TS Module 15 autoregressive forecasts: practice problems

(The attached PDF file has better formatting.)

[Note: You may want to complete all the modules on forecasting before solving the practice problems. The logic is similar for Modules 15, 16, and 17, and the problems may be easier after module 17.]

** Exercise 15.1: AR(2) process

An AR(2) process has a mean μ of 100 and the expected and actual values below in periods T-4 through T.

Period	*Expected Value*	*Actual Value*
T-4	100	100
T-3	100	100
T-2	100	101
T-1	100.2	101
T	99.8	99

A. What is the ϕ_1 parameter for this AR(2) process?
B. What is the ϕ_2 parameter for this AR(2) process?
C. What is the forecast for period T+1?
D. What is the forecast for period T+2?

Part A: Use the fitted value from period T-1 to determine ϕ_1.

$$100 + \phi_1 \times (101 - 100) + \phi_2 \times (100 - 100) = 100.2 \Rightarrow$$
$$\phi_1 = 0.2$$

Part B: Use the fitted value from period T to determine ϕ_2.

$$100 + \phi_1 \times (101 - 100) + \phi_2 \times (101 - 100) = 99.8 = 100 - 0.2 \Rightarrow$$
$$0.2 \times 1.0 + \phi_2 \times 1 = -0.2 \Rightarrow$$
$$0.2 + \phi_2 = -0.2 \Rightarrow$$
$$\phi_2 = -0.4$$

Jacob: Do we always determine ϕ_1 from Period T-1 and ϕ_2 from Period T?

Rachel: In this exercise, the actual value equals the mean in Period T-3. In general, we solve for ϕ_1 and ϕ_2 by regression analysis from all the past values.

Part C: The forecast for period T+1 = 100 + (99 – 100) × 0.2 + (101 – 100) × –0.4 = 100 – 0.6 = 99.4

Part D: The fitted value is the best estimate, so the expected residual in Period T+1 = 0 and the actual value is the expected value. The forecast for period T+2 = 100 + (99.4 – 100) × 0.2 + (99 – 100) × –0.4 = 100.280.

Note: Cryer and Chan generally convert the ARMA process to a mean of zero, which simplifies the arithmetic. The table below subtracts 100 from each expected value and actual value.

Period	*Expected Value*	*Actual Value*
T-4	0	0
T-3	0	0
T-2	0	1
T-1	0.2	1
T	-0.2	-1

The format with a mean of zero is simpler, but remember to add the mean to the forecasts for your solution.

** Exercise 15.2: AR(2) Forecast for interest rates

An actuary estimates the Period T interest rate (y_t) by a weighted average of

- the long-term average interest rate (μ), with a weight of 20%
- the interest rate in Period T-2 (y_{t-2}), with a weight of 30%
- the interest rate in Period T-1(y_{t-1}), with a weight of 50%

Illustration: if the long-term average interest rate is 10.5%, the 20X3 interest rate is 8%, and the 20X4 interest rate is 11%, the forecast for 20X5 is

$$20\% \times 10.5\% + 30\% \times 8.0\% + 50\% \times 11.0\% = 10.00\%.$$

- Interest rates are 9% in 20X5 and 8% in 20X6, and the actuary predicts a rate of 9% for 20X7.
- The actual interest rate in 20X7 is 9%.

A. What ARIMA model is the actuary using?
B. What is the long-term average interest rate?
C. What is the constant parameter θ_0 of the ARIMA process?
D. What is the forecast interest rate for 20X8?
E. What is the forecast interest rate for 20X9?

Part A: The actuary is using an AR(2) model.

$Y_t = 20\% \times \mu + 30\% \times Y_{t-2} + 50\% \times Y_{t-1} + \epsilon_t \Rrightarrow$

$(Y_t - \mu) = 20\% \times \mu - 20\% \times \mu + 30\% \times Y_{t-2} - 30\% \times \mu + 50\% \times Y_{t-1} - 50\% \times \mu + \epsilon_t \Rrightarrow$

$(Y_t - \mu) = 30\% \times (Y_{t-2} - \mu) + 50\% \times (Y_{t-1} - \mu) + \epsilon_t$

Part B: Solve for the long-term average interest rate μ by the first equation above.

$$20\% \times \mu + 30\% \times 9\% + 50\% \times 8\% = 9\%$$
$$\Rrightarrow \mu = (9\% - 30\% \times 9\% - 50\% \times 8\%) / 20\% = 11.50\%$$

Part C: Solve for θ_0 as $\mu \times (1 - \phi_1 - \phi_2) = 11.50\% \times (1 - 80\%) = 2.30\%$.

Part D: The forecast for 20X8 is:

$$20\% \times 11.5\% + 30\% \times 8\% + 50\% \times 9\% = 9.20\%$$

Part E: The forecast for 20X9 is:

$$20\% \times 11.5\% + 30\% \times 9\% + 50\% \times 9.2\% = 9.60\%$$

The forecasts move gradually to the long-term average interest rate of 11.5%.