Time series Mod15: MA(1) forecasts practice problems

(The attached PDF file has better formatting.)

** Exercise 15.1: MA(1) Process

An MA(1) model of 100 observations has $\mu = 50$, $\theta = 0.5$, $\epsilon_{99} = 6$, $y_{100} = 57$, and $\sigma^2_\varepsilon = 16$.

A. What was the estimate for Period 100, based on the observations through period 99?
B. What is the one period ahead forecast (for Period 101)?
C. What is the variance of the one period ahead forecast?
D. What is the two periods ahead forecast (for Period 102)?
E. What is the variance of the two periods ahead forecast?

Part A: The estimate for period *t* is $\mu - \theta \times \epsilon_{t-1}$, so the estimate for period 100 = 50 – 0.5 × 6 = 47.

Part B: The estimate for period 100 was 47 and the actual value was 57, so the residual in period 100 = 10.

The estimate for period 101 is 50 – 0.5 × 10 = 45.

Part C: The value in period *t* is $\mu + \epsilon_t - \theta \times \epsilon_{t-1}$. μ and θ are parameters, not random variables. In period *T+1*, the value of ϵ_t is known, so its variance is zero.

The variance of Y_{T+1} (the one period ahead forecast) = variance $(\mu + \epsilon_{T+1} - \theta \times \epsilon_T) = 0 + \sigma^2_t + \theta^2 \times 0 = \sigma^2_t = 16$.

Part D: The fitted value is the best estimate, so the expected residual in Period 101 is zero, and the forecast for Period 102 is $\mu = 50$.

Part E: In period *T+2*, the values of ϵ_{t+1} and ϵ_{t+2} are both unknown, so their variances are σ^2_t. The residuals in different periods are independent, so the variance of the sum of the sum of the variances. The variance of the two periods ahead forecast = variance $(\mu + \epsilon_{T+2} - \theta \times \epsilon_{T+1}) = 0 + \sigma^2_t + \theta^2 \times \sigma^2_t = (1 + \theta^2) \times \sigma^2_\varepsilon = (1 + 0.5^2) \times 16 = 20$.

** Exercise 15.2: MA(1) Process

An MA(1) process is $y_t - \mu = \varepsilon_t - 0.5\ \varepsilon_{t-1}$, with $\mu = 50$, $\epsilon_{T-1} = 6$, $y_T = 57$, and $\sigma^2_t = 16$.

A. What was the fitted value for Period *T*?
B. What is the residual for Period *T*?
C. What is the one period ahead forecast?
D. What is the variance of the one period ahead forecast?
E. What is the 95% confidence interval for the one period ahead forecast, assuming a z-value of 1.96 standard deviations?
F. What is the two periods ahead forecast?
G. What is the variance of the two periods ahead forecast?
H. What is the 95% confidence interval for the two periods ahead forecast?

Part A: The estimate for period *t* is $\mu - \theta \times \epsilon_{t-1}$, so the fitted value for period T is $\hat{y}_T = 50 - 0.5 \times 6 = 47$.

Part B: The residual for Period *T* is 57 – 47 = 10.

Part C: The one period ahead forecast (Period T+1) is 50 – 0.5 × 10 = 45.

Part D: The value in period *t* is $\mu + \epsilon_t - \theta \times \epsilon_{t-1}$.

μ and θ are parameters, not random variables. In period *T+1*, the value of ϵ_t is known, so its variance is zero.

The variance of Y_{T+1} (the one period ahead forecast) = variance $(\mu + \epsilon_{T+1} - \theta \times \epsilon_T) = 0 + \sigma^2_t + \theta^2 \times 0 = \sigma^2_t = 16$.

Part E: The standard deviation of the forecast is $16^{0.5} = 4$, so the 95% confidence interval is

$$(45 - 1.96 \times 4,\ 45 + 1.96 \times 4) = (37.16,\ 52.84).$$

Jacob: Do we always use 1.96 standard deviations for the 95% confidence interval?

Rachel: We generally do not know σ^2_t. We estimate its value from the observed values and we use *t*-values instead of *z*-values for the 95% confidence interval. The regression analysis course deals with *t*-values and confidence intervals; the time series course gives you the figures (usually a *z*-value).

Part F: The fitted value is the best estimate, so the expected residual in Period 101 is zero, and the forecast for Period *T+2* is $\mu = 50$.

Part G: In period *T+2*, the values of ϵ_{t+1} and ϵ_{t+2} are both unknown, so their variances are σ^2_t. The residuals in different periods are independent, so the variance of the sum of the sum of the variances. The variance of the two periods ahead forecast = variance $(\mu + \epsilon_{T+2} - \theta \times \epsilon_{T+1}) = 0 + \sigma^2_t + \theta^2 \times \sigma^2_t = (1 + \theta^2) \times \sigma^2_\varepsilon = (1 + 0.5^2) \times 16 = 20$.

Part H: The standard deviation of the forecast is $20^{0.5} = 4.472$, so the 95% confidence interval is

$$(50 - 1.96 \times 4.472,\ 50 + 1.96 \times 4.472) = (41.23,\ 58.77).$$