## Module 10: Advanced multiple regression

(The attached PDF file has better formatting.)

Homework assignment: Two correlated independent variables

Do this homework assignment after module 12, which gives the equations for the standard errors of the least squares estimators.

We regress the Y values on the  $X_1$  and  $X_2$  values in the table below.

| X <sub>1</sub> | X <sub>2</sub> | Y      | X <sub>1</sub> | X <sub>2</sub> | Y      |
|----------------|----------------|--------|----------------|----------------|--------|
| 1              | 1              | 1.016  | 6              | 8              | -1.076 |
| 2              | 6              | -3.429 | 7              | 4              | 3.461  |
| 3              | 2              | 0.049  | 8              | 9              | -2.525 |
| 4              | 7              | -3.099 | 9              | 5              | 4.195  |
| 5              | 3              | 0.359  | 10             | 10             | -0.746 |

A. What is the correlation of  $X_1$  and  $X_2$ ?

- B. What is the least squares estimator of  $\alpha$ ?
- C. What is the least squares estimator of  $\beta_1$ , the coefficient of  $X_1$ ?
- D. What is the least squares estimator of  $\beta_2$ , the coefficient of  $X_2$ ?
- E. What is the standard error of the least squares estimator of  $\beta_1$ , the coefficient of  $X_1$ ?
- F. What is the standard error of the least squares estimator of  $\beta_2$ , the coefficient of  $X_2$ ?

Show the formulas and the computations. You can check your work with Excel or other statistical software.

*Jacob:* When using the equation on page 106 and page 107, what is  $R^2$ ? Is  $R^2$  the value from the Excel regression add-in using these two explanatory variables?

*Rachel:* No, the  $R^2$  in the equation on page 106 and page 107 is the correlation of the two explanatory variables, or the  $R^2$  from a regression on one on the other. If one have more than two explanatory variables, it is the  $R^2$  from a regression of the explanatory variable under consideration on all the other explanatory variables.

*Illustration:* In this homework assignment, the  $R^2$  from the regression on the response variable on the two explanatory variables is 0.8712. The correlation of the two explanatory variables is 0.6364. Use 0.6364 in the equation, not 0.8712.