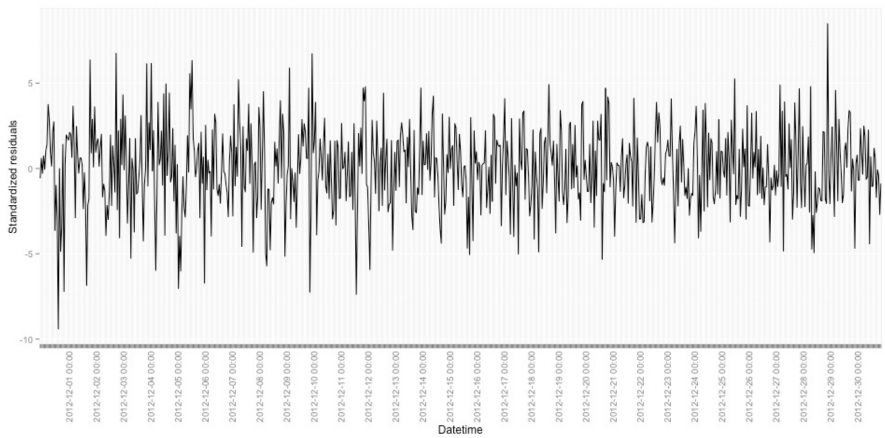

Figure 8: Residuals from the ARMA(2,4) model.

The plot does not show any major irregularities. Moreover, a look to the sample autocorrelation function (Figure 9) of the residuals shows that the residuals are almost uncorrelated. This is also confirmed by the Box-Ljung test.

```
> Box.test(tide_model$residuals,type="Ljung-Box")

Box-Ljung test

data:  tide_model$residuals
X-squared = 0.1646, df = 1, p-value = 0.685
```

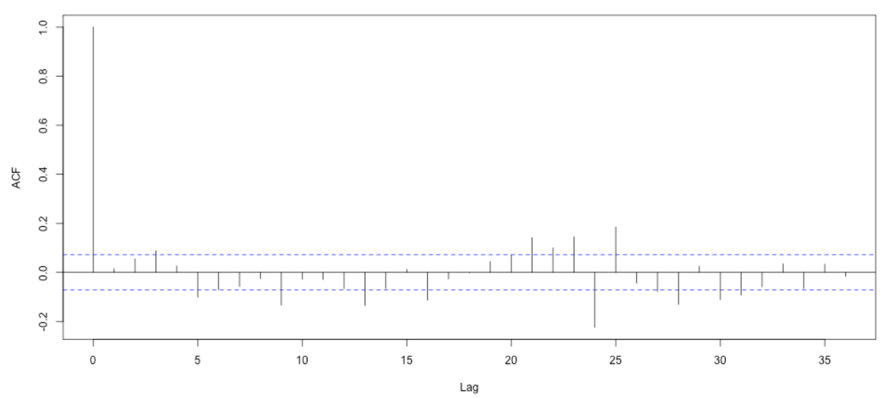

Figure 9: Sample autocorrelation function of the residuals.

Finally, we check the normality of the residuals in Figure 10. We notice that the distribution of the residuals is slightly heavy-tailed.

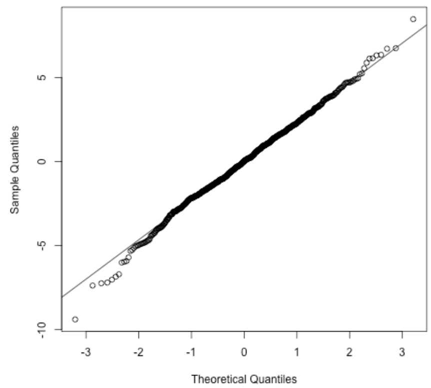

Figure 10: QQ-normal plot of the residuals.

5 Forecast

Since Z_t is an ARMA(2,4) model we can easily forecast its values for the day 2012-12-31.

```
> predict(tide_model,n.ahead=24)
$pred
Time Series:
Start = 744
End = 767
Frequency = 1
 [1]  2.62040895  3.85268067  3.26944699  1.65448108 -0.37052977 -2.29940037
 [7] -3.65038285 -4.08648590 -3.49953393 -2.03699857 -0.06494752
[12]  1.92358042  3.43184129  4.08347463  3.71641731  2.42320808  0.52765944
[18] -1.49622264 -3.14276962 -4.00099172 -3.85716347 -2.74804756
[23] -0.95150298  1.08312445

$se
Time Series:
Start = 744
End = 767
Frequency = 1
 [1] 2.411309 2.937038 3.020407 3.021168 3.021189 3.021569
 [7] 3.023066 3.025389 3.027475 3.028482 3.028580 3.028787
 [13] 3.030016 3.032220 3.034437 3.035694 3.035920 3.036004
 [19] 3.036967 3.039004 3.041300 3.042800 3.043197 3.043212
```

Finally, we can calculate the forecast values of our original time series by means of the following recursive formula:

$$Y_t = Y_{t-1} + Y_{t-24} - Y_{t-25} + Z_t.$$

The following plot shows the forecast tide values with the observed values.

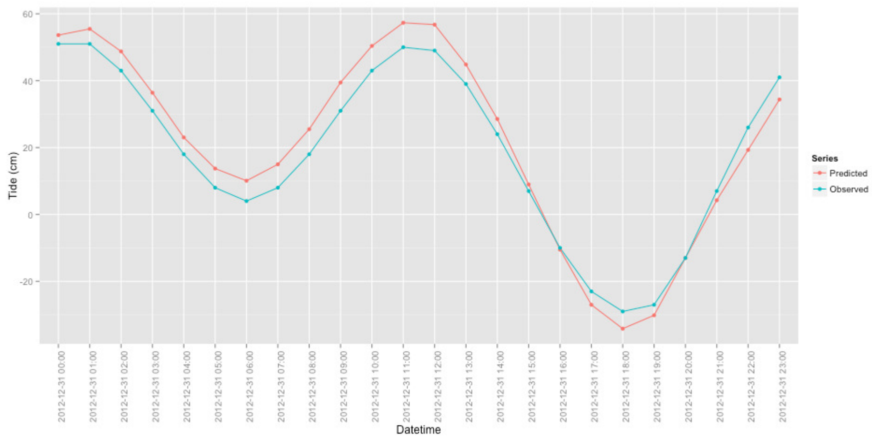

Figure 11: Predicted vs. observed values.

Appendix

Below we report the values in centimetres of the tide at Punta della Salute. Measures had been taken on the stroke of every hour.
2012-11-29: 112, 103, 89, 73, 58, 50, 47, 52, 63, 72, 78, 84, 83, 73, 61, 50, 41, 43, 50, 62, 77, 88, 101, 108. **2012-11-30**: 107, 97, 83, 67, 53, 46, 45, 53, 70, 86, 97, 105, 106, 99, 84, 68, 52, 40, 38, 42, 51, 62, 71, 77. **2012-12-01**: 79, 74, 64, 51, 39, 32, 33, 43, 58, 75, 89, 100, 104, 100, 87, 69, 50, 33, 20, 14, 16, 31, 46, 60. **2012-12-02**: 68, 71, 67, 58, 48, 40, 39, 48, 60, 74, 86, 94, 96, 90, 76, 60, 40, 22, 7, -3, 2, 17, 34, 46. **2012-12-03**: 57, 64, 68, 64, 59, 53, 48, 51, 60, 67, 75, 82, 82, 78, 66, 51, 31, 13, 0, -11, -13, -5, 8, 25. **2012-12-04**: 40, 53, 63, 70, 72, 71, 65, 59, 58, 62, 69, 76, 79, 78, 74, 62, 49, 35, 24, 16, 13, 17, 28, 42. **2012-12-05**: 57, 68, 79, 82, 79, 70, 57, 46, 39, 35, 35, 40, 43, 49, 55, 57, 55, 48, 39, 30, 25, 27, 32, 43. **2012-12-06**: 56, 68, 75, 80, 80, 75, 64, 53, 40, 32, 27, 31, 36, 43, 49, 52, 51, 46, 41, 34, 28, 26, 25, 28. **2012-12-07**: 36, 48, 58, 64, 70, 70, 64, 55, 47, 43, 39, 36, 37, 41, 45, 50, 54, 58, 59, 58, 55, 48, 41, 39. **2012-12-08**: 40, 45, 55, 66, 75, 79, 82, 81, 72, 59, 46, 35, 25, 19, 16, 17, 23, 32, 40, 44, 46, 45, 39, 37. **2012-12-09**: 37, 34, 35, 41, 50, 63, 72, 76, 70, 57, 42, 24, 6, -5, -12, -13, -4, 11, 28, 42, 52, 56, 56, 48. **2012-12-10**: 37, 31, 31, 37, 50, 64, 75, 83, 84, 77, 64, 46, 28, 13, -1, -7, -2, 14, 34, 50, 61, 69, 69, 61. **2012-12-11**: 49, 38, 30, 32, 43, 57, 71, 82, 87, 86, 76, 57, 37, 16, -2, -14, -21, -15, 4, 24, 43, 58, 68, 70. **2012-12-12**: 67, 57, 44, 36, 33, 37, 50, 65, 76, 80, 78, 66, 47, 24, 3, -16, -26, -24, -7, 15, 36, 53, 65, 72. **2012-12-13**: 68, 54, 37, 23, 15, 16, 26, 42, 59, 72, 79, 75, 59, 38, 16, -4, -18, -24, -17, 2, 22, 39, 53, 63. **2012-12-14**: 64, 58, 43, 28, 16, 12, 19, 33, 52, 68, 80, 85, 81, 66, 46, 25, 7, -7, -11, -4, 13, 32, 48, 60. **2012-12-15**: 66, 65, 56, 43, 30, 23, 23, 33, 52, 70, 84, 95, 99, 90, 72, 51, 31, 12, -3, -7, -2, 14, 31, 43. **2012-12-16**: 54, 60, 58, 48, 35, 25, 17, 19, 32, 47, 63, 76, 83, 79, 64, 46, 25, 6, -9, -17, -16, -2, 16, 32. **2012-12-17**: 45, 55, 60, 60, 51, 42, 32, 29, 32, 42, 55, 64, 71, 70, 60, 42, 24, 7, -9, -18, -17, -7, 8, 24. **2012-12-18**: 39, 50, 56, 58, 54, 44, 32, 25, 22, 21, 28, 37, 44, 44, 39, 29, 16, 3, -7, -13, -13, -7, 5, 18. **2012-12-19**: 28, 39, 48, 53, 56, 54, 45, 35, 26, 20, 19, 20, 25, 29, 28, 24, 16, 10, 4, -1, -5, -6, -4, 5. **2012-12-20**: 17, 30, 42, 51, 58, 59, 54, 44, 32, 24, 16, 11, 10, 14, 17, 20, 22, 19, 14, 8, 7, 5, 7, 17.
2012-12-21: 27, 38, 50, 61, 68, 70, 68, 61, 50, 39, 27, 19, 13, 12, 13, 18, 22, 24, 25, 24, 23, 23, 25, 29.
2012-12-22: 34, 41, 48, 54, 59, 60, 57, 51, 43, 35, 24, 12, 2, -6, -12, -11, -5, 5, 15, 24, 32, 36, 37, 37.
2012-12-23: 36, 38, 42, 48, 55, 60, 66, 69, 66, 56, 41, 26, 10, -3, -11, -12, -5, 8, 21, 33, 43, 49, 48, 43.
2012-12-24: 35, 29, 28, 33, 44, 54, 61, 66, 63, 54, 43, 27, 11, -3, -16, -20, -15, 0, 18, 34, 46, 53, 54, 48.
2012-12-25: 40, 30, 25, 27, 36, 45, 55, 63, 66, 63, 53, 39, 24, 8, -4, -11, -11, -1, 17, 33, 46, 58, 64, 62.
2012-12-26: 52, 42, 33, 28, 31, 40, 51, 62, 69, 74, 69, 59, 44, 28, 12, 0, -7, -3, 11, 28, 44, 56, 63, 62.
2012-12-27: 52, 41, 28, 18, 15, 19, 32, 45, 58, 64, 65, 60, 48, 31, 15, 1, -8, -10, -2, 16, 37, 52, 63, 71.
2012-12-28: 67, 56, 43, 31, 22, 20, 29, 42, 54, 66, 69, 64, 48, 29, 9, -11, -27, -36, -35, -23, -2, 18, 33, 43.
2012-12-29: 50, 47, 35, 21, 11, 6, 8, 20, 34, 47, 55, 57, 47, 30, 9, -12, -29, -41, -42, -29, -4, 18, 36, 50.
2012-12-30: 57, 55, 45, 31, 18, 11, 11, 20, 34, 50, 61, 66, 62, 46, 26, 4, -16, -31, -35, -27, -6, 14, 30, 44.
2012-12-31: 51, 51, 43, 31, 18, 8, 4, 8, 18, 31, 43, 50, 49, 39, 24, 7, -10, -23, -29, -27, -13, 7, 26, 41.