

TS module 15 MA(2) forecasting practice problems

(The attached PDF file has better formatting.)

** Exercise 15.1: MA(2) parameters and forecasts

An MA(2) process has a mean μ of 100 and the expected and actual values below in periods T-4 through T.

<i>Period</i>	<i>Expected Value</i>	<i>Actual Value</i>
T-4	100	100
T-3	100	100
T-2	100	101
T-1	100.2	101
T	99.8	99

- A. What is the value of θ_1 ?
- B. What is the value of θ_2 ?
- C. What is the forecast for period T+1?
- D. What is the forecast for period T+2?

Solution 15.1: Add residuals to the table and use periods T-1 and T to determine θ_1 and θ_2 .

<i>Period</i>	<i>Expected Value</i>	<i>Actual Value</i>	<i>Residual</i>
T-4	100	100	0
T-3	100	100	0
T-2	100	101	1
T-1	100.2	101	0.8
T	99.8	99	-0.8

Part A: From the expected value in Period T-1 we solve for θ_1 :

$$100 - \theta_1 \times 1 - \theta_2 \times 0 = 100.2 \Rightarrow \theta_1 = -0.2$$

Part B: From the expected value in Period T and the value of θ_1 we solve for θ_2 :

$$\begin{aligned} 100 - \theta_1 \times 0.8 - \theta_2 \times 1 &= 99.8 = 100 - 0.2 \Rightarrow \\ -0.2 \times -0.8 - \theta_2 \times 1 &= -0.2 \Rightarrow \\ 0.16 - \theta_2 &= -0.2 \Rightarrow \\ \theta_2 &= +0.36 \end{aligned}$$

Part C: The forecast for period T+1 = $100 - (-0.8) \times -0.2 - 0.8 \times 0.36 = 99.552$

Part D: The fitted value in Period T+1 is the best estimate, so the expected residual is zero. The forecast for Period T+2 is

$$100 - \theta_2 \times -0.8 = 100 - 0.36 \times (-0.8) = 100.288$$

** Exercise 15.2: MA(2) process

An MA(2) process has a mean μ of 100 and the expected and actual values below in periods T-4 through T.

<i>Period</i>	<i>Expected Value</i>	<i>Actual Value</i>
T-4	100	100
T-3	100	100
T-2	100	101
T-1	100.2	101
T	99.8	99

- A. What is the forecast for period T+1?
 B. What is the forecast for period T+2?

Part A: Add residuals to the table and use periods T-1 and T to determine ϕ_1 and ϕ_2 .

<i>Period</i>	<i>Expected Value</i>	<i>Actual Value</i>	<i>Residual</i>
T-4	100	100	0
T-3	100	100	0
T-2	100	101	1
T-1	100.2	101	0.8
T	99.8	99	-0.8

- Period T-1: $100 - \theta_1 \times 1 - \theta_2 \times 0 = 100.2 \Rightarrow \theta_1 = -0.2$
- Period T: $100 - \theta_1 \times 0.8 - \theta_2 \times 1 = 99.8 = 100 - 0.2 \Rightarrow$

$$-0.2 \times -0.8 - \theta_2 \times 1 = -0.2 \Rightarrow$$

$$0.16 - \theta_2 = -0.2 \Rightarrow$$

$$\theta_2 = +0.36$$

The forecast for period T+1 = $100 + (-0.8) \times -0.2 - 0.8 \times 0.36 = 99.872$

Part B: For Period T+2, we assume the residual in Period T+1 is zero. The forecast for Period T+2 is

$$100 - \theta_2 \times -0.8 = 100 - 0.36 \times (-0.8) = 100.288$$