Federal Spending in the U.S.

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Time Series
Summer 2014

INTRODUCTION:

This project looks at the history of government spending in the United States at the Federal level. During the initial investigation I looked at data going back to 1792 and adjusted for inflation and population growth. For the purposes of building a stationary time series no single series fit the entire period, however, so the time series models investigated here are restricted to the post-WWII era.

Traditionally Government spending is considered on a percent of GDP basis as opposed to a per resident basis, but there are some possible issues with this approach: 1. Data on pre-20th century GDP is suspect whereas the decennial census going back to 1790 provides an accurate and reliable source for population data. 2., whether government spending should be expected to grow with GDP or with population is an open question. For example, historically a large (~30-80%) of Federal outlays have been on defense, and it's not clear why a larger economy would necessitate a larger, more expensive military in real terms whereas it does make sense why a growing population - which until ~1900 corresponded with a geographically growing country - would require a larger military. Similar debates could be had over other Federal government roles. Given that, spending on a per-resident basis seemed a novel way to assess how the Federal government has evolved over time and may provide new insights not captured by spending/GDP.

DATA COLLECTION:

Population and Federal outlays data for the period 1792-1899 was sourced from "Historical Statistics of the United States 1789-1945", pages 26 and 299-301, respectfully, which can be found on the Census Bureau's website:
http://www2.census.gov/prod2/statcomp/documents/HistoricalStatisticsoftheUnitedStates1789-1945.pdf

Population data post 1900 was obtained here:
http://www.census.gov/popest/data/historical/index.html

Post 1900 Federal outlay data was obtained here:
http://www.whitehouse.gov/omb/budget/historicals

CPI data was sourced from www.measuringworth.com, a collaborative effort between a number of economists from recognized institutions. They have compiled a number of inflation time series that date back further than what is available from the BLS or Federal Reserve.

<u>DATA ASSESSMENT[1]:</u>

Looking at inflation adjusted Federal outlays (Figure 1) and inflation adjusted outlays/resident (Figure II) the data exhibits a clear pattern of exponential growth with increased volatility around the Civil War, WWI, WWII, and the recent financial crisis.

Figure I:

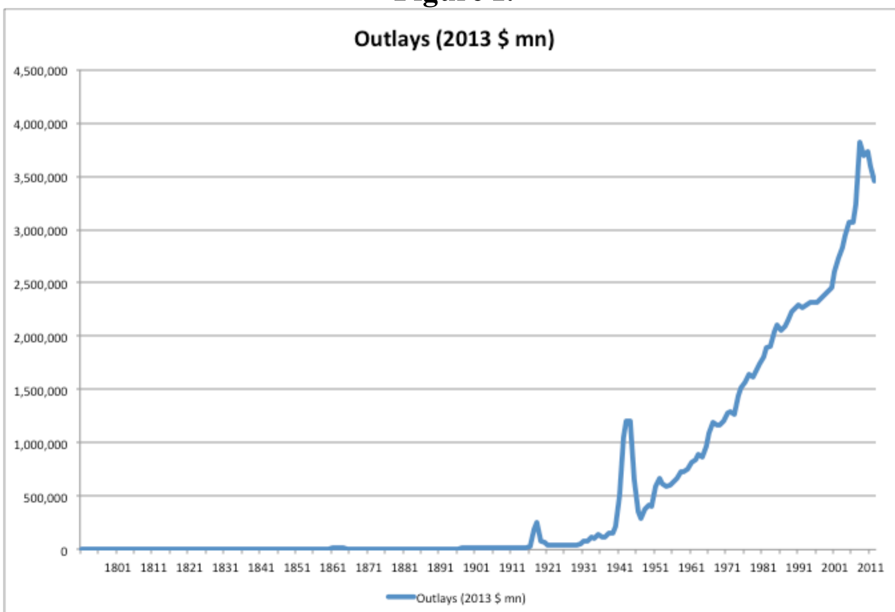
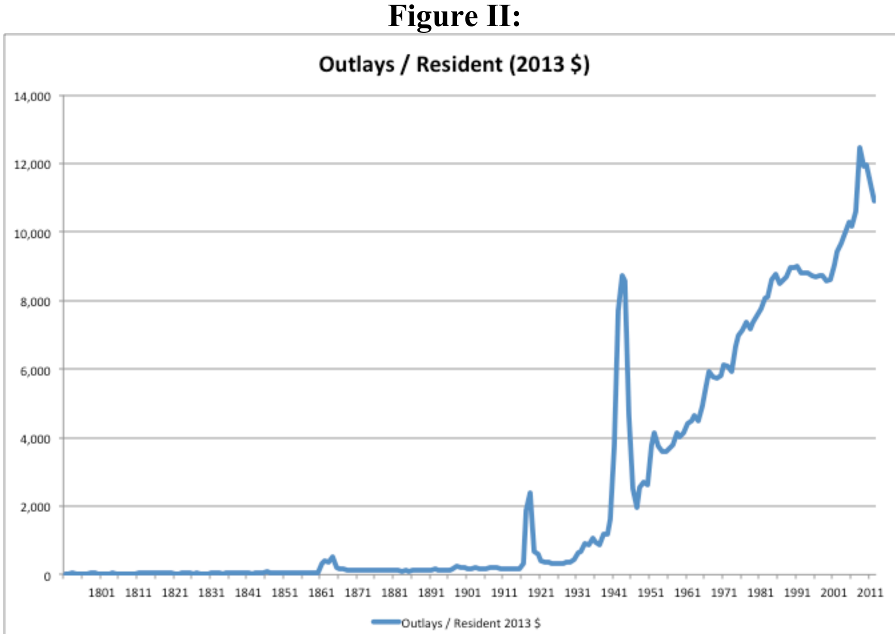


Figure II:



[1] See Excel sheet "Data"

Clearly neither of these series is stationary, in general, though, exponential growth can be made stationary by taking logs and first differences, shown in Figures III and IV. If spending per resident is indeed growing exponentially a linear trend is expected in Figure III and a series in Figure IV with stationary mean, which appears reasonable.

Given that the per-resident series in Figures II and III are showing an obvious upward growth - a far larger and sustained deviation from the mean than would be expected from any stationary series - it's immediately clear real Federal outlays per resident have not been stationary over time[2].

Figure III:

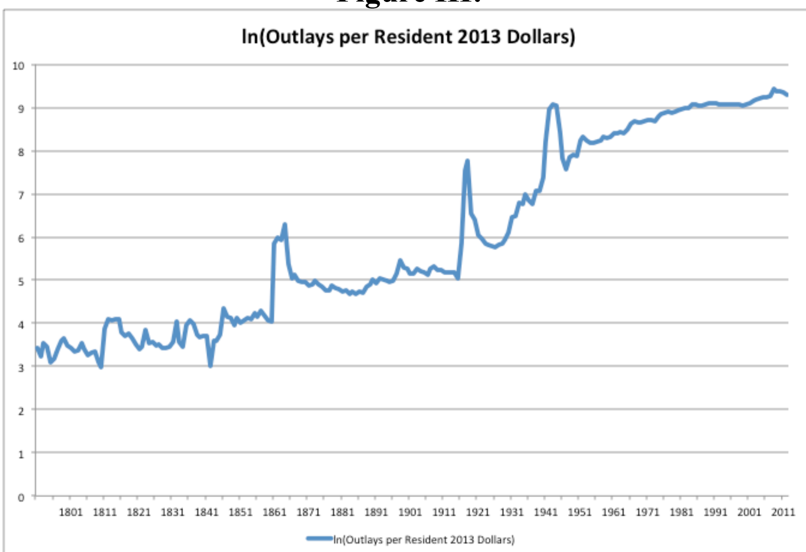
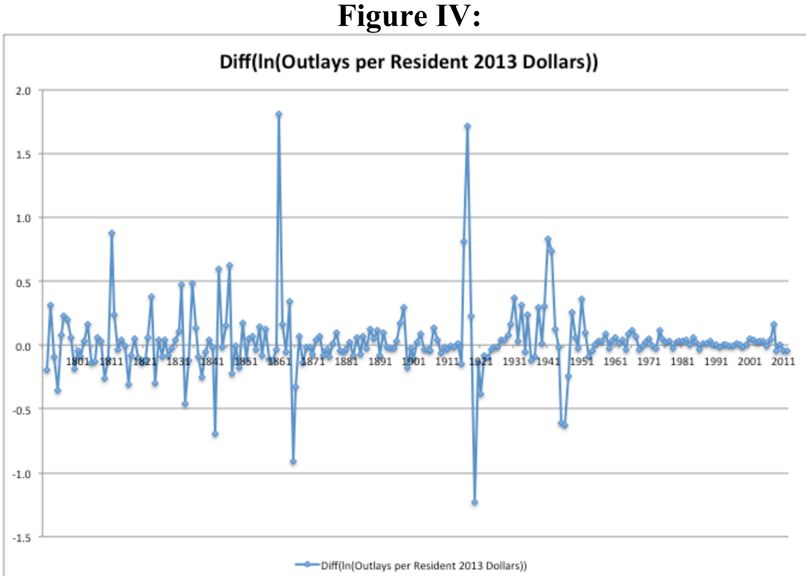


Figure IV:



[2] This was confirmed by looking at the sample ACF, see Excel sheet "Sample Correlogram-LN"

Looking at Figures I, II, III, and IV, and combining them with general knowledge of American History the data breaks down into four well defined eras with distinct characteristics: The Founding through the Civil War, characterized by very low levels of spending with little growth per resident (<1%/year) and large year-to-year volatility; Post Civil-War through WWI, with higher spending per resident but still little growth (<1%/year) and lower volatility; WWI through WWII, characterized by very high volatility and rapidly growing spending per resident (~7.5%); and Post-WWII, characterized by a steady growth rate of Federal spending per resident and much lower volatility (~2.5%).

Given the evolution of the data: varying growth rates during different periods, changing volatility, and the dramatic effects of two World Wars and the Civil War, it's unreasonable to expect any single model to fit with any degree of precision, therefore, the remainder of the study will focus only on the Post-WWII period. The start of this period will be 1950 after war efforts had ramped down and spending stabilized.

MODEL SPECIFICATION:

This study looks at two possible models for the data. The first is a simple mean plus white noise model of first differences of the logs:

$$1: \nabla \ln(Y_t) = \mu + X_t$$

Where:

Y_t = Federal outlays per resident

μ = Mean growth rate

X_t = White noise process of 0 mean and constant variance

The second model uses a more sophisticated approach while keeping parsimony in mind. This model regresses spending on population, which is itself a linear regression[3], where the residuals are modeled as an AR(1) process:

$$2: Y_t = \mu_t + \varphi * r_{t-1} + \varepsilon_t$$

Where:

Y_t = Total Federal outlays

μ_t = Linear estimate of outlays at t; $\beta_0 + \beta_1 * Pop_t$

[3] The reason the population regression was required rather than just using the actual data is for forecasting. In order to generate an estimate of future Federal outlays that depends on population or outlays per resident an estimate of future population is needed. Given population tends to grow exponentially a linear regression isn't optimal, however, over the given time frame a linear approximation is sufficiently accurate (the population on year regression has an R^2=0.995). The extra noise introduced by the population regression is dwarfed by the volatility in outlays.

Pop_t = Linear estimate of population at t; $\xi_0 + \xi_1 * t$

$r_t = Y_t - (\beta_0 + \beta_1 * Pop_t)$

ε_t = Stochastic component

Which gives: $Y_t = \beta_0 + \beta_1(\xi_0 + \xi_1 * t) + \varphi(Y_{t-1} - (\beta_0 + \beta_1 * Pop_{t-1}))$

$= (1-\varphi)(\beta_0 + \beta_1\xi_0 + \beta_1\xi_1 * t) + \varphi\beta_1\xi_1 + \varphi * Y_{t-1}$

Model 1 Parameters[4]:

With Model 1 as: $\nabla \ln(Y_t) = \mu + X_t$

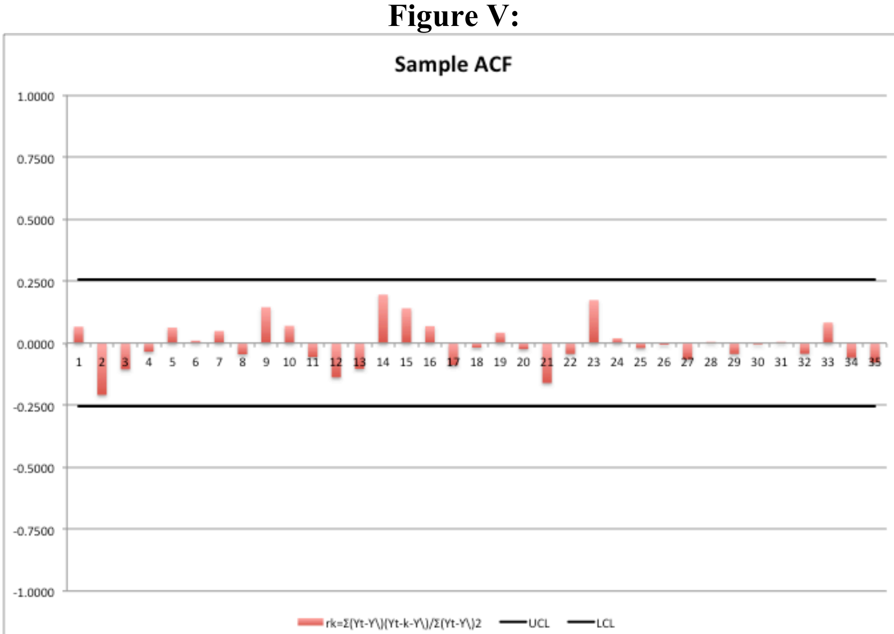
The following were generated:

$\mu = 0.0242$

$\sigma_\varepsilon^2 = 0.0035$

Figure V shows the sample ACF of $\nabla \ln(Y_t)$ and with no values beyond the significance bounds for a white noise process $\left(0 \pm 1.96\sqrt{1/n}\right)$ there is no evidence to reject this model.

Figure V:



[4] See Excel sheets "Sample ACF-Diff_LN" and "Model 1"

$$ACF:$$

$$r_k = \frac{\sum_{t=k+1}^{n}\left(Y_t - \bar{Y}\right)\left(Y_{t-k} - \bar{Y}\right)}{\sum_{t=1}^{n}\left(Y_t - \bar{Y}\right)^2}$$

$$UCL/LCL = 0 \pm 1.96\sqrt{\frac{1}{n}}$$

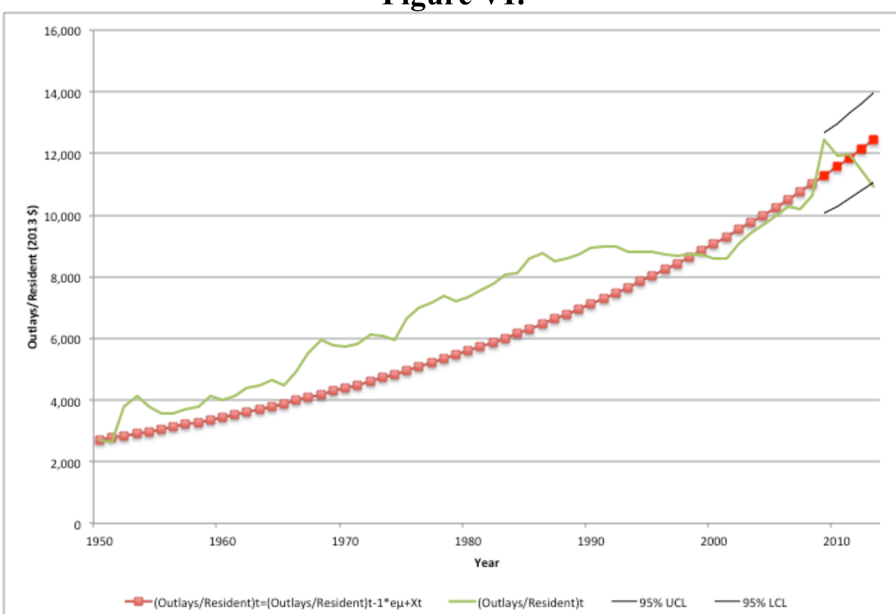
MODEL 1 FORECASTS:

Based on the above model forecasts are generated for Federal spending per resident for 2009-2013 and the results compared to actual data. All years fall within the 95% bounds except 2013 which is below by < $200.

$$\nabla \ln\left(Y_t\right) = 0.0242 + X_t$$
$$\Rightarrow Y_t = Y_{t-1}e^{0.0242+X_t}$$
$$\text{Where } X_t \sim N\left(0, 0.0035\right)$$

$$\hat{Y}_t\left(\ell\right) = Y_t e^{0.0242*\ell}$$
$$UCL/LCL = Y_{2008}e^{0.0242*\ell \pm 1.96*\sqrt{0.0035}}$$

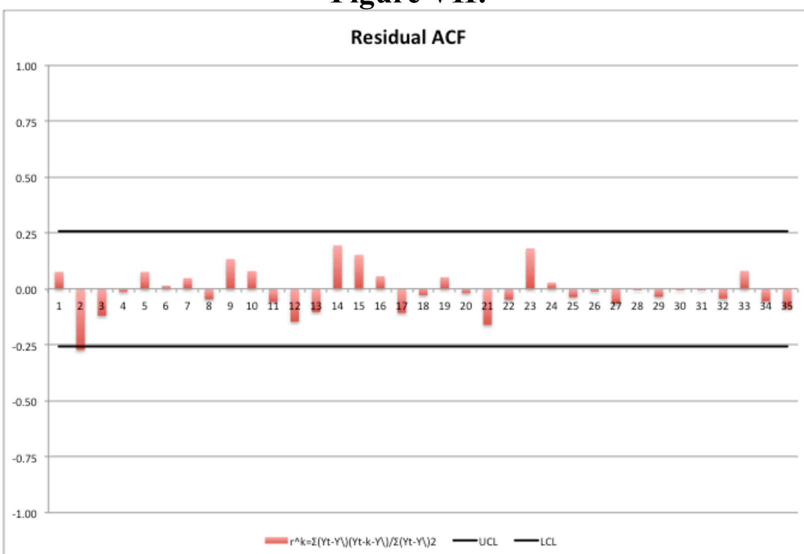
Figure VI:



Model 1 Diagnostics:

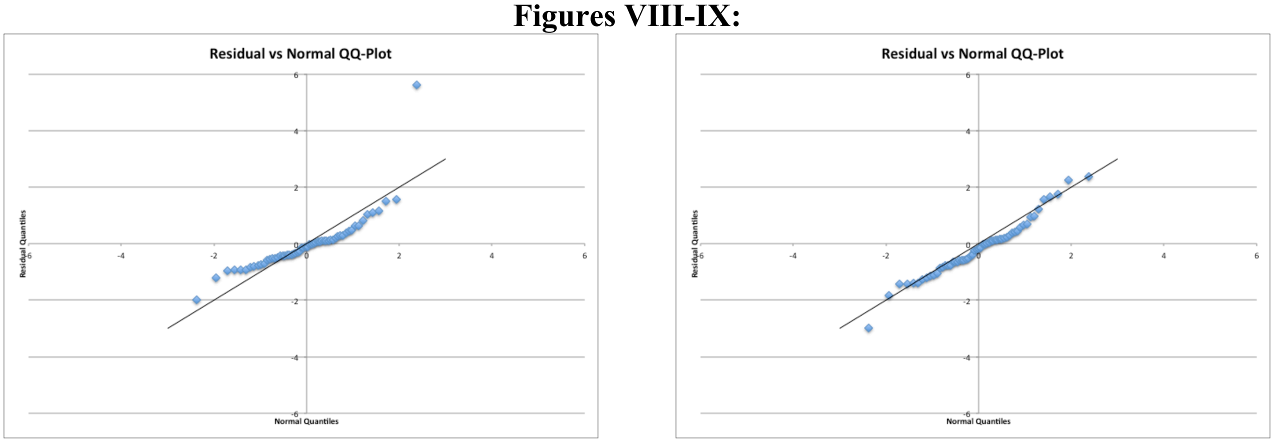
To assess the accuracy of the model the first checks are the residual ACF and qq-plot. Looking at Figure VII there doesn't appear to be any reason to question the model with all lags within the white-noise bounds.

Figure VII:



Turning to the qq-plot in Figure VIII, the slope is slightly below what would be expected for a normal distribution, however, correcting for the 5.6σ outlier at t=1952 this is eliminated and the residuals closely follow a normal plot, seen in Figure IX.

Figures VIII-IX:



[5]

[5] See Excel Sheets "Mod 1 Resids ACF" and "Model 1"

As a final check of the residuals the Ljung-Box test is performed to ensure the total spread of residuals is within expected bounds, and as Table I shows all lags fall within the 95% confidence bound.

Table I:

k=Lag	Ljung-Box Q*	Ljung-Box Q* P-value	Reject at 95%
1	0.3537	0.5520	No
2	4.9819	0.0828	No
3	5.8952	0.1168	No
4	5.9073	0.2062	No
5	6.2861	0.2794	No
6	6.2995	0.3905	No
7	6.4562	0.4876	No
8	6.6080	0.5795	No
9	7.8774	0.5465	No
10	8.3415	0.5955	No
11	8.5906	0.6596	No
12	10.2119	0.5974	No
13	11.0378	0.6077	No
14	14.0291	0.4475	No
15	15.8987	0.3888	No
16	16.1661	0.4414	No
17	17.1405	0.4449	No
18	17.2056	0.5090	No
19	17.4519	0.5593	No
20	17.4838	0.6214	No
21	19.9056	0.5273	No
22	20.1317	0.5748	No
23	23.3779	0.4389	No
24	23.4600	0.4928	No
25	23.5993	0.5426	No
26	23.6140	0.5980	No
27	24.1000	0.6248	No
28	24.1001	0.6762	No
29	24.2483	0.7166	No
30	24.2501	0.7606	No
31	24.2501	0.8002	No
32	24.5023	0.8257	No
33	25.4047	0.8249	No
34	25.8453	0.8410	No
35	26.9640	0.8326	No

$$Q^* = n*(n+2)\left(\frac{\hat{r}_1}{n-1} + \frac{\hat{r}_2}{n-2} + \ldots + \frac{\hat{r}_k}{n-k}\right)$$

$$P - value = 1 - \chi^2(Q^*, df = k)$$

MODEL 2 PARAMETERS:

With Model 2 specified as: $Y_t = \mu_t + \varphi * r_{t-1} + \varepsilon_t$

Where:

$Y_t =$ Federal outlays

$\mu_t =$ Linear estimate of outlays at t; $\beta_0 + \beta_1 * Pop_t$

$Pop_t =$ Linear estimate of population at t; $\xi_0 + \xi_1 * t$

$r_t = Y_t - (\beta_0 + \beta_1 * Pop_t)$

The following values were generated:

$\beta_0 = -2473879.10$

$S.E_{\beta_0} = 70035.02$

$\beta_1 = 18220.74$

$S.E_{\beta_1} = 304.36$

$R^2 = 0.9843$

$\hat{\varphi} = 0.8886$

$S.E_{\hat{\varphi}} = 0.0777$

$Intercept = 0.0263$

$S.E_{Inter} = 0.0728$

$R^2 = 0.7000$

$\xi_0 = -4710.66$

$S.E_{\xi_0} = 44.2012$

$\xi_1 = 2.49$

$S.E_{\xi_1} = 0.0223$

$R^2 = 0.9955$

$\hat{\sigma}_\varepsilon = 49129.74$

The intercept of the φ regression is expected to be 0 and with an intercept of 0.0263 and S.E. of 0.0728 we do not reject H_0 where H_0: Intercept=0 at α=0.05. It is also of note r_1=0.7786, the method of moments estimator of φ, is within 1.96 standard errors (0.0777) of $\hat{\varphi}$, the least squares estimator. [6]

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[6] See Excel sheets "Population Regression", "Out vs Pop Regress", "Samp ACF-PACF Out vs Pop", "Out vs Pop ACF Output", "Phi_Hat Regression", "LS Est", "Model 2"

With R^2 of 0.9843 and 0.9955 both the population and outlays regressions are close fits to the observed data - See Figures X and XI.

Figure X:

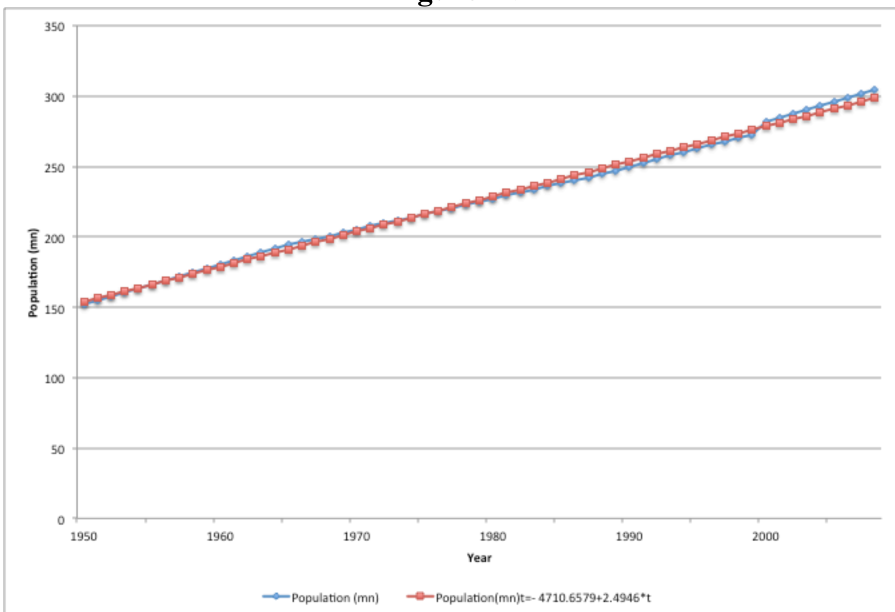
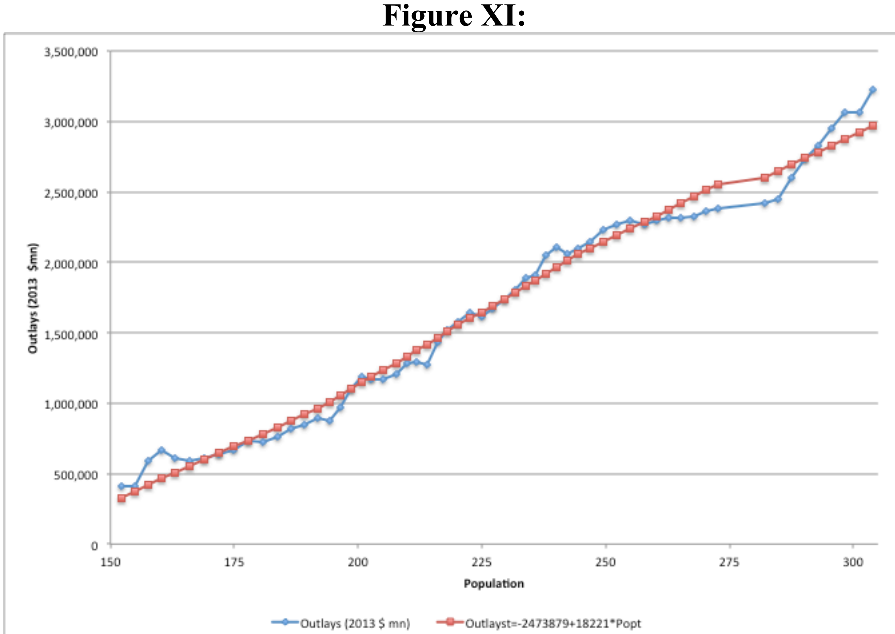
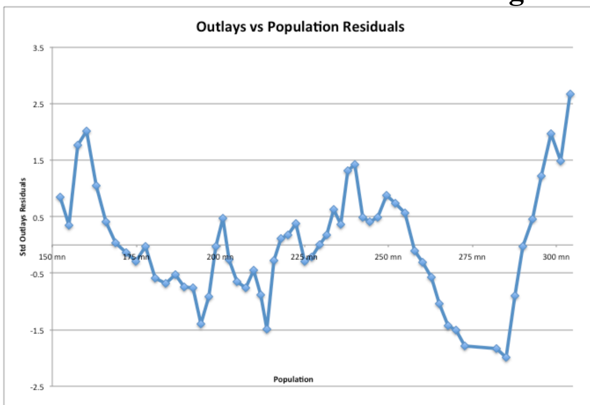


Figure XI:



Next the time and *t* vs *t-1* plots and the sample ACF and PACF of the outlays regression residuals are assessed to determine if there is an identifiable pattern - See Figures XII, XIII, XIV, and XV respectively.

Figures XII-XIII:



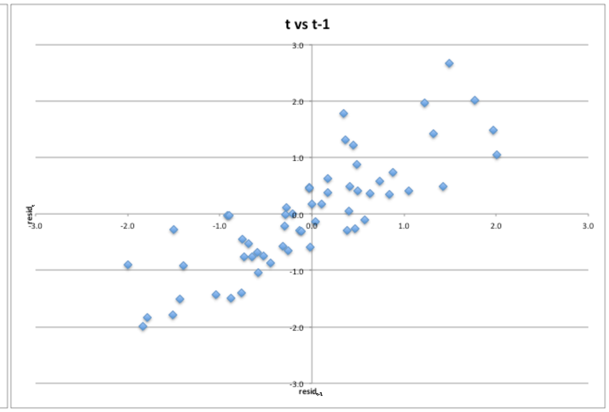


Figure XIV:

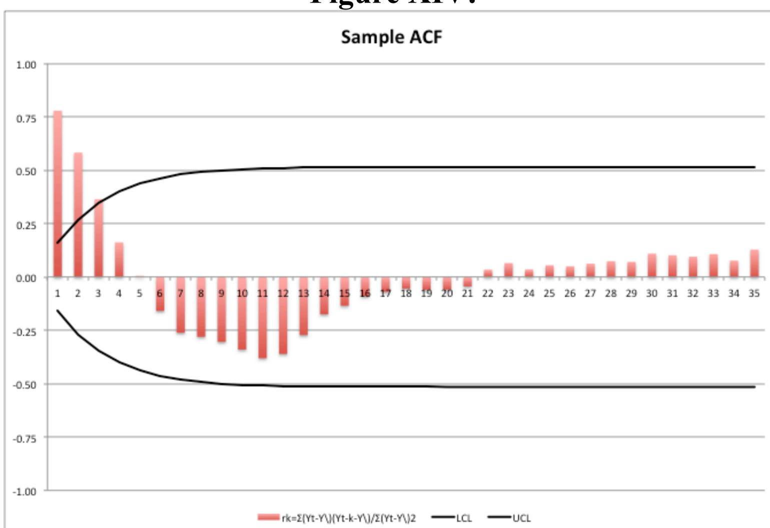
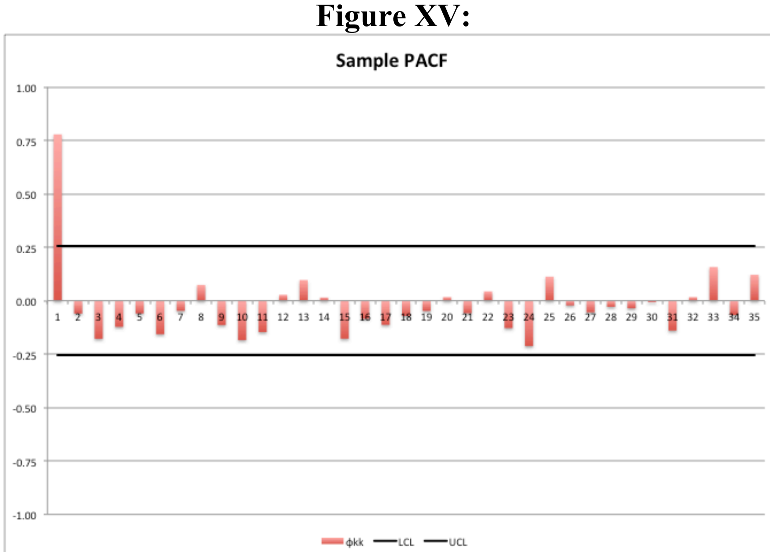


Figure XV:



Looking at Figures XII and XIII it appears there is a strong correlation among neighboring values, with Figure XII in particular showing a strong resemblance to an AR(1) model with high φ. Looking at the ACF the first three lags fall within the significance bounds for an AR(1) model and the exponential decay is consistent with an AR(1) model, although the negative values at lags at 6-20 indicate the possibility of an AR(2) model.

Turning to the PACF[7], there is a very strong indication of an AR(1) model where only the first lag falls outside the significance bounds with all others well inside. Due to the strength of the AR(1) seen in the PACF and the fact that none of the negative values in the ACF fall outside the significance bounds the AR(2) model is abandoned. The added complexity would likely outweigh any possible gains in precision and could add unnecessary noise. The ACF, PACF, and significance bounds were generated using the following functions - Note the UCL/LCL bounds are those expected of an AR(1) process with $\varphi = \hat{\varphi}$:

$$ACF:$$

$$r_k = \frac{\sum_{t=k+1}^{n} \left(Y_t - \bar{Y}\right)\left(Y_{t-k} - \bar{Y}\right)}{\sum_{t=1}^{n} \left(Y_t - \bar{Y}\right)^2}$$

$$UCL / LCL = 0 \pm 1.96 \sqrt{\frac{1}{n} \frac{\left(1+\varphi^2\right)\left(1-\varphi^{2k}\right)}{1-\varphi^2} - 2k\varphi^{2k}}$$

$$PACF:$$

$$\varphi_{kk} = \frac{\rho_k - \sum_{j=1}^{k-1} \varphi_{k-1,j}\rho_{k-j}}{1 - \sum_{j=1}^{k-1} \varphi_{k-1,j}\rho_j}$$

$$UCL / LCL = 0 \pm 1.96 \sqrt{\frac{1}{n}}$$

MODEL 2 FORECASTS:

From the parameters outlined above the modeled values in Table 2 were generated - The first three columns are actual data and the final four columns the modeled data. Forecasts for 2009-2013 are graphed in Figures XIV-XV.

The forecasts in Figure XV are 1, 2, 3, 4, and 5 year forecasts based on the last known residual from 2008. Given the large deviation in 2009 due to the financial crises the 2009 actual is well outside the 95% bounds (The 2009 residual was an 11σ event). The forecasts in Figure XVI are a

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[7] PACF values were calculated using the PACF() function in R with max lag set to 35, the output was then exported back to Excel.

series of 1 year forecasts updated by the actual residual in *t-1*. As shown by the charts a model incorporating the auto-regression inherent in the residuals improves significantly on the naïve regression, although as 2009 demonstrates caution is needed, especially when forecasting for more than 1 year.

Table 2:

Year	Population (mn)	$Outlays_t$		$Pop_t=\xi_0+\xi_1*t$	$Outlays*_t=\beta_0+\beta_1*Pop_t$	$Outlays_t$ - $Outlays*_t$ (res_t)	$Outlays'_t=\beta_0+\beta_1*Pop_t+\varphi*res_{t-1}$
1950	152	411,763		154	328,556	83,207	
1951	155	408,119		156	374,009	34,110	447,943
1952	158	593,903		159	419,463	174,440	449,771
1953	160	662,747		161	464,916	197,831	619,916
1954	163	614,077		164	510,370	103,707	686,154
1955	166	595,396		166	555,823	39,573	647,973
1956	169	605,456		169	601,276	4,180	636,440
1957	172	633,734		171	646,730	-12,996	650,444
1958	175	663,799		174	692,183	-28,384	680,636
1959	178	735,773		176	737,636	-1,863	712,416
1960	181	725,078		179	783,090	-58,012	781,434
1961	184	760,881		181	828,543	-67,663	776,997
1962	187	822,373		184	873,997	-51,623	813,875
1963	189	846,903		186	919,450	-72,547	873,580
1964	192	889,858		189	964,903	-75,046	900,441
1965	194	872,699		191	1,010,357	-137,657	943,674
1966	197	965,514		194	1,055,810	-90,297	933,494
1967	199	1,098,288		196	1,101,264	-2,976	1,021,030
1968	201	1,192,474		199	1,146,717	45,757	1,144,073
1969	203	1,166,642		201	1,192,170	-25,528	1,232,828
1970	205	1,173,491		204	1,237,624	-64,133	1,214,941
1971	208	1,208,632		206	1,283,077	-74,446	1,226,092
1972	210	1,284,097		209	1,328,531	-44,434	1,262,382
1973	212	1,287,737		211	1,373,984	-86,247	1,334,502
1974	214	1,272,043		214	1,419,437	-147,395	1,342,802
1975	216	1,437,966		216	1,464,891	-26,925	1,333,922
1976	218	1,521,122		219	1,510,344	10,777	1,486,420
1977	220	1,572,866		221	1,555,798	17,069	1,565,374
1978	223	1,638,600		224	1,601,251	37,349	1,616,417
1979	225	1,618,001		226	1,646,704	-28,703	1,679,891
1980	227	1,671,105		229	1,692,158	-21,053	1,666,654
1981	229	1,737,634		231	1,737,611	22	1,718,904
1982	232	1,800,293		234	1,783,065	17,229	1,783,084
1983	234	1,890,728		236	1,828,518	62,210	1,843,827
1984	236	1,909,880		239	1,873,971	35,908	1,929,248
1985	238	2,048,888		241	1,919,425	129,463	1,951,331
1986	240	2,105,104		244	1,964,878	140,226	2,079,913
1987	242	2,058,942		246	2,010,332	48,610	2,134,930
1988	244	2,096,081		249	2,055,785	40,296	2,098,978
1989	247	2,148,763		251	2,101,238	47,525	2,137,043
1990	249	2,233,340		254	2,146,692	86,648	2,188,920
1991	252	2,264,990		256	2,192,145	72,845	2,269,137
1992	255	2,293,949		259	2,237,598	56,350	2,302,326
1993	258	2,272,184		261	2,283,052	-10,868	2,333,122
1994	260	2,297,773		264	2,328,505	-30,732	2,318,848
1995	263	2,316,977		266	2,373,959	-56,982	2,346,651
1996	265	2,316,956		269	2,419,412	-102,456	2,368,780
1997	268	2,323,963		271	2,464,865	-140,903	2,373,827
1998	270	2,361,697		274	2,510,319	-148,622	2,385,119
1999	273	2,379,719		276	2,555,772	-176,054	2,423,713
2000	282	2,420,173		279	2,601,226	-181,053	2,444,792
2001	285	2,450,416		281	2,646,679	-196,263	2,485,804
2002	288	2,603,990		284	2,692,132	-88,142	2,517,741
2003	290	2,734,620		286	2,737,586	-2,966	2,659,267
2004	293	2,827,635		289	2,783,039	44,596	2,780,404
2005	296	2,948,628		291	2,828,493	120,136	2,868,118
2006	298	3,068,058		294	2,873,946	194,112	2,980,693
2007	301	3,065,857		296	2,919,399	146,457	3,091,879
2008	304	3,227,187		298	2,964,853	262,335	3,094,988

Figure XV:

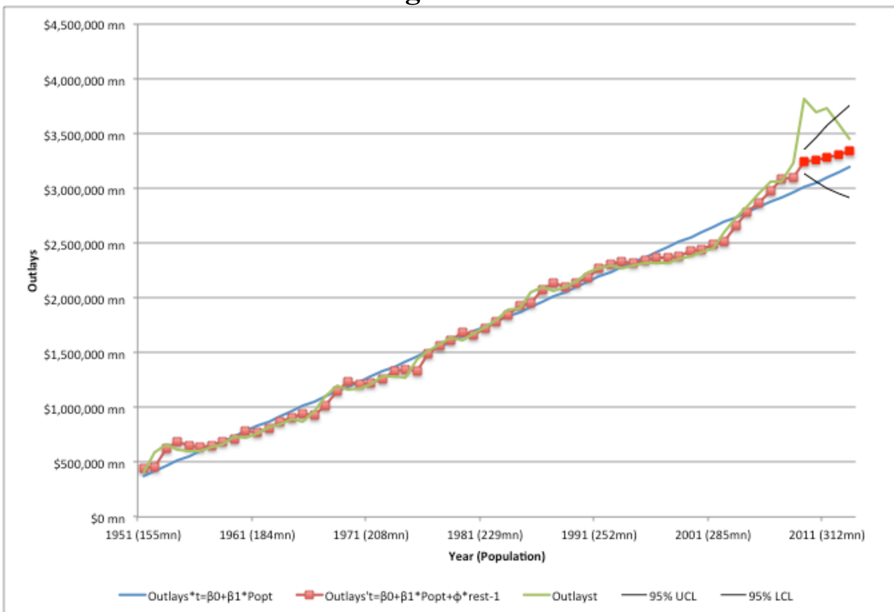


Figure XVI:

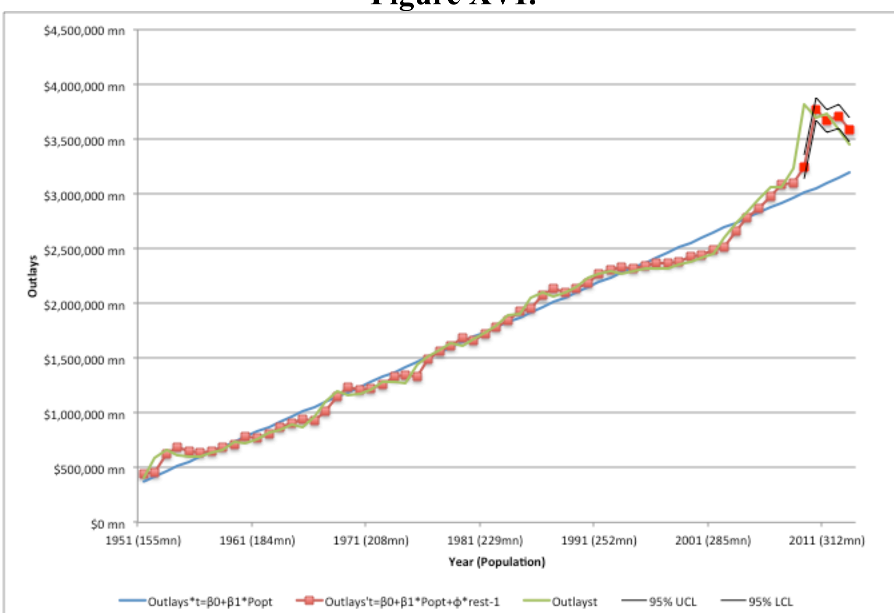


Figure XV:

$$\hat{Y}_{2008}(\ell) = (1-\varphi)(\beta_0 + \beta_1\xi_0 + \beta_1\xi_1 * t) + \varphi\beta_1\xi_1 + \varphi^{\ell} * Y_{2008}$$

Figure XVI:

$$\hat{Y}_t(1) = (1-\varphi)(\beta_0 + \beta_1\xi_0 + \beta_1\xi_1 * t) + \varphi\beta_1\xi_1 + \varphi * Y_{t-1}$$

MODEL 2 DIAGNOSTICS[8]:

As with Model 1, if the model is a correct fit for this data it is expected the residuals are normally distributed with mean 0. To check this a simple time plot, qq-plot, and Barletts test using AR(1) bounds were performed - See Figures XVII, XVIII, and XIX, respectively.

Figure XVII:

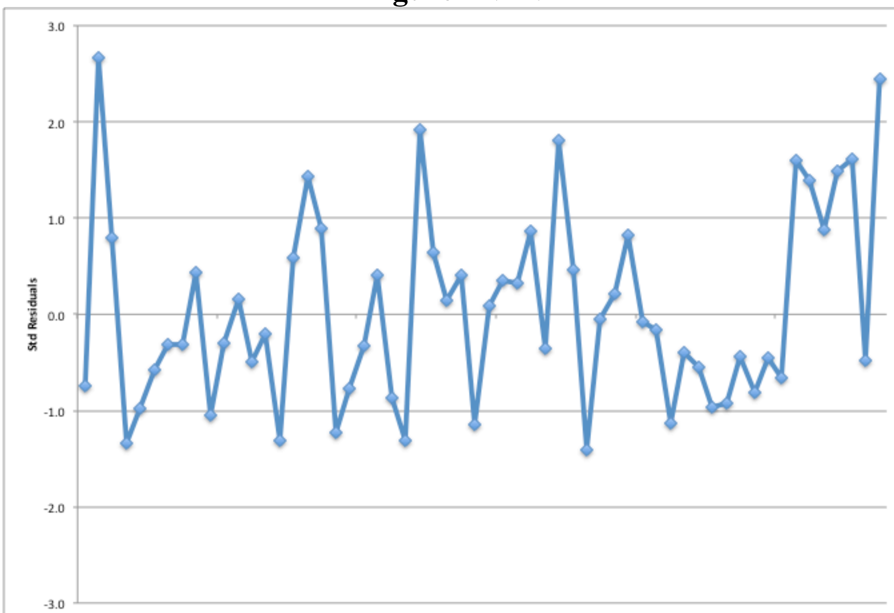
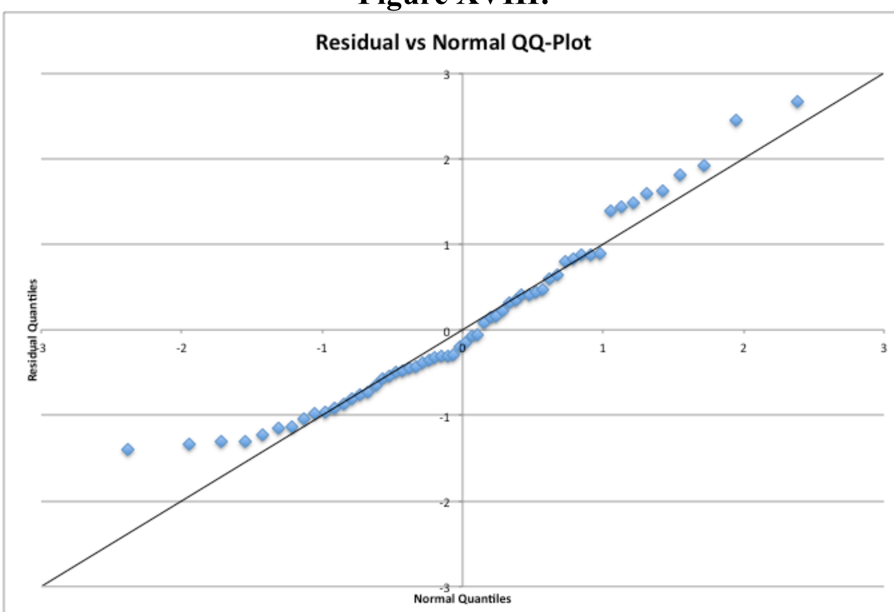
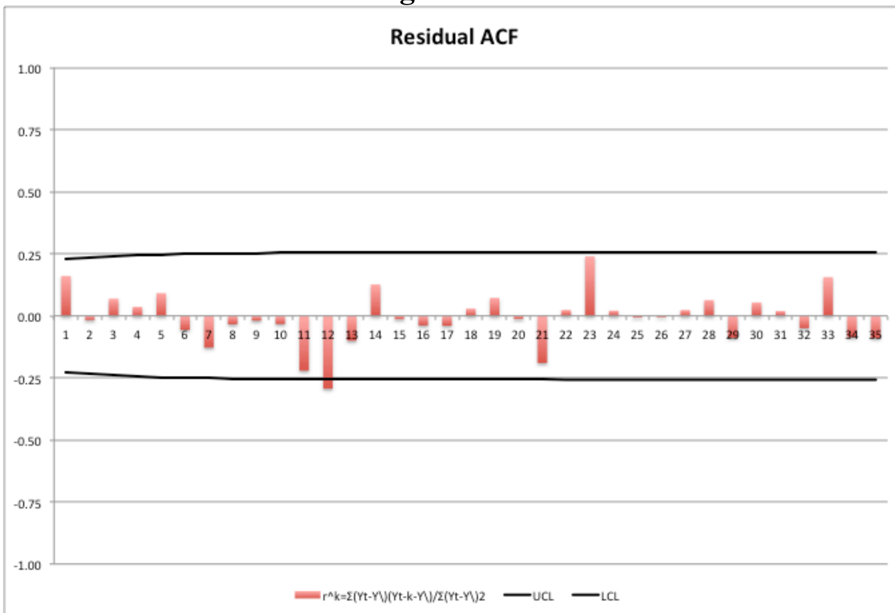


Figure XVIII:



[8] See Excel sheet "Mod 2 Resids ACF", "Mod 2 Resids ACF Output"

Figure XIX:



$$UCL / LCL = 0 \pm 1.96\sqrt{\frac{1}{n}\left(1-\left(1-\hat{\varphi}^2\right)\hat{\varphi}^{2k-2}\right)}$$

Only the lag 12 value of the $\hat{r}_k$ ACF is significant and falls outside the significance bounds by a small margin. As expected, comparing Figure XIX to Figure XIV much less autocorrelation is evident. These observations combined with a qq-plot that fairly closely tracks normality indicates the residuals are independent and normally distributed, thus based on the evidence from these tests the model is efficiently specified.

Next the standard deviation of the observed residuals is compared against the method of moments estimator, $\hat{\sigma}_\varepsilon$:

$$\hat{\gamma}_0 = \frac{\hat{\sigma}_\varepsilon^2}{1-\hat{\varphi}^2}$$

$$s = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}\left(res_t - \overline{res}\right)^2} = 98455.72$$

$$\Rightarrow \hat{\sigma}_\varepsilon^2 = 98455.72^2\left(1-0.8666^2\right) = 2413731710.65$$

$$\Rightarrow \hat{\sigma}_\varepsilon = 49129.74$$

$$\hat{e}_t = 54056.46$$

The observed value closely matches the expected value.

Lastly the Ljung-Box test is performed and with no p-values falling below the 5% significance level there is not sufficient evidence to indicate the residuals are anything other than white noise.

Table 3:

k=Lag	Ljung-Box Q*	Ljung-Box Q* P-value	Reject at 95%
1	1.5761		
2	1.5926	0.2069	No
3	1.9005	0.3866	No
4	1.9856	0.5754	No
5	2.5376	0.6379	No
6	2.7494	0.7385	No
7	3.8576	0.6959	No
8	3.9359	0.7871	No
9	3.9615	0.8606	No
10	4.0383	0.9089	No
11	7.6331	0.6646	No
12	14.1196	0.2264	No
13	14.8870	0.2477	No
14	16.1522	0.2410	No
15	16.1662	0.3033	No
16	16.2895	0.3631	No
17	16.4222	0.4239	No
18	16.4974	0.4889	No
19	16.9627	0.5257	No
20	16.9761	0.5915	No
21	20.3585	0.4357	No
22	20.4142	0.4952	No
23	26.1146	0.2468	No
24	26.1602	0.2934	No
25	26.1648	0.3449	No
26	26.1651	0.3988	No
27	26.2288	0.4506	No
28	26.6995	0.4801	No
29	27.6304	0.4842	No
30	27.9915	0.5184	No
31	28.0406	0.5683	No
32	28.3666	0.6022	No
33	31.7569	0.4788	No
34	32.8249	0.4758	No
35	34.0208	0.4667	No

$$Q^* = n*(n+2)\left(\frac{\hat{r}_1}{n-1}+\frac{\hat{r}_2}{n-2}+\ldots+\frac{\hat{r}_k}{n-k}\right)$$

$$P-value = 1-\chi^2\left(Q^*, df = k-1\right)$$

CONCLUSION:

Comparing the modeled data in Figure VI with Figure XX (which is Figure XVI on a per-resident basis) it's evident Model 2 more closely tracks actual spending. While the actual 2009 value fell within the 95% bounds in Model 1, whereas it did not in Model 2, this is largely due to the fact that the significance bounds are much wider for Model 1 ($2624 vs $691) not that the actual projection was notably more accurate. Furthermore, looking at actual outlays for 2013 being outside the significance bounds with the trend looking further downward it appears there is cause to doubt the accuracy of Model 1 in the future. In contrast the AR component in Model 2 should capture the downward trend.

Figure XX:

Even with the more accurate and sophisticated approach in Model 2 updates will definitely be necessary going forward, though. Just as in the past, wars, demographics, and the role of government will continue to evolve and thus no single model is appropriate forever.