Time series Mod 17 moving average forecasts

(The attached PDF file has better formatting.)

** Exercise 17.1: MA(1) Forecast

A time series of 60 interest rate observations y_t, t = 1, 2, ..., 60, is represented by an MA(1) model, with μ = 8.55%, θ = –0.50, and σ^2_ε = 0.04.

A. What is the variance of the *one period* ahead forecast?
B. What is the variance of the *two periods* ahead forecast?
C. What is the variance of the *three periods* ahead forecast?

Part A: The only random variable in the one period ahead forecast is the standard error, so the variance is σ^2_ε = 0.04.

Part B: The two periods ahead forecast is $\mu + \varepsilon_{t+2} + 50\% \times \varepsilon_{t+1}$.

The residuals are independent, so the variance of a sum of residuals is the sum of their variances.

If the variance of random variable Y is σ^2, the variance of $\alpha \times Y$ is $\alpha^2 \times \sigma^2$.

The variances of both residuals are σ^2, so the combined variance is $\sigma^2 \times (1 + 0.5^2) = 0.04 \times (1.25) = 0.05$.

Part C: The three periods ahead forecast is $\mu + \varepsilon_{t+3} + 50\% \times \varepsilon_{t+2}$.

For more than one period ahead, the variance of the forecast error is

$(1 + \theta^2) \times \sigma^2_\varepsilon = (1 + 0.25) \times 0.04 = 0.050$.