Module 17 forecast confidence intervals practice problems

(The attached PDF file has better formatting.)

[Know the relations for confidence intervals of autoregressive and moving average processes. Final exam problems are multiple choice.]

** Exercise 17.1: Width of Confidence Interval for AR(1) process

We use an AR(1) process to model a time series of N observations, y_t, t = 1, 2, …, N, for which we estimate μ, ϕ, and σ^2_ε.

- W = the width of the P% confidence interval for the *k* periods ahead forecast, $k \geq 2$.
- *Note:* For a 95% confidence interval (a significance level of 5%), P = 95%, not 5%.

What is the effect on W as

A. The estimate of μ increases?
B. The estimate of ϕ increases?
C. The estimate of σ^2_ε increases?
D. *k* increases?
E. *P* increases?
F. N increases?

Part A: The mean μ is a location parameter. It does not affect the shape of the time series or the width of the confidence interval for forecasts.

Part B: W depends on ϕ^2, so it depends on the *absolute value* of ϕ.

The variance of the forecast is $\sigma^2_\varepsilon \times (1 - \phi^{2k}) / (1 - \phi^2) = \sigma^2_\varepsilon \times (1 + \phi^2 + \phi^4 + \ldots + \phi^{2k-2})$. As $|\phi|$ increases, the variance increases, so the standard deviation increases and the width of the confidence interval increases.

Part C: The variance of the forecast is proportional to σ^2_ε, so as σ^2_ε increases, the variance of the forecast increases, the standard deviation of the forecast increases, and the confidence interval becomes wider.

Part D: The variance of the forecast increases with *k*, reaching a limit of $\sigma^2_\varepsilon / (1 - \phi^2)$ as $k \to \infty$.

Part E: The *z*-value (or *t*-value if the parameters are estimated) increases with P. The confidence interval is the mean of the forecast ± (*t*-value) × the standard deviation of the forecast, so as the *t*-value increases, the confidence interval becomes wider.

Part F: As N increases, the *t*-value has more degrees of freedom, so it is smaller, and the confidence interval becomes narrower.

Jacob: The textbook does not mention the relation of the *t*-value to N.

Rachel: This topic is covered in the regression analysis course; it will not be tested on the time series final exam. The time series textbook does not want to get into discussions of the degrees of freedom.

Take heed: For a moving average process, the width of the confidence interval reaches a maximum for lags > q (the moving average order). If the ARIMA process has an autoregressive part, the width of the confidence interval widens as *k* increases. The increase is asymptotic to γ_0.

** Exercise 17.2: Width of Confidence Interval for MA(1) process

We use an MA(1) process to model a time series of N observations, y_t, t = 1, 2, …, N, for which we estimate μ, θ, and σ^2_ε.

- W = the width of the P% confidence interval for the *k* periods ahead forecast, $k \geq 2$.
- Note: For a 95% confidence interval (a significance level of 5%), W = 95%, not 5%.

What is the effect on W as

A. The estimate of μ increases?
B. The estimate of ϕ increases?
C. The estimate of σ^2_ε increases?
D. *k* increases?
E. *P* increases?
F. N increases?

Part A: The mean μ is a location parameter. It does not affect the shape of the time series or the width of the confidence interval for forecasts.

Part B: W depends on θ^2, so it depends on the absolute value of θ.

The variance of the forecast is $\sigma^2_\varepsilon \times (1 + \theta^2)$. As $|\phi|$ increases, the variance increases, so the standard deviation increases and the width of the confidence interval increases.

Part C: The variance of the forecast is proportional to σ^2_ε, so as σ^2_ε increases, the variance of the forecast increases, the standard deviation of the forecast increases, and the confidence interval becomes wider.

Part D: The variance of the forecast does not change with *k*, as long as $k \geq 2$.

Part E: The *z*-value (or *t*-value if the parameters are estimated) increases with P. The confidence interval is the mean of the forecast ± (*t*-value) × the standard deviation of the forecast, so as the *t*-value increases, the confidence interval becomes wider.

Part F: As N increases, the *t*-value has more degrees of freedom, so it is smaller, and the confidence interval becomes narrower.

Take heed: For a moving average process, the width of the confidence interval reaches a maximum for lags > q (the moving average order). If the ARIMA process has an autoregressive part, the width of the confidence interval widens as *k* increases. The increase is asymptotic to γ_0.

** Exercise 17.3: Width of Confidence interval

A time series of 250 observations is modeled by an AR(1) process with $\mu = 0$.

A. If the parameters ϕ and σ^2_ε are known (not estimated from the data), what is the 95% confidence interval?
B. What is the width of this confidence interval?
C. If the parameters ϕ and σ^2_ε are estimated from the data, is the 95% confidence interval wider or narrower?

Part A: The forecast is $y_T \times \phi$. The variance of this forecast is $\sigma^2_\varepsilon \times (1 + \phi^2)$. The standard deviation of this forecast is $\sigma_\varepsilon \times (1 + \phi^2)^{0.5}$. The z-value for a two-sided 95% confidence interval is 1.960. The 95% confidence interval is $y_T \times \phi \pm 1.960 \times \sigma_\varepsilon \times (1 + \phi^2)^{0.5}$.

[Note: Final exam problems give the z-values for significance levels.]

Part B: The width of the confidence interval is $2 \times 1.960 \times \sigma_\varepsilon \times (1 + \phi^2)^{0.5}$. The width depends on the standard deviation of the forecast, not on the expected value.

Part C: If ϕ is estimated from the data, the forecast is uncertain. It may be higher or lower, so the confidence interval is wider. If σ^2_ε is estimated from the data, the variance of the forecast is uncertain. It may be higher or lower, so the confidence interval is wider.