TS Module 12: MA(1) parameter estimation (Yule-Walker equations) practice problems

(The attached PDF file has better formatting.)

** Exercise 12.1: MA(1) model and Yule-Walker equations

If the autocorrelation of lag 1 for an MA(1) process is ρ_1,

A. What is the Yule-Walker initial estimate for θ?
B. What is the relation of the two roots of the Yule-Walker solution?

Part A: An MA(1) model has $\rho_1 = -\theta / (1 + \theta^2)$.

We write this as a quadratic equation in θ, where ρ_1 is a parameter:

$$\rho_1 \theta^2 + \theta + \rho_1 = 0 \Rightarrow \theta = [-1 \pm (1 - 4 \rho_1^2)^{0.5}] / 2 \rho_1$$

$$\theta = \frac{-1 \pm \sqrt{1 - 4\rho_1^2}}{2\rho_1}$$

Part B: The expression $-\theta / (1 + \theta^2)$ has the same value $1/\theta$ as for θ. Taking the reciprocal of θ gives

$$-(1/\theta) / (1 + 1/\theta^2).$$

Multiplying the numerator and denominator of this fraction by θ^2 gives the original expression.

Illustration: We fit an MA(1) process with $-1 \leq \theta \leq 1$ to a time series. The sample autocorrelation of lag 1 is -0.400.

The Yule-Walker initial estimate for θ is $[-1 \pm (1 - 4 \times 0.16)^{0.5}] / (2 \times 0.4) = (-1 \pm 0.6) / 0.8 = \pm0.5$.