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Time Series
Fall 2012

Analysis of Inflation (Part 2)

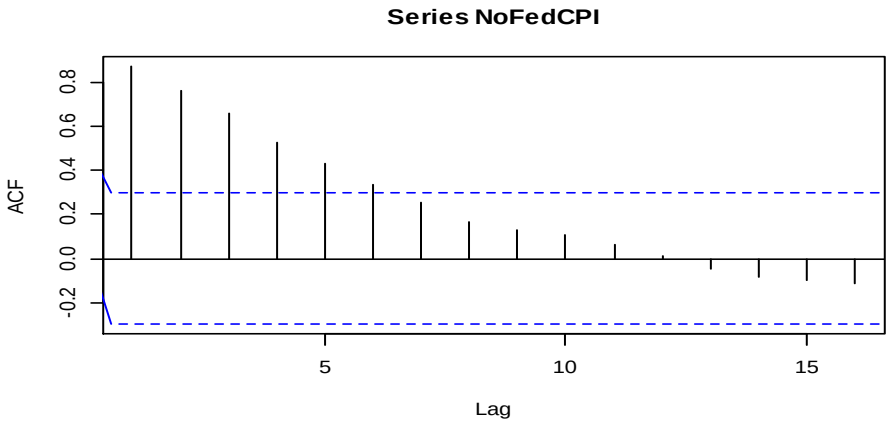
I wanted to look at the value of money under a few different scenarios. In particular there are two things that are discussed regarding the US government and the US Dollar: whether there should be a federal reserve bank (The Fed), and whether we should be on a gold standard. To check the value of these, I want to test the predictability of inflation before and after The Fed was established, and before and after the US went off the Gold standard.

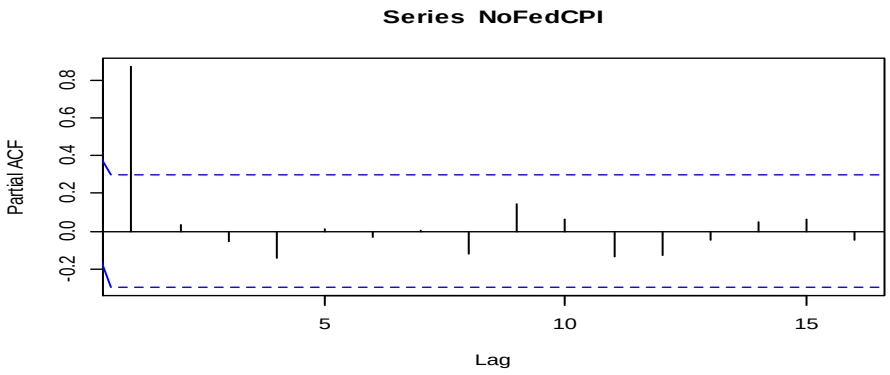
When considering inflation, it is natural to go to a time series model,. The data I used is pulled from several sources (noted below in "Sources") and includes data between 1870 and 2013. There is interpolation between missing data (e.g. Treasury gold was only available every 5 years until 1950, and population data is from the census every 10 years). I selected 1870 as the start which would hopefully avoid the impact of both the CA Gold Rush and the Civil War.

The pre-Fed data then runs 1870-1913. The Fed data runs 1914-1972 (until the US came off the gold standard in 73). And the Modern data runs 1973-2012.

ARIMA Pre-Fed (Consumer Price Index between 1870 and 1913)

For the time series analysis, we first look at the auto-correlation and partial auto-correlation functions. They show significant correlation on the first term of an auto regressive process, and so we model on AR(1).



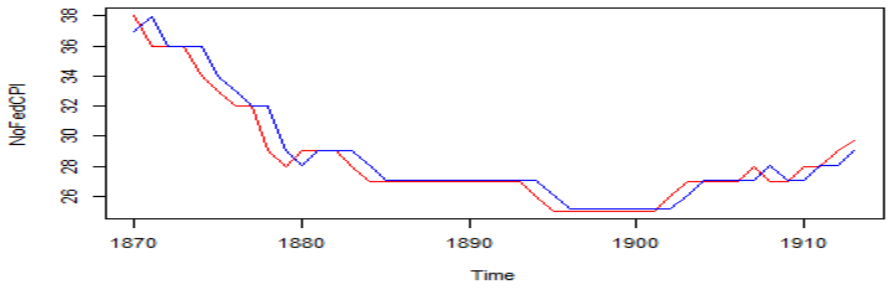


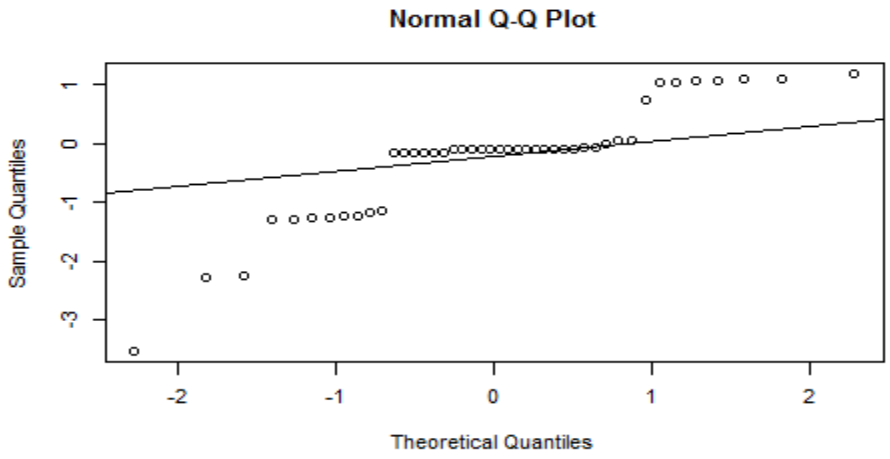
Using ARIMA to compute AR(1) : CPI(t) = CPI(t-1) * X_1 + X_0.

Coefficients:		
	AR1	mu
	0.984	32.398
s.e.	0.018	4.234
sigma^2		0.7252
log likelihood		-57.08
aic		118.16

Further, I compute (ignoring the first term) RSS = $\sum_2^n (y_t - \hat{y}_t)^2 = 30.89$ and (also ignoring the first term), TSS = 380.26. This implies an R^2 of .919, which is significantly better than we saw on the regression model on treasury reserves and population. Also, note the error term s_e = .852, and error/$\bar{Y}$ = .030. This seems lower than the regression model that we saw in (Part 1). The only place we see this model may not be so great is when we look at the QQ plot, which shows significant deviation at the extreme values. In fact, I would generally say this is a poor fit based on this plot. I don't really understand why the QQ plot looks like a step function. However, I did also test the AR(2), but found the coefficient of Y_{n-2} to be not significantly different from 0 (if the coefficients errors follow a t-distribution of 41 degrees of freedom for an AR(2), this would mean a value as extreme as -.279 would occur 7.9% of the time).

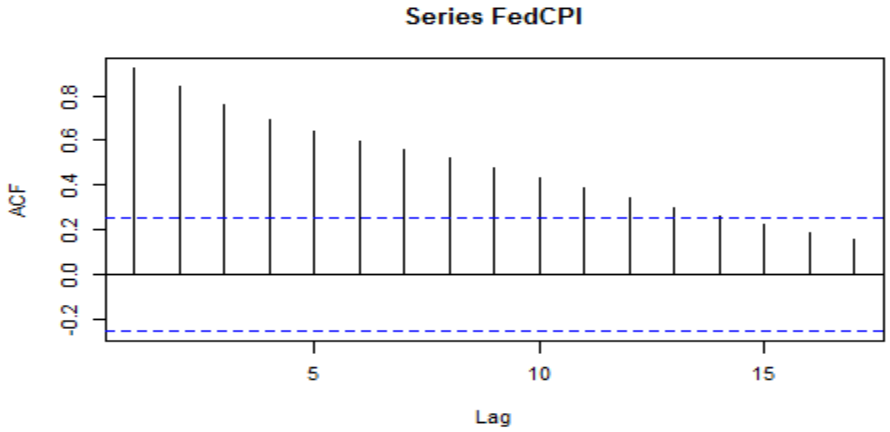
This Model can be rewritten as: Y_n=(.9816) Y_{n-1}+.528. Hence, there is little point to graphing the actual vs. expected… the expected looks very much like the actual, shifted to the right.

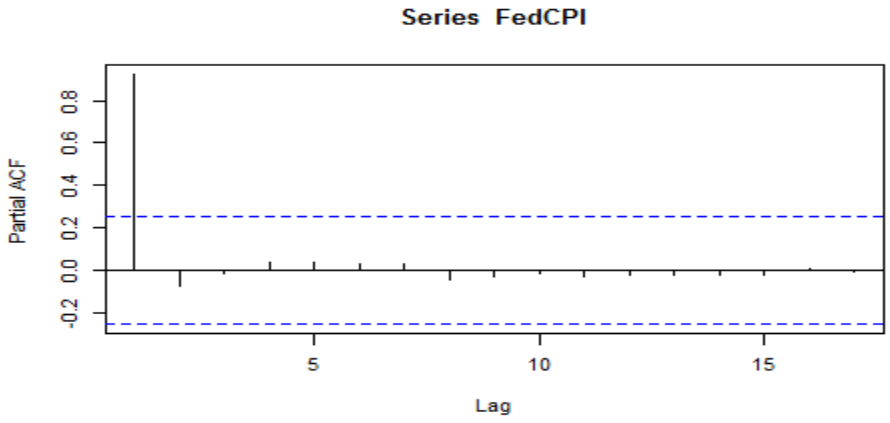




ARIMA Fed (Consumer Price Index between 1914 and 1972)

Again in the post Fed Era, I begin by looking at the ACF and Partial-ACF.



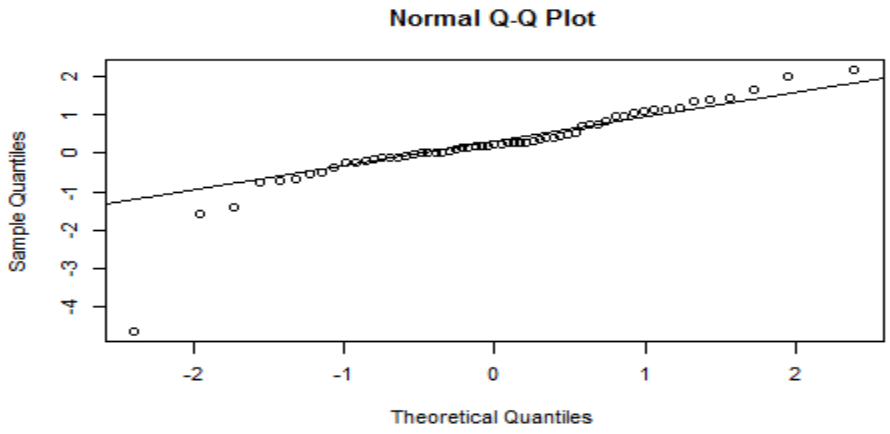


This would suggest that we want an AR(1) again, expect that $\phi>1$, so that it would not be a stationary process. So, I dodge the issue, and instead fit ARIMA(1,1,0). (Note I tried Fitting CPI/Prior for both Fed and Modern, but ended up with a worse fit. Also, The ARIMA(1,1,0) comes to mind because of possible trending, i.e. if there was a decrease last time, there might be again.) By rewriting the formula for this, we note that it is similar to fitting $Y_t = (1+\phi)Y_{t-1} - \phi Y_{t-2}$. Oddly though, AR(2) did not converge.

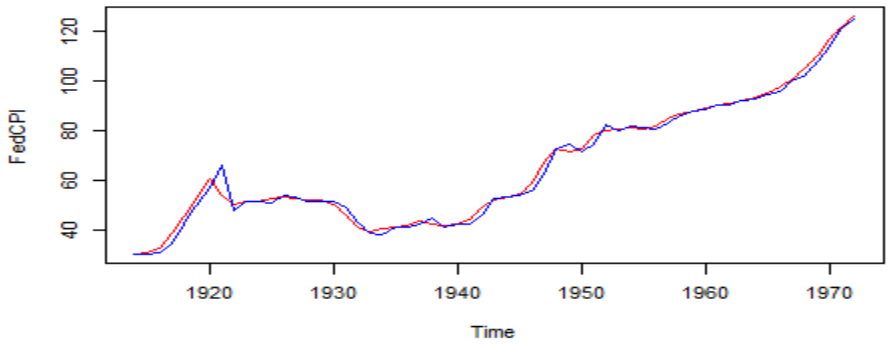
Using ARIMA to compute ARIMA(1,1,0) : $\nabla CPI(t) = \nabla CPI(t-1) * X_1$

Coefficients:	
	AR1
	0.656
s.e.	0.098
sigma^2	6.614
log likelihood	-137.36
aic	276.73

Further, I compute (ignoring the first two terms) RSS = $\sum_2^n (y_t - \hat{y}_t)^2 = 383.53$ and (also ignoring the first two terms), TSS = 41887.34. This implies an R^2 of .991, which is higher than expected, and a better fit than the regression on gold price and population. Also note that in this case the standard error is 2.57, and s.e./mean(Y_n) = .039. This is slightly worse than for the pre-Fed era. Again, looking at the QQ plot, we see the fit is overall pretty good: some thickness in the tails, and a single extreme outlier.

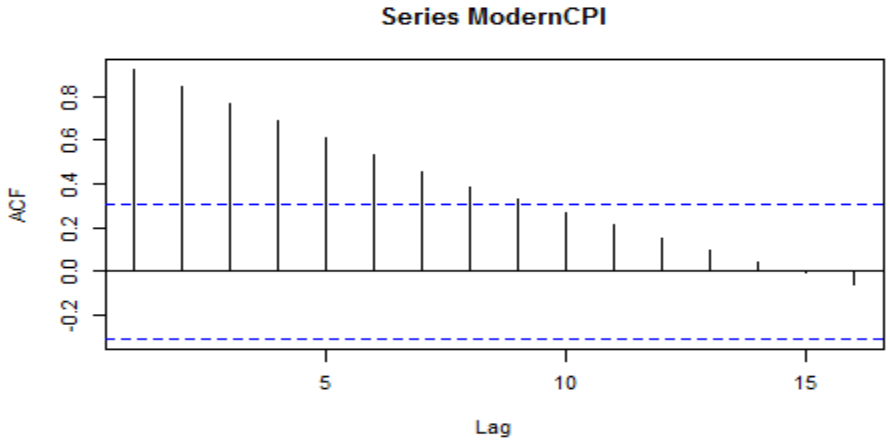


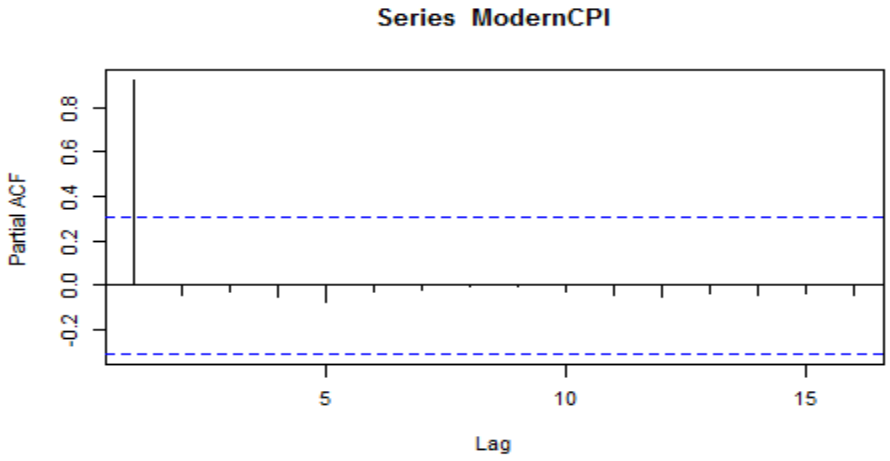
Also, looking at the actual vs expected, we what we might expect, the two period trending carries forward another term causing some errors. It is worth noting here, that I tried increasing the number of terms (e.g. ARIMA(3,1,0)) and did not improve the model.



ARIMA Modern (Consumer Price Index between 1973 and 2012)

Finally, in the Modern Era, I begin by looking at the ACF and Partial-ACF.





Again, we see a one lag auto regressive process being suggested, but the ϕ is greater than 1. So, again, we instead fit an ARIMA(1,1,0) and get:

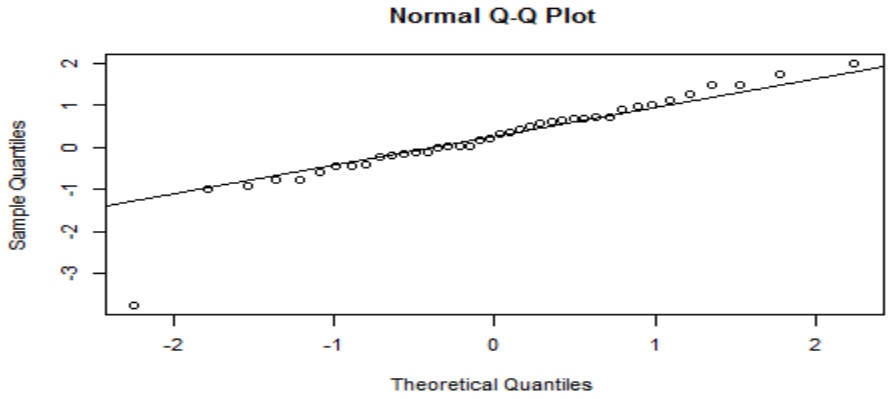
Coefficients:

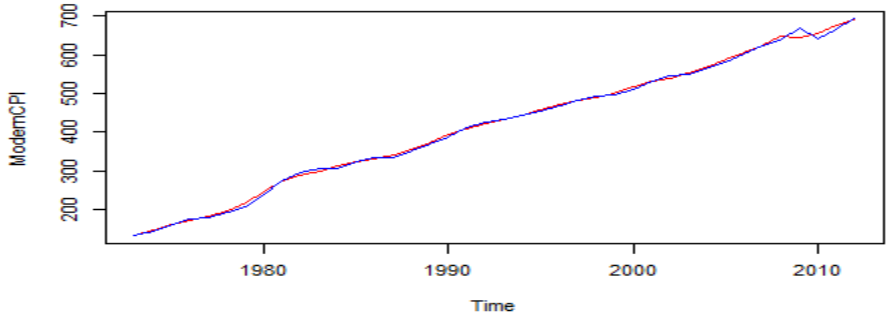
	AR1
	0.907
s.e.	0.061

sigma^2	40.87
log likelihood	-128.55
aic	259.11

This implies $Y_t = 1.907*Y_{t-1} – 0.907*Y_{t-2} + e_t$. With this model (again ignoring the first two samples), I get RSS=1554.8 and TSS=891748.5, implying an R^2 of .998, which is pretty good. This model has a standard error of 6.39, and $\sigma/\mu = .015$. This is the best evidence I have that this model is a better fit of inflation than the other models.

Looking at the QQ plot, we see it is very good, except for a single outlier, which occurred in 2009: the only time in the past 50 years there has been negative inflation.





The Actual vs. expected plot shows the same as the QQ plot: a good fit, expect in 2009 and 2010 when the trend nature of the model over (and under) estimated the actual CPI.

Conclusion

Overall time series models do a good job at predicting the consumer price index, much better than the regressive models. Further, as more controls have been introduced (introducing the Fed, and removing the gold standard) there has been an improvement in the predictable nature of the CPI. It is also worth noting that I left out the traditional model for CPI entirely which would be $Y_t = b * a^t$. Though this model works well currently, it seemed like an unfair advantage, since it was not how money was expected to work 100 years ago.

Sources

Data was compiled from information found in the following websites:

CPI data from Federal Reserve Bank:
http://www.minneapolisfed.org/community_education/teacher/calc/hist1800.cfm

Gold Prices:

http://www.onlygold.com/TutorialPages/PicesSince1972FS.htm

http://www.nma.org/pdf/gold/his_gold_prices.pdf

http://useconomy.about.com/od/monetarypolicy/p/gold_history.htm

Population:

http://en.wikipedia.org/wiki/Demographics_of_the_United_States