TS module 16 ARMA forecasts practice problems

(The attached PDF file has better formatting.)

** Exercise 16.1: ARMA (1,1) Process

A time series of 45 observations y_t, t = 1, 2, …, 45, is represented by an ARMA(1,1) model, with $\mu = 22$, $y_{45} = 21$, $\epsilon_{45} = 1$, $\phi = -0.8$, $\theta = 0.5$

A. What is θ_0 (the constant term in the ARMA process)?
B. What is the one period ahead forecast (for Period 46)?
C. What is the expected residual in Period 46?
D. What is the two periods ahead forecast (for Period 47)?

Solution 16.1: We solve this exercise two equivalent ways. Use the method that is clearer for you.

Part A: From the mean and ϕ parameter, we solve for θ_0, the constant term in the time series:

$$\mu = \theta_0 / (1 - \phi) = 22 \Rightarrow$$

$$\phi = -0.8, \text{ so } \theta_0 = 22 \times (1 - (-0.8)) = 39.600$$

Part B: The ARMA(1,1) forecast is $\theta_0 + \phi \times Y_t - \theta \times \epsilon_t$.

The residual in Period 45 is 1, so the one period ahead forecast is $39.6 - 0.8 \times 21 - 0.5 \times 1 = 22.300$

Part C: The forecast is the best estimate, so the expected residual in all future periods is zero.

Part D: The two periods ahead forecast is $39.6 - 0.8 \times 22.3 - 0.5 \times 0 = 21.760$

Alternatively, we subtract the mean from the observed value, generate the forecasts, and add the mean to the forecasts.

Part A: Subtracting the mean gives $y_{45} = 21 - 22 = -1$.

Part B: The one period ahead forecast (with a mean of zero) is $\phi \times Y_t - \theta \times \epsilon_t$.

The residual in Period 45 remains 1, so the one period ahead forecast is $-0.8 \times -1 - 0.5 \times 1 = 0.300$.

Adding the mean of 22 gives a forecast of 22.300.

Part C: The expected residual in future periods remains zero.

Part D: The two periods ahead forecast (with a mean of zero) is $\phi \times Y_{t+1} - \theta \times \epsilon_{t+1}$.

The time series value residual in Period 46 is 0.300 and the residual in Period 46 is zero, two periods ahead forecast is $-0.8 \times 0.300 - 0.5 \times 0 = -0.240$. Adding the mean of 22 gives $22 - 0.240 = 21.760$.

** Exercise 16.2: ARMA (1,1) Process

A time series of 45 observations y_t, t = 1, 2, …, 45, is represented by an ARMA(1,1) model, with μ = 22, y_{45} = 21, and ϵ_{45} = 1.

- The one period ahead forecast is 22.300.
- The two periods ahead forecast is 21.760.

A. What is ϕ for this time series?
B. What is θ for this time series?

Solution 16.2: This exercise uses the same values as the previous exercise, but it derives the time series parameters from the forecasts. Final exam problems may ask either derivation.

We show both solution methods. From the mean and ϕ parameter, we solve for θ_0:

$$\mu = \theta_0 / (1 - \phi) = 22 \Rightarrow$$
$$\phi = -0.8, \text{ so } \theta_0 = 22 \times (1 - (-0.8)) = 39.600$$

We have two linear equations for the two forecasts:

- One period ahead forecast: $39.6 + \phi \times 21 - \theta \times 1 = 22.300$
- Two periods ahead forecast: $39.6 + \phi \times 22.3 - \theta \times 0 = 21.760$

Part A: From the two periods ahead forecast, we obtain

$$39.6 + \phi \times 22.3 = 21.760 \Rightarrow$$
$$\phi = (21.760 - 39.6) / 22.3 = -0.800$$

Part B: From the one period ahead forecast, we obtain

$$39.6 + -0.8 \times 21 - \theta \times 1 = 22.300 \Rightarrow$$
$$\theta = (22.300 - 39.6 + 0.8 \times 21) / 1 = -0.500$$

Alternatively, we subtract the mean from the observed values and forecasts and derive the parameters. Subtracting the mean gives y_{45} = 21 – 22 = –1 and forecasts of 0.300 and –0.240.

We have two linear equations for the two forecasts:

- One period ahead forecast: $\phi \times -1 - \theta \times 1 = 0.300$
- Two periods ahead forecast: $\phi \times 0.300 - \theta \times 0 = -0.240$

Part A: From the two periods ahead forecast, we obtain

$$\phi \times 0.300 = -0.240 \Rightarrow$$
$$\phi = -0.240 / 0.300 = -0.800$$

Part B: From the one period ahead forecast, we obtain

$$-0.8 \times -1 - \theta \times 1 = 0.300 \Rightarrow$$
$$-\theta = (0.300 + 0.8 \times -1) / 1 = -0.500 \Rightarrow \theta = 0.500$$