TS Module 4: Variance of mean homework assignment

(The attached PDF file has better formatting.)

Homework assignment: Variance of mean

An MA(2) process $Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}$ has N observations, with $\sigma^2_e = 1$, $-1 \le \theta_1 \le +1$, $-1 \le \theta_2 \le +1$.

A. What values of θ_1 and θ_2 maximize the variance of $\bar{y}$, the average of the Y values?
B. What values of θ_1 and θ_2 minimizes the variance of $\bar{y}$, the average of the Y values?

Your answer should give a line of values for each part, such as $\theta_1 + \theta_2 = k$.

Jacob: How should we reason through this homework assignment?

Rachel: Write the value of $\bar{y}$ in terms of the ϵ's: $\sum y_j = \epsilon_n + (1 - \theta_1) \epsilon_{n-1} + (1 - \theta_1 - \theta_2) \epsilon_{n-2} + \ldots$

Most of the terms have $(1 - \theta_1 - \theta_2) \epsilon_{n-2}$; only the two terms at the beginning and the two terms at the end have fewer θ's. Ignore these beginning and end terms (assuming n is large).

All the ϵ's are independent. We choose θ_1 and θ_2 to maximize or minimize $(1 - \theta_1 - \theta_2) \epsilon_{n-2}$, which is easy.

For the homework assignment, ignoring the end terms is fine. If N is small, the answer differs slightly, and the calculations are messy.