Real Currency Effective Exchange Rate Ratio Series Analysis of CNY to USD

September 13th, 2014

1. Introduction

In this article, time series models were used to analyze the real currency effective exchange rate ratio series of Chinese Yuan (CNY) to United States Dollar (USD) from January, 2011 to July, 2014. The end of this time period is due to the last available datum but the beginning is arbitrary. However, I hope to avoid the effect of subprime mortgage crisis. The currency effective exchange rate data are released by the Bank for International Settlements (BIS) and available on http://www.bis.org/statistics/eer/. The ratio is positive so that a logarithm transfer is used to avoid some possible negative values in the time series model.

The sample autocorrelation function (SACF) plot and sample partial autocorrelation function (SPACF) plot are used to determine the appropriate time series model of the logarithm data, which is IMA(1,1). The parameter of the time series model is estimated by the method of moment. Then I made some diagnostics on the model and verified that the model is appropriate.

2. Model Selection and Parameter Estimation

The logarithm of real effective exchange rate ratio series of CNY to USD from January, 2011 to July, 2014 is plotted in Figure 1. The series does not seem stationary, which is verified by the figure of SACF, (see Figure 2). Let $\{X_t, t \in \mathbb{N}, 1 \leq t \leq 43\}$ denote the logarithm series. The SACF r is

$$r_k = \frac{\sum_{t=k+1}^{n} (X_t - \bar{X})(X_{t-k} - \bar{X})}{\sum_{t=1}^{n} (X_t - \bar{X})^2} \tag{2.1}$$

where k is the lag and n is equal to sample size of 43. The dashed horizontal lines in Figure 2, plotted at $\pm 2/\sqrt{n} = \pm 0.305$ (n is the sample size), are intended to give critical values for testing whether or not the autocorrelation coefficients are significantly different from zero.

So we take the first difference of the logarithm series, and let $\{Y_t\}$ denote the first difference series:

$$Y_t = \nabla X_t \stackrel{\text{d}}{=} X_t - X_{t-1}, 2 \leq t \leq n \tag{2.2}$$

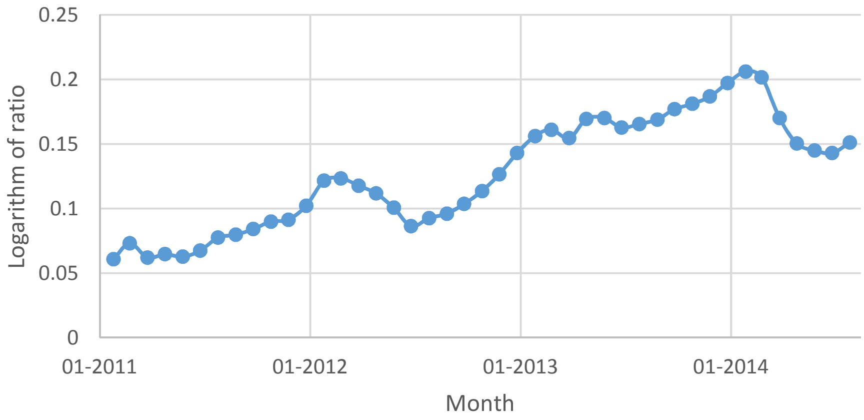

Figure 1. Logarithm of the real effective exchange rate ratio series of CNY to USD

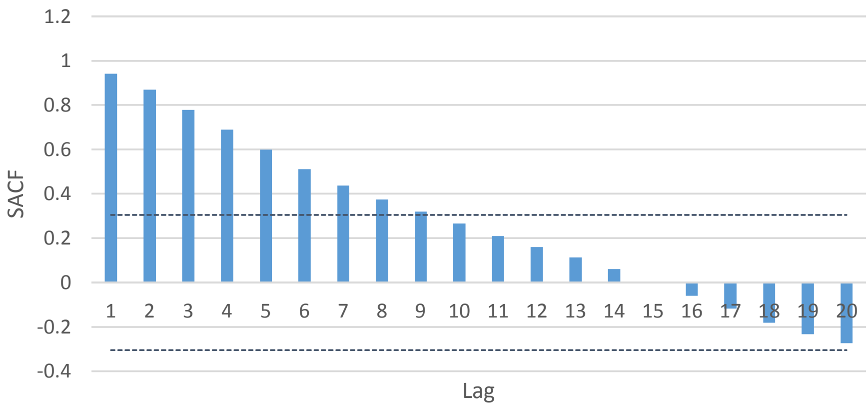

Figure 2. SACF's for the logarithm series

For convenience, let $\{Z_t, t \in \mathbb{N},\ 1 \le t \le n-1\}$ denote the zero-mean normalized first difference series and rewrite the time label t by $t-1$, that is

$$Z_t = Y_{t+1} - \bar{Y} \tag{2.3}$$

where $\bar{Y} = 0.00215$. $\{Z_t\}$ is plotted in Figure 3 and seem to be stationary now. The SACF for $\{Z_t\}$ series

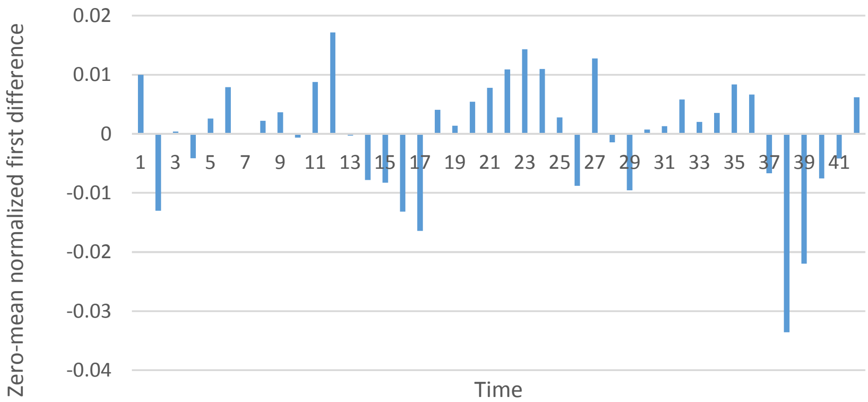

Figure 3. Zero-mean normalized first difference series

is plotted in Figure 4, which suggests an MA(1) model at first glance as only the lag 1 autocorrelation is

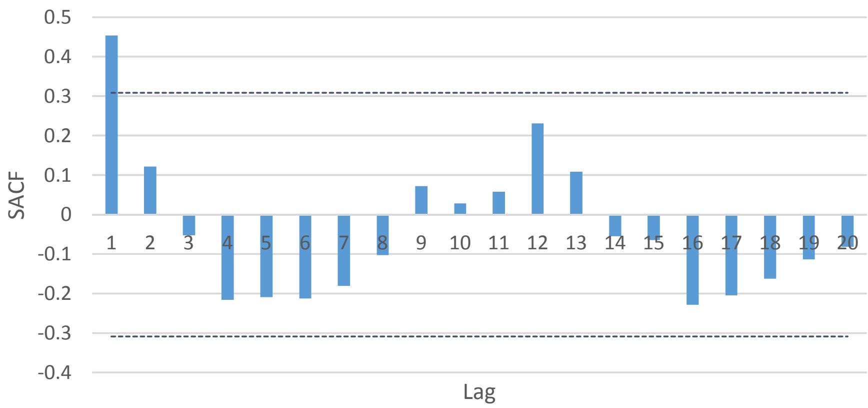

Figure 4. SACF for $\{Z_t\}$ series

significantly different from zero. But $r_1 = 0.454$ and $r_1^2 = 0.206 > 0.122 = r_2$, which impliy an AR(1) term may be needed. However, the sample size is so small that there is a large error of the estimation of r_1:

$$\sigma_{r_1} \approx \frac{1}{\sqrt{n-1}} = \frac{1}{\sqrt{42}} = 15.4\%$$

So we look further at the SPACF $\phi_{n,n}$ as there seems to be a sin wave appearance of the SACF figure. $\phi_{n,n}$

is calculated by the Yule-Walker formula:

$$\begin{bmatrix} \phi_{n,1} \\ \phi_{n,2} \\ \phi_{n,3} \\ \vdots \\ \phi_{n,n} \end{bmatrix} = \begin{bmatrix} 1 & r_1 & r_2 & \dots & r_{n-1} \\ r_1 & 1 & r_1 & \dots & r_{n-2} \\ r_2 & r_1 & 1 & \dots & r_{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{n-1} & r_{n-2} & r_{n-3} & \cdots & 1 \end{bmatrix}^{-1} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ r_n \end{bmatrix} \tag{2.4}$$

The SPACF's are plotted in Figure 5. As is shown in Figure 5, the AR(1) model seems fairly good.

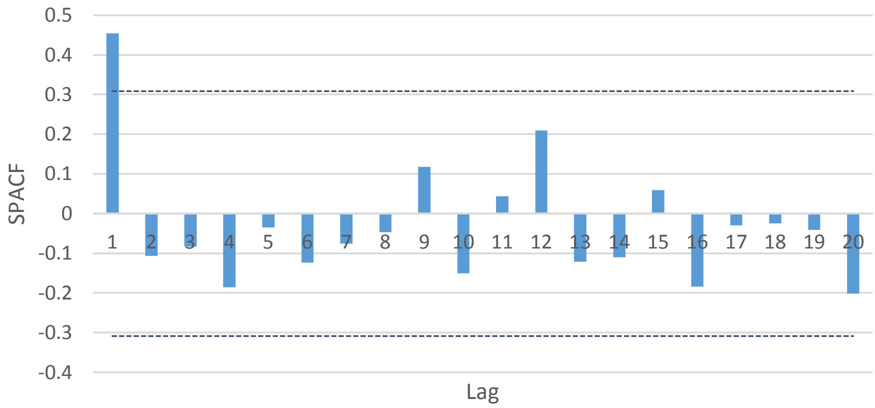

Figure 5. SPACF's for $\{Z_t\}$ series

The moment estimation of the AR(1) model parameter ϕ is equal to $r_1 = 0.4542$. In other words, for $\{Z_t\}$ it follows that:

$$Z_t = 0.4542Z_{t-1} + \varepsilon_t,\ 2 \le t \le n-1 \tag{2.5}$$

So the model for $\{X_t\}$ series is

$$X_t - X_{t-1} - 0.00215 = 0.4542(X_{t-1} - X_{t-2} - 0.00215) + \varepsilon_t \tag{2.6}$$

3. Residual Analysis

As the time series model is determined, we can calculate the one-step estimations $\{\hat{Z}_t(1)\}$:

$$\hat{Z}_t(1) = 0.4542Z_t,\ 2 \le t \le n-1 \tag{3.1}$$

and the residual series $\{\varepsilon_t, 2 \le t \le n-1\}$:

$$\varepsilon_t = Z_t - \hat{Z}_{t-1}(1) \tag{3.2}$$

The estimation series and residual series versus actual series are plotted in Figure 6.

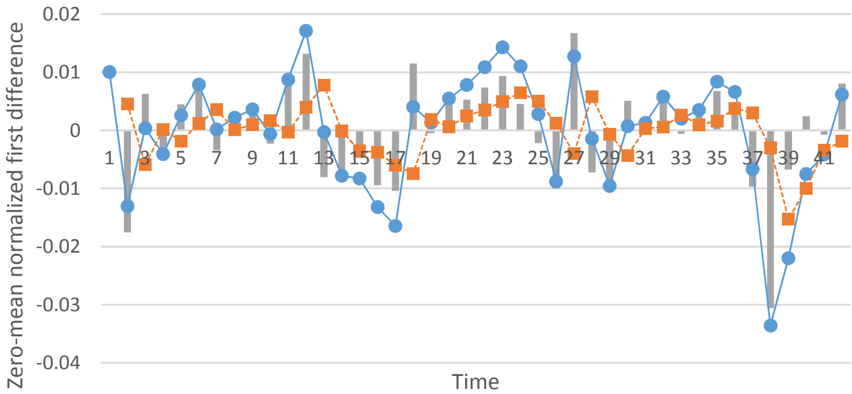

Figure 6. Estimations and residuals compared with the actual data
Note: squares with orange dashed line are the estimations; circles with blue full line are the actual data; and the gray columns are residuals.

The residuals are normalized by

$$\delta_t = \varepsilon_t / \sigma_\varepsilon \tag{3.3}$$

where

$$\sigma_\varepsilon = \sqrt{\frac{1}{n-3}\sum_{t=2}^{n-1}\varepsilon_t^2} \tag{3.4}$$

As is shown in Figure 7, there is a significant outlier at the time of 38 where the corresponding time is from February to April in 2014. And we can find that the exchange rate for USD to CNY was increasing dramatically during that time.

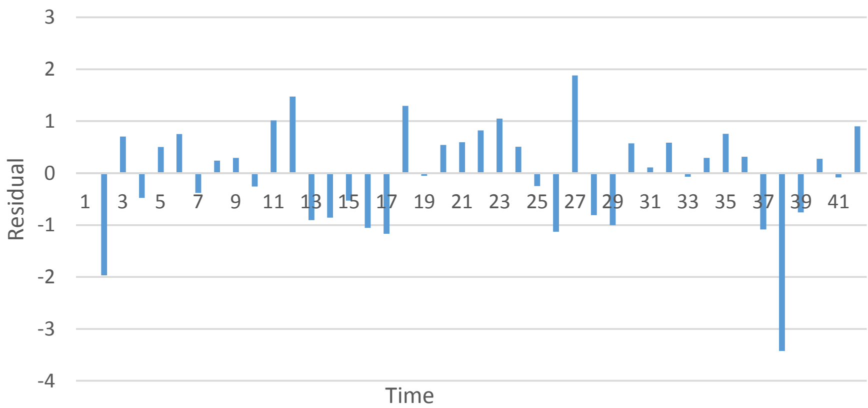

Figure 7. Normalized residual series

The SACF's of $\{\varepsilon_t\}$ are plotted in Figure 8 and the Ljung-Box statistics compared with the corresponding critical values at 0.05 significance level are plotted in Figure 9. So the IMA(1,1) series seems to be capturing the characteristics of the real currency effective exchange rate ratio series of CNY to USD quite well.

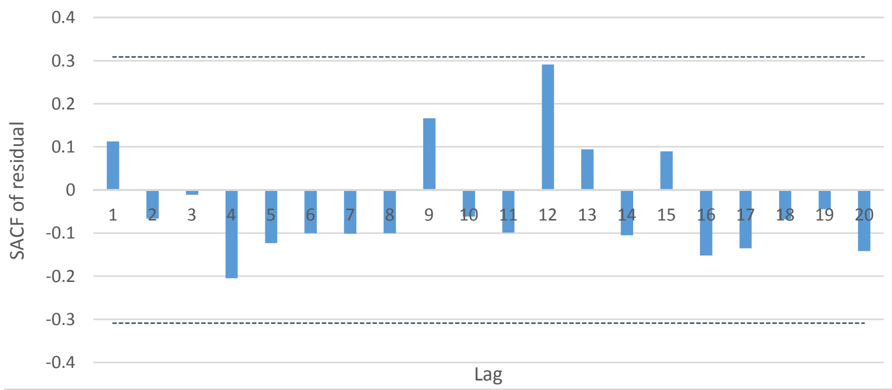

Figure 8. SACF's for $\{\varepsilon_t\}$ series

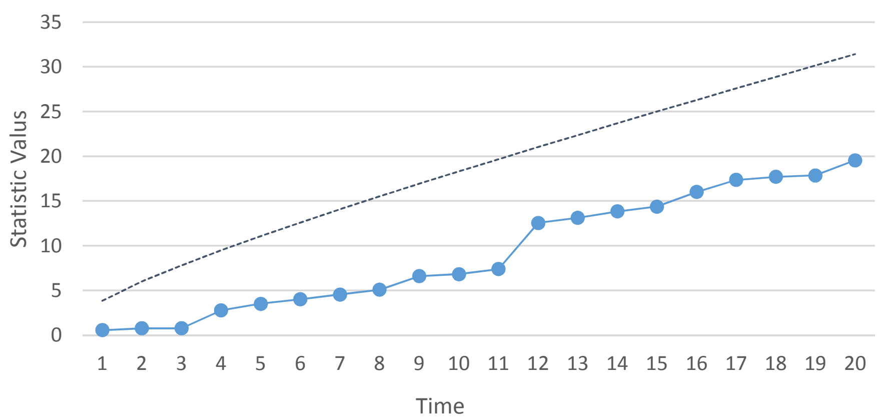

Figure 9. Ljung-Box test for $\{Z_t\}$ series
Note: circles with blue full line are the Ljung-Box statistics; and the dashed line is the critical bound.

4. Conclusion

The logarithm of real currency effective exchange rate ratio series of CNY to USD from January, 2011 to July, 2014 follows an IMA(1,1) model quite well. Let $\{X_t, t \in \mathbb{N}, 1 \leq t \leq 43\}$ denote the logarithm series then the model is

$$X_t - X_{t-1} - 0.00215 = \phi(X_{t-1} - X_{t-2} - 0.00215) + \varepsilon_t \tag{4.1}$$

where the parameter $\phi = 0.4542$ is estimated by the method of moments. The first difference of the logarithm series has a significant outlier at the time of 38 where the corresponding time is from February to April in 2014. The exchange rate for USD to CNY was increasing dramatically during that time, which also implies that the model works fairly well.

REFERENCES

[1] J. D. Cryer, and K.-S. Chan, *Time Seris Analysis with Applications in R*, Springer, 2nd edition, 2008.