Time series Mod 15 AR(1) forecasts.wpd

(The attached PDF file has better formatting.)

** Exercise 15.1: AR(1) forecasts

An AR(1) model with *T* observations has $\mu = 2$, $\phi = 0.5$, $y_T = 1.6$, and $\sigma^2_t = 2$,

A. What is the *one* period ahead forecast (for Period 51)?
B. What is the *two* periods ahead forecast (for Period 52)?
C. What is the variance of the one period ahead forecast?
D. What is the variance of the two periods ahead forecast?

Solution 15.1: We solve this problem two equivalent ways; use whichever way is clearer to you.

Method 1: An AR(1) process is $Y_t = \theta_0 + \phi \times Y_{t-1} + \epsilon_t$, where $\theta_0 = \mu \times (1 - \phi)$.

$$\theta_0 = \mu \times (1 - \phi) = 2 \times (1 - 0.5) = 1.$$

Part A: The one period ahead forecast is $1 + 0.5 \times 1.6 = 1.800$.

Part B: The two periods ahead forecast is $1 + 0.5 \times 1.8 = 1.900$.

Part C: The one period ahead forecast is a fixed amount $\theta_0 + \phi \times Y_T$ plus the stochastic term ϵ_t. The fixed amount has a variance of zero, so the variance of the one period ahead forecast is $\sigma^2_t = 2$.

Part D: The two periods ahead forecast is $\theta_0 + \phi \times Y_{T+1} + \epsilon_{T+2} = \theta_0 + \phi \times (\theta_0 + \phi \times Y_T + \epsilon_{T+1}) + \epsilon_{T+2} =$

$$[\theta_0 + \phi \times \theta_0 + \phi^2 \times Y_T] + [\phi \times \epsilon_{T+1} + \epsilon_{T+2}]$$

The terms in the first set of brackets are fixed, with no variance. The variance of the stochastic terms in the second set of brackets is $\phi^2 \times \sigma^2_t + \sigma^2_t = (1 + 0.5^2) \times 2 = 2.50$.

Jacob: How do we get the relation $\theta_0 = \mu \times (1 - \phi)$?

Rachel: The mean μ does not depend on *t*: $\mu = E(Y_t) = E(Y_{t-1})$. Take the expectation of the AR(1) equation:

$$E(Y_t) = E(\theta_0 + \phi \times Y_{t-1} + \epsilon_t) \Rrightarrow$$
$$\mu = \theta_0 + \phi \times \mu + 0 \Rrightarrow$$
$$\mu \times (1 - \phi) = \theta_0.$$

Method 2: An AR(1) process is $Y_t - \mu = \phi \times (Y_{t-1} - \mu) + \epsilon_t \Rrightarrow Y_t = \mu + \phi \times (Y_{t-1} - \mu) + \epsilon_t$

Part A: The one period ahead forecast is $2 + 0.5 \times (1.6 - 2) = 1.800$.

Part B: The two periods ahead forecast is $2 + 0.5 \times (1.8 - 2) = 1.900$.

Parts C and D are the same as for Method 1.

** Exercise 15.2: AR(1) forecasts: deriving the mean and the ϕ parameter

We can ask the same exercise in reverse, deriving the mean and ϕ from the first two forecasts.

An AR(1) process of T observations has $y_T = 1.600$. The one period ahead forecast is 1.800, and the two periods ahead forecast is 1.900.

A. What is the ϕ parameter of this AR(1) process?
B. What is the mean μ of this AR(1) process?

Solution 15.2: The arithmetic is slightly simpler with the AR(1) process written as $Y_t = \theta_0 + \phi \times Y_{t-1} + \epsilon_t$, though we could use the version with μ instead of θ_0 as well.

We write two linear equations:

- $1.8 = \theta_0 + \phi \times 1.6 + 0$
- $1.9 = \theta_0 + \phi \times 1.8 + 0$

Part A: Subtracting the first from the second give $(1.9 - 1.8) = \phi \times (1.8 - 1.6) \Rightarrow \phi = 0.5$

Part B: Using the first equation and the value of ϕ gives $1.8 = \theta_0 + 0.5 \times 1.6 \Rightarrow \theta_0 = 1.8 - 0.8 = 1$.

We derive μ as $\theta_0 / (1 - \phi) = 1 / (1 - 0.5) = 2$.