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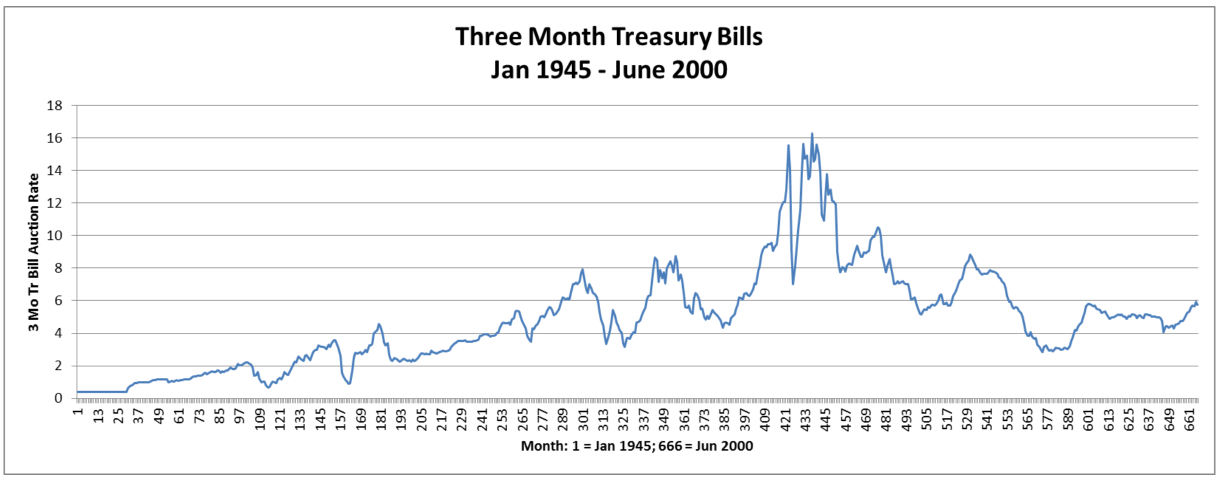
Time Series Project: Interest Rates

Part 1: Data and Objective

This project covers the stability of nominal interest rates and real interest rates. For nominal interest rates part, I used data from January 1945 to June 2000. According to the trend of the nominal interest rates and background information about interest rates policies, the whole period is finally divided into three sub-periods: from January 1945 to May 1979, the nominal interest rates kept an increasing trend; from May 1979 to October 1982, the nominal interest rates varied frequently; from October 1982 to June 2000, the nominal interest rates kept decreasing.



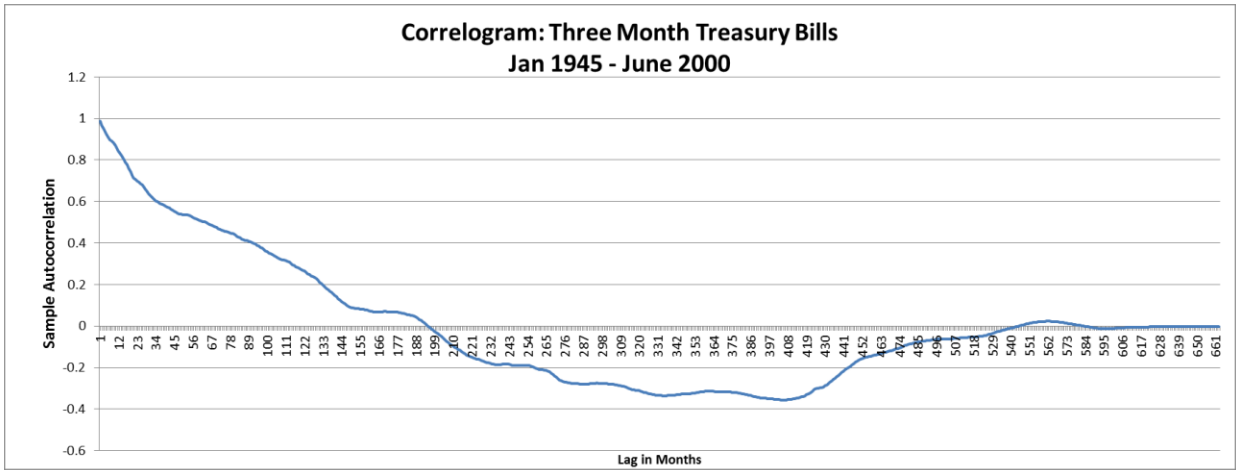
I took the nominal interest rates along with actual inflation (calculated from CPI index) and calculated the real interest rates. This was done by subtracting the inflation rate from the nominal interest rate to remove the impact of inflation.

Part 2: Techniques and Results

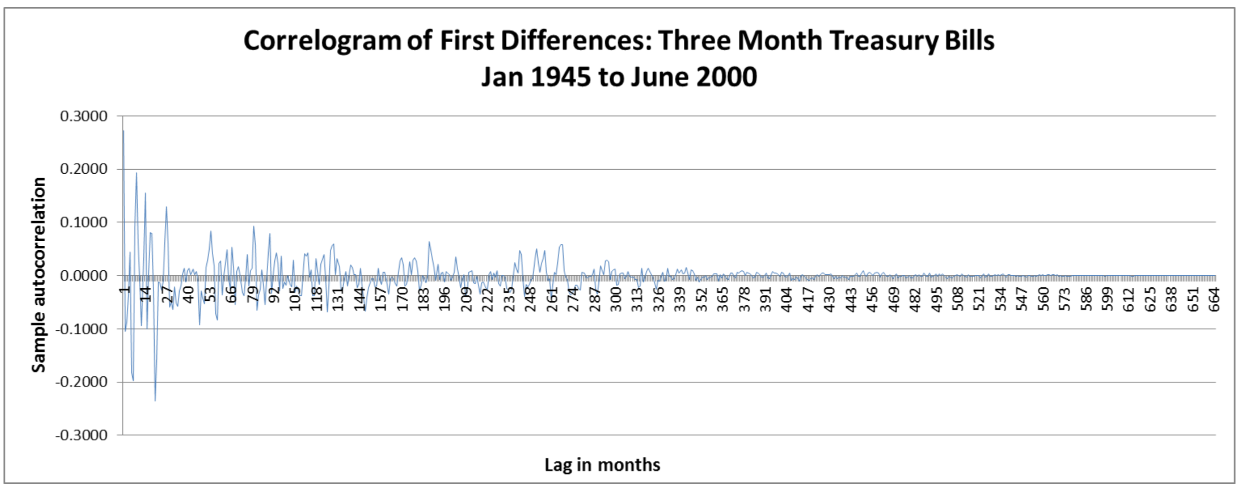
Using the techniques described in the instructions, I calculated the sample ACF, got the correlogram and did the goodness-of-fit tests.

Nominal Interest Rates

The correlogram for three month treasury bills from January 1945 to June 2000 is shown below. According to the correlogram, the sample autocorrelations are positive for 196 lags, negative for 349 lags, positive for 38 lags, and then negative for the remaining lags. It suggests that the nominal interest rates is a non-stationary time series.



In order to obtain a stationary time series, take the first differences of the nominal interest rates. The correlogram is displayed below. With 665 obeservations, the standard deviation of a white noise process is 1/sqrt(665)=3.88%. For the first 20 lags, there are many values of sample autocorrelations greater than two standard deviations. Meanwhile, there is no clear pattern of this correlogram. Therefore, it is not good enough to fit an ARIMA model to the first differences.

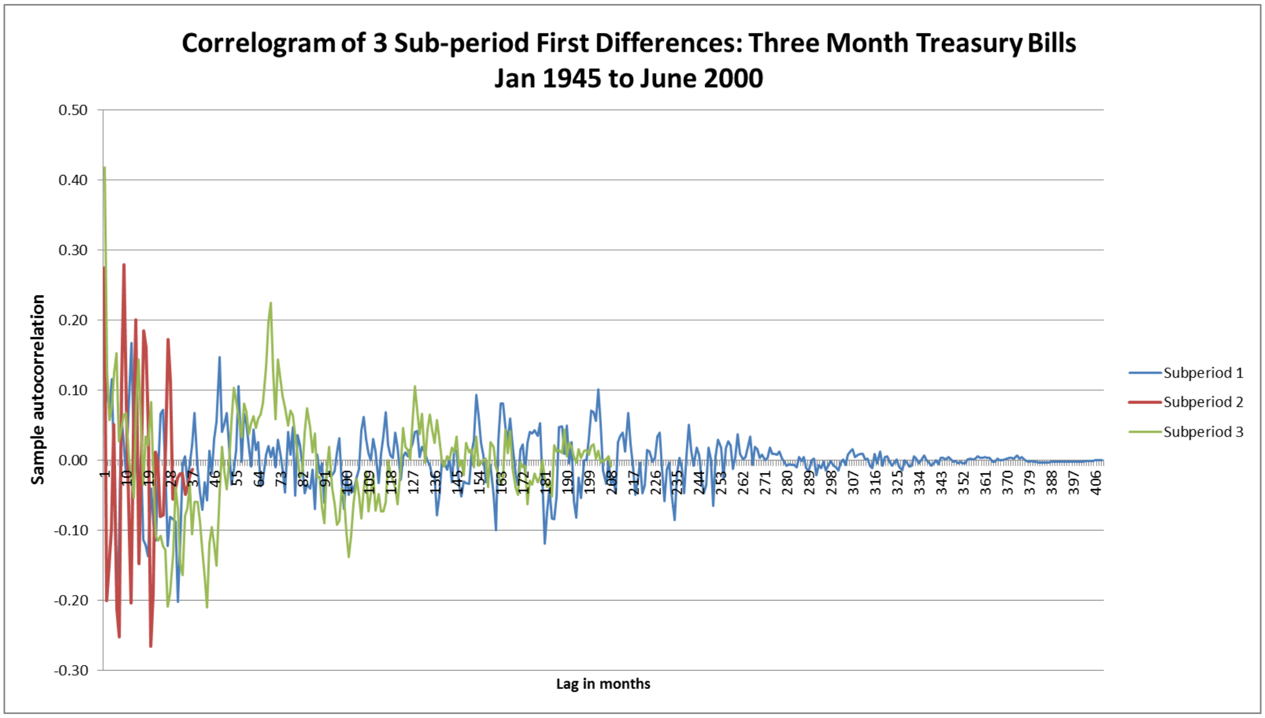


Considering exogenous factors and the general trend of interest rates of the three month treasury bills, the whole period is divided into three sub-periods as discussed in Part 1. The correlogram of three sub-periods after taking the first differences is showed below.

Period 1(January 1945 to May 1979): The auto-correlation drops off dramatically immediately and finally hovers around zero. There are 409 observations and the standard deviation is 4.9%. Most sample auto-correlations are below two standard deviations. This is evidence that the first difference of the nominal interest rate (AR(1)) is stationary. By Box-Pierce Q statistics, the Q-statistic is approximately distributed as a chi-square distribution with 408 degrees of freedom. The p-value for all Q-statistics is 1. By Bartlett's test, the 95% confidence interval is (-0.0969,0.0969) and the sample auto-correlation of lag 1 is 0.2190. Therefore, it is evident that we can reject the null hypotheses that the autocorrelation coefficient equals zero.

Period 2 (May 1979 to October 1982): There are 37 observations and the standard deviation is 16.44%. There is no auto-correlation exceeding two standard deviations. The second period is volatile and has much less data than the other two periods. Therefore, I think the model of this period will be less perfect than the other two periods. By Box-Pierce Q statistics, the p-value will drop below 95% after lag 19.

Period 3 (October 1982 to June 2000): For the first ten lags, the auto-correlation drops off dramatically immediately from lag 1 to lag 2 and then has a declining trend. The auto-correlation finally hovers around zero. There are 211 observations and the standard deviation is 6.9%. Most of them are under two standard deviations. It is appropriate to simulate the first difference of the nominal interest rate with AR(1). By Box-Pierce Q statistics, the p-value is lower than 95% after lag 68. By Bartlett's test, the 95% confidence interval is (-0.1349, 0.1349) and the sample auto-correlation of lag 1 is 0.4174. Therefore, it is evident that we can reject the null hypotheses that the autocorrelation coefficient equals zero.

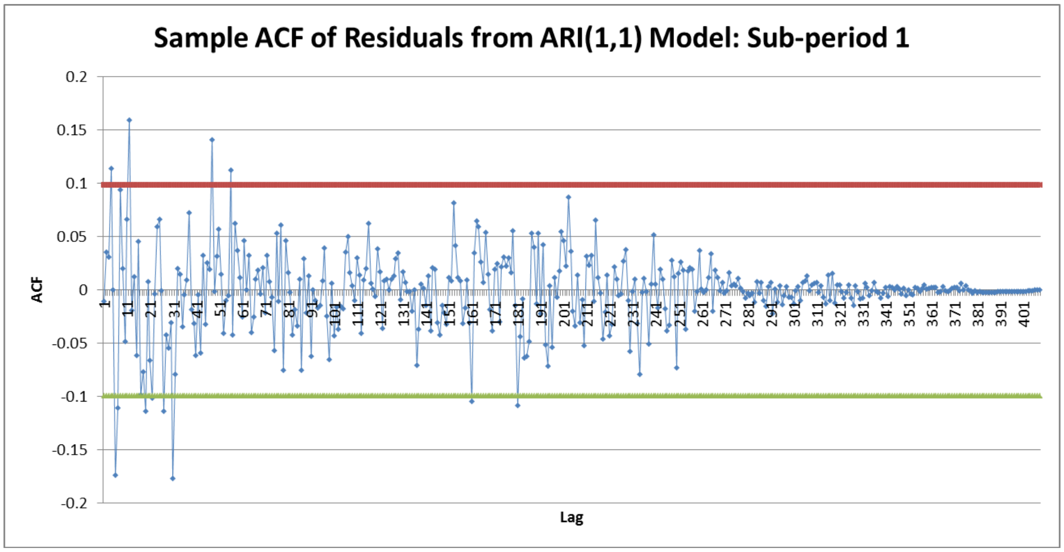


The fitted results for Sub-period 1:

$$M_t=0.017528+0.21908*M_{t-1}$$
$$M_t=Y_t-Y_{t-1}$$
$$M_{t-1}=Y_{t-1}-Y_{t-2}$$

Where, Y_t is the nominal interest rate.

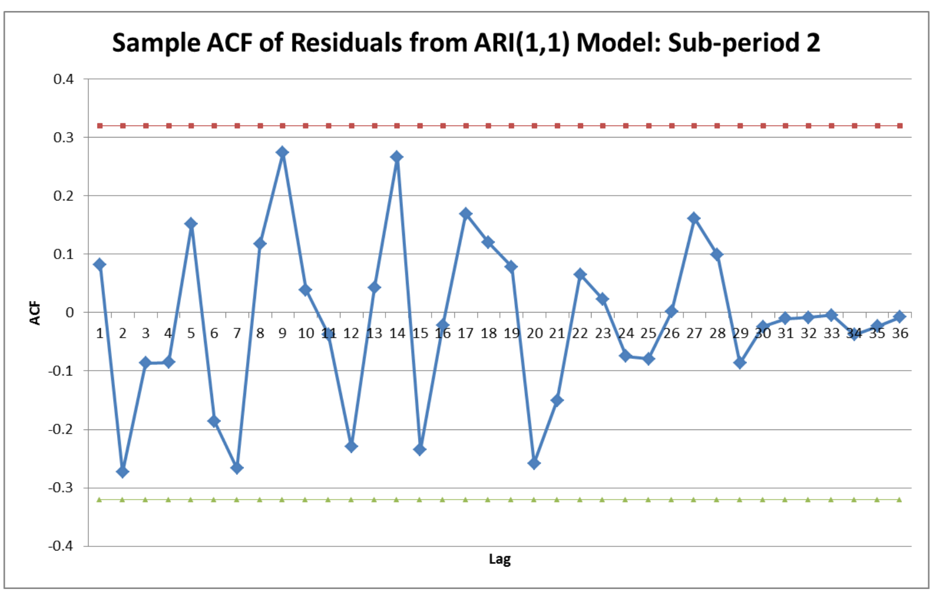
The R^2 for this ARI(1,1) model is 4.8%, while the R^2 for this ARI(2,1) model is 5.0%. There is not a great improvement in R^2, therefore, I choose to use ARI(1,1) to fit all these model. The p-value of M_{t-1} is 7.38705E-06 which means that this coefficient is significantly different from zero. The residuals are calculated by Excel. The sample ACF of residual is displayed below. It can be seen that most of them are below two standard deviations. There are 411 observations so I had 410 degrees of freedom for Box-Pierce Q statistics. For the whole range from K=1 to 408, the p-value is around 1, so we have no evidence to reject the null hypothesis that the error terms are uncorrelated.



The fitted results for Sub-period 2:

$$M_t = -0.03297 + 0.2754 * M_{t-1}$$
$$M_t = Y_t - Y_{t-1}$$
$$M_{t-1} = Y_{t-1} - Y_{t-2}$$
Where, Y_t is the nominal interest rate.

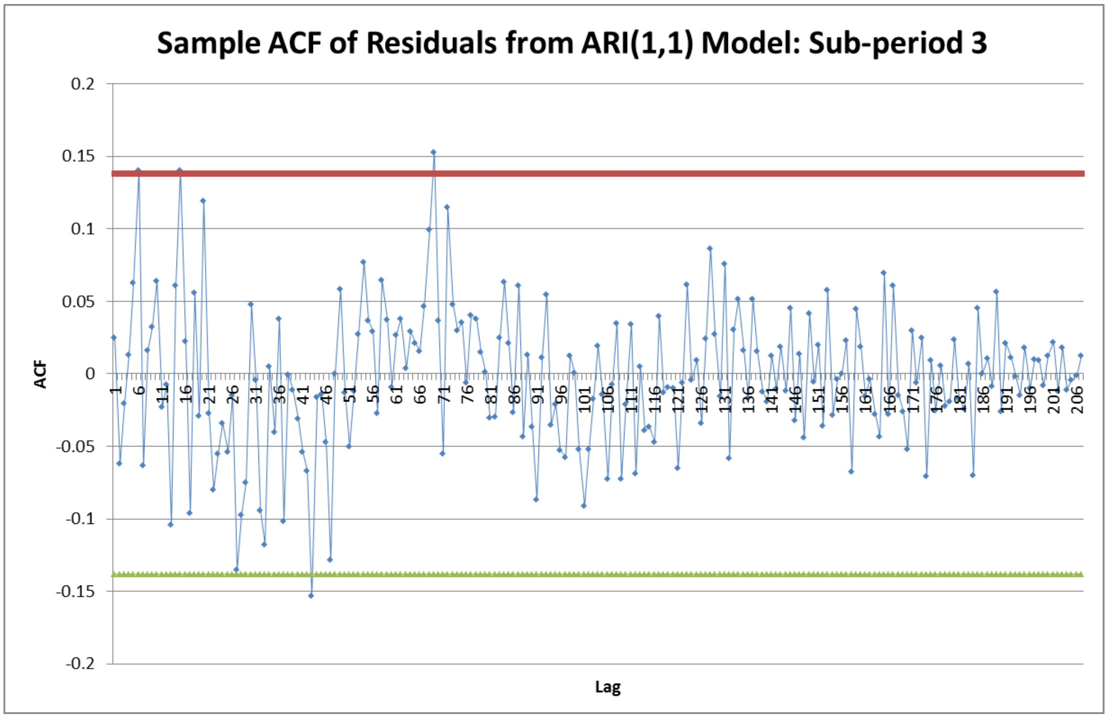
The R^2 for this ARI(1,1) model is 7.6%. The p-value of M_{t-1} is 0.08987 which means that this coefficient is not significantly different from zero. The residuals are calculated by Excel. The sample ACF of residual is displayed below. It can be seen that all of them are below two standard deviations. There are 39 observations so I had 38 degrees of freedom for Box-Pierce Q statistics. For the whole range from K=1 to 21, the p-values are well above 0.95, so we have no evidence to reject the null hypothesis that the error terms are uncorrelated.



The fitted results for Sub-period 3:

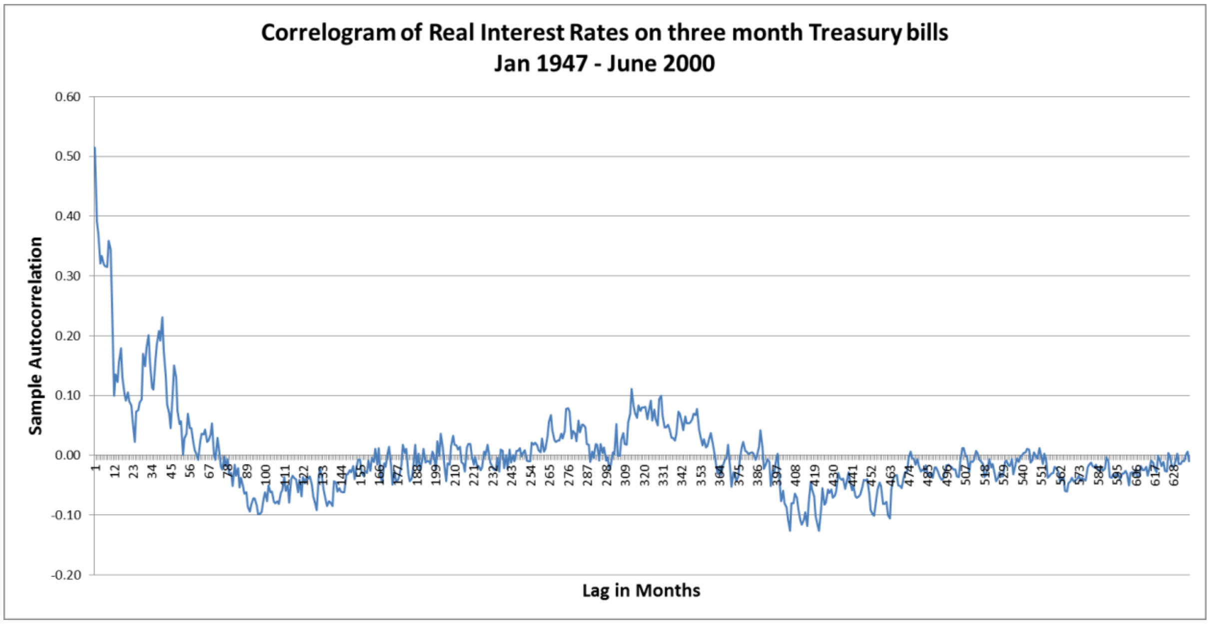
$$M_t = -0.0066 + 0.41837 * M_{t-1}$$
$$M_t = Y_t - Y_{t-1}$$
$$M_{t-1} = Y_{t-1} - Y_{t-2}$$
Where, Y_t is the nominal interest rate.

The R^2 for this ARI(1,1) model is 7.6%. The p-value of M_{t-1} is 2.78352E-10 which means that this coefficient is significantly different from zero. The residuals are calculated by Excel. The sample ACF of residual is displayed below. It can be seen that most of them are below two standard deviations. There are 210 observations so I had 209 degrees of freedom for Box-Pierce Q statistics. For the whole range, the p-values are around 1, so we have no evidence to reject the null hypothesis that the error terms are uncorrelated.

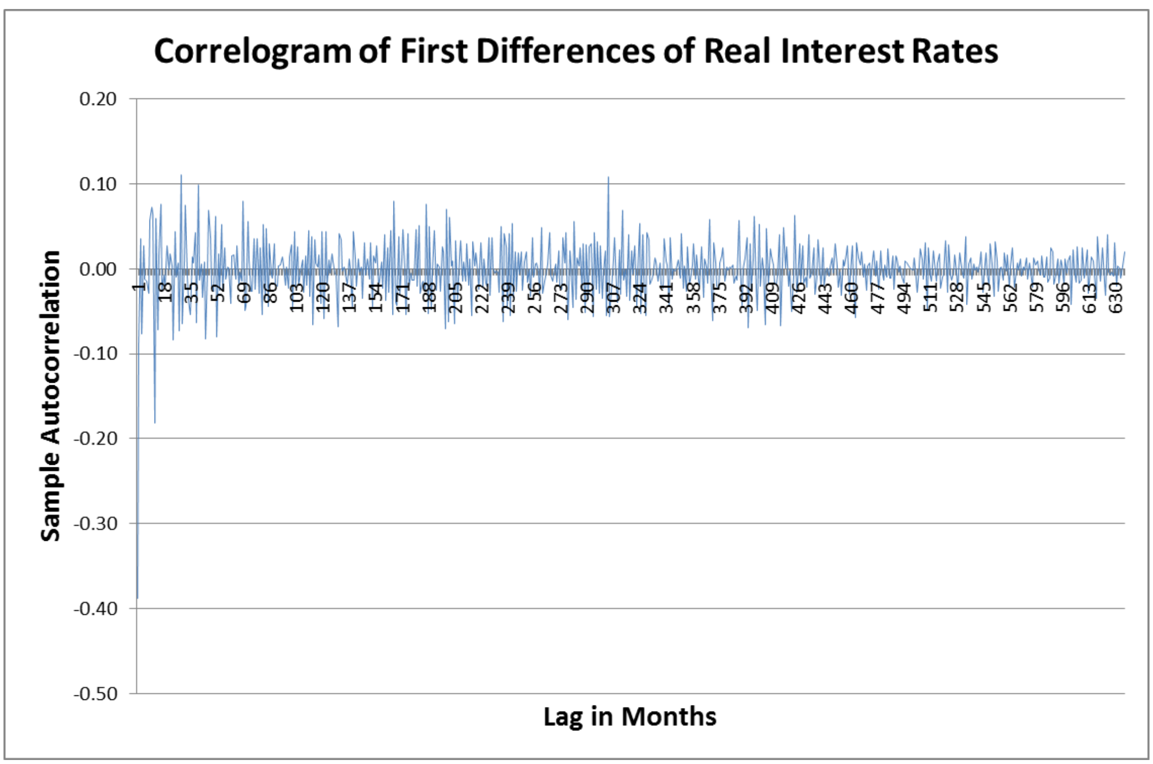


Real Interest Rates

Correlogram of real interest rates from January 1947 to June 2000 is displayed below. Sample autocorrelations are firstly positive with declining trend, then negative, positive, negative though values are close to zero. The series is not stationary.



The correlogram of first differences of real interest rates is showed below. The two standard deviations is 8%. The sample autocorrelation of lag 1 is -38.8%, suggesting that we should take first differences of a stationary autoregressive process. The following sample autocorrelations hover around zero to be a white noise process.



The fitted results for real interest rates model:

$$M_t = 0.043 - 0.3882 * M_{t-1}$$
$$M_t = Y_t - Y_{t-1}$$
$$M_{t-1} = Y_{t-1} - Y_{t-2}$$
Where, Y_t is the nominal interest rate.

The R^2 for this ARI(1,1) model is 15.3%. The p-value of M_{t-1} is 9.95861E-25 which means that this coefficient is significantly different from zero. The residuals are calculated by Excel. The sample ACF of residual is displayed below. It can be seen that most of them are below two standard deviations. There are 638 observations so I had 637 degrees of freedom for Box-Pierce Q statistics. For the whole range, the p-values are well above 0.99, so we have no evidence to reject the null hypothesis that the error terms are uncorrelated.