Time Series Analysis of Final Jeopardy Questions

Introduction

It is rare that a television program not only stretches beyond three decades, but continues to enthral audiences of all ages with various backgrounds, interests, and intellects. Since 1984 Alex Trebek has entered our hearts and living rooms to praise contestants on correct responses while smugly fixing their errors.

Many interesting Jeopardy time series could be modelled. One that easily comes to mind would be for the expected nightly winnings since each episode's champion remains and can last weeks before being eliminated—a clear correlation between neighbouring payouts.

However, my interest in fitting Jeopardy data to autoregressive integrated moving average (ARIMA) models is more selfish. With the high-level of game theory involved, I believe the most interesting part of the game is Final Jeopardy when contestants wager any amount of their daily purse to surpass their two opponents and be crowned champion. Since I began regularly watching during university, I have always felt confident at the unveiling of certain Final Jeopardy categories, while other categories guarantee my struggle through a clueless reasoning process as the theme song whistles away.

It is well known by the Jeopardy faithful that certain categories have higher repetition rates than others, and I think it will be interesting to explore ARIMA(p,d,q) models for the annual percentages of Final Jeopardy questions falling in popular categories. If a suitable model exists, I will be able to predict the successive years' percentage for that Final Jeopardy category.

Data

I obtained the database, filled with 216,931 lines from Jeopardy's history, on Reddit's web site (http://www.reddit.com/r/datasets/comments/1uyd0t/200000_jeopardy_questions_in_a_json_file/). With the help of archived episodes the available data includes the show number, date the show aired, round of the clue (Jeopardy, Double Jeopardy, or Final Jeopardy), category name, value of the clue, question/clue, and the correct answer.

Based on the interest of this time series model (and time), I first extracted only the lines associated with the Final Jeopardy round. I then proceeded to sort the remaining 3,631 lines into broader groupings than what the database listed in the category column. Although not from the exact same-named category, certain clues should generally be grouped together: 'WORLD CITIES' and 'THE 50 STATES', 'AMERICAN LITERATURE' and 'FICTIONAL CHARACTERS', 'PUNCTUATION' and 'ANAGRAMS', 'ROMAN EMPERORS' and 'HISTORIC NAMES', etc. The

categories, question/clues, and answers from the original database were taken into consideration during this further grouping process. This process sifted the 1,948 unique Final Jeopardy categories into the 12 broader categories listed here:

- Famous Individuals & Leaders

- Geography, Places & Landmarks

- History & Empires

- Language & Words

- Literature, Poetry & Authors

- Religion, Mythology & the Bible

- Science, Industry & Organizations

- Sports & Games

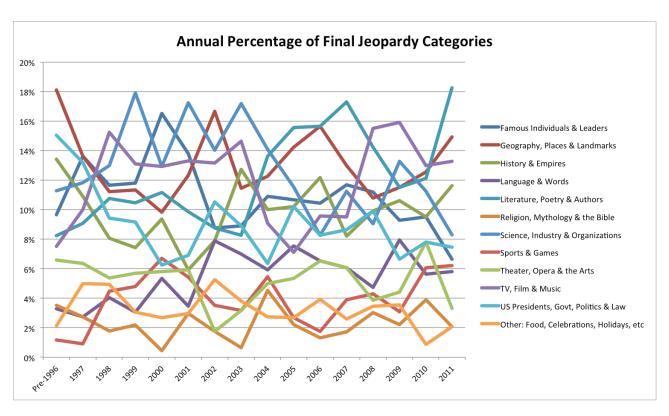
- Theater, Opera & the Arts

- TV, Film & Music

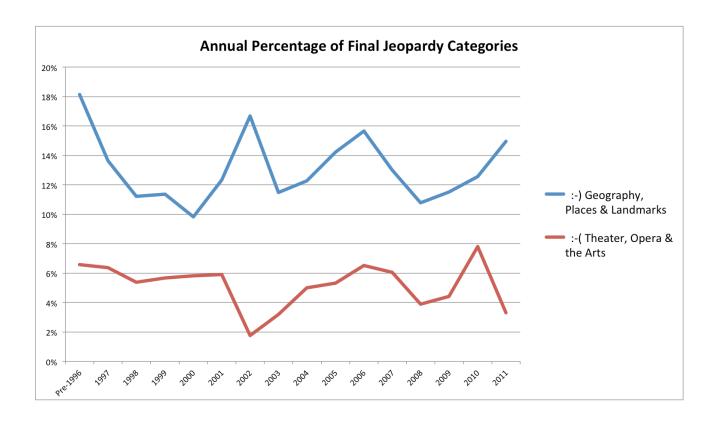
- U.S. Presidents, Govt, Politics & Law

- Other: Food, Celebrations, Holidays, etc.

The original intention was to track how many Final Jeopardy questions fell into each of these broader categories on an annual basis dating back to 1984. However, a problem quickly became apparent. The number of episodes per year increased as the show gained in popularity, so the pure number of Final Jeopardy questions asked is naturally skewed towards the later years. Further, archived episodes are more difficult to locate from the earliest years, thus contributing to the data being heavily weighted towards the later years. The solution was to focus on the annual percentages of each category instead of the raw number of questions. Also, combining the years prior to 1997 eliminated the need to rely on the limited data (~20 data pts/year) in calculating percentages for years from 1984 to 1996.



Once again selfishly, I will focus on building a model for my favourite and least favourite categories, 'Geography, Places & Landmarks' and 'Theater, Opera & the Arts' respectively. Side note: these categories may or may not also be my most and least successful.



Test of Stationarity and Adjustments (If Necessary)

For each category, I must examine the data to see if we have a stationary time series, which is necessary to correctly model the time series. The test of stationarity will be based on autocorrelation function, which is the ratio of the sample covariance to the sample variance.

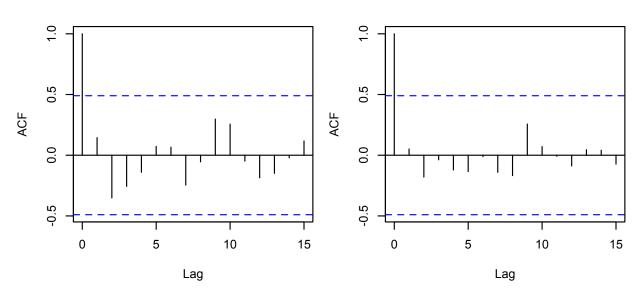
The autocorrelation formula used to estimate ρ_k can be expressed as:

$$r_{k} = \frac{\sum_{t=k+1}^{n} (Y_{t} - \overline{Y})(Y_{t-k} - \overline{Y})}{\sum_{t=1}^{n} (Y_{t} - \overline{Y})^{2}}$$

Instead of manually calculating and plotting the "Correlogram" graph of r_k versus k, it is easily graphed using R. Below are two graphs depicting the autocorrelation function for the categories 'Geography, Places & Landmarks' (denoted X) and 'Theater, Opera & the Arts' (denoted Y).

Autocorrelation Function X

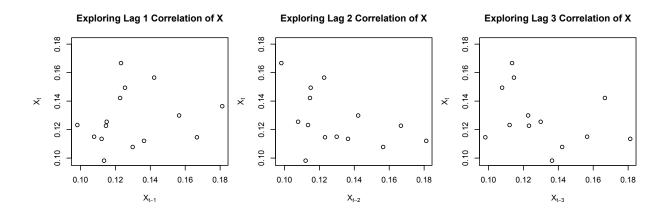
Autocorrelation Function Y

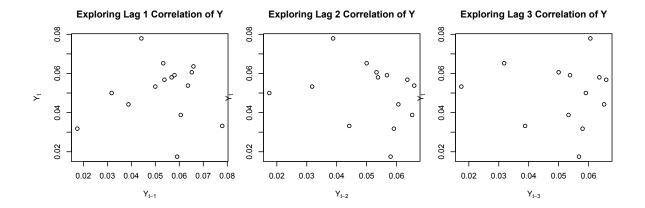


In order to suppose that after a certain amount of lag the result becomes white noise, the autocorrelation needs to be close to zero. All values are within the horizontal dashed lines placed at plus or minus two approximate standard errors of the sample autocorrelations. Since the autocorrelations oscillate around zero starting at lag 1 and continuing forever, the correlogram graphs very clearly indicate that each category can be considered a stationary process. No adjustments (differences, logarithms, diff of logs, transformations, etc.) are necessary to create a stationary process. I conclude that ARIMA(p,d,q) will have d=0, and we can move ahead with estimating the p and q in our ARIMA(p,0,q) model.

Model Estimation

Now that the stationarity of these time series have been determined, the question becomes which model is a best fit. The graphs of X_t vs. X_{t-1} , X_t vs. X_{t-2} , and X_t vs. X_{t-3} (Similarly for the Ys) give a general idea if AR(1), AR(2), or AR(3) models would be a good fit for the data. A correlation in these graphs would suggest an autoregressive model is appropriate, where as a general scatter with little correlation might insinuate a moving average model.





For 'Geography, Places & Landmarks' (X) one can see a slight positive correlation in the Lag 1 graph and slight negative correlation in the Lag 2 graph, however neither are strong. The graphs for 'Theater, Opera & the Arts' (Y) are more randomly scattered with very little, if any, trend. I will proceed by fitting and testing AR(1), AR(2), AR(3) and MA(1) models for these data using R.

Model/Parameter Fitting

A pth-order autoregressive process and qth-order moving average model satisfy the equations:

Moving Average [MA(q)]

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + ... + \phi_p Y_{t-p} + \delta + e_t$$

$$Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - ... - \theta_q e_{t-q} + \delta$$

Note: δ is simply a constant corresponding to the intercept of the model.

Using R's built-in ARIMA(p,d,q) function I was able to find the values of the unknown parameters for the AR(1), AR(2), AR(3) and MA(1) models of X ('Geography, Places & Landmarks') and Y ('Theater, Opera & the Arts'). Please refer to the Appendix: R output.

X: Geography, Places & Landmarks								
	R ²	Adj R ²	φ1	ф2	ф3	θ_1	δ	Σφ
AR(1)	0.0299	-0.0447	0.2179	-	-	-	0.1321	0.2179
AR(2)	0.2551	0.1196	0.3053	-0.5283	1	-	0.1301	-0.2230
AR(3)	0.2562	0.0083	0.2050	-0.4759	-0.1944	-	0.1297	-0.4653
MA(1)	-	-	-	-	-	0.5483	0.1330	-

Y: Theater, Opera & the Arts								
	R^2	Adj R ²	ф1	ф2	ф3	θ_1	δ	Σφ
AR(1)	0.0038	-0.0728	0.0568	-	-	-	0.0518	0.0568
AR(2)	0.0578	-0.1135	0.1028	-0.2702	ı	ı	0.0513	-0.1674
AR(3)	0.0781	-0.2292	0.1244	-0.2898	0.0594	-	0.0514	-0.1060
MA(1)	-	-	-	-	-	0.1002	0.0518	-

Clearly none of the AR(p) models are stellar. In regards to the 'Geography, Places & Landmarks' category, one might conclude the AR(2) model is the best autoregressive option due to it having the highest coefficient of determination (R²) in combination with the principle of parsimony, which aims to simplify the model with the least amount of terms. That said, a model that only explains 25.5% of variation in a data set means it is not a good model for the data. None of the autoregressive models have any clout for the 'Theater, Opera & the Arts' category. Looking even further into the associated p-values of each AR(p) models' coefficients, it becomes clear to select the MA(1) models since the p-values are almost all considered insignificant (>0.05).

AR(p)	X : Geogra	phy, Places & L	andmarks	Y : Theater, Opera & the Arts		
р	p-value φ ₁	p-value φ ₂	p-value φ ₃	p-value φ ₁	p-value φ ₂	p-value φ ₃
1	0.4664	-	-	0.8282	1	-
2	0.2662	0.0284	-	0.6947	0.3457	-
3	0.4995	0.0542	0.4771	0.6624	0.3386	0.8458

Note: calculation of p-value ϕ_1 for X: z = 0.2179/0.2992_(s.e.) = 0.7283 → p-value = 0.4664

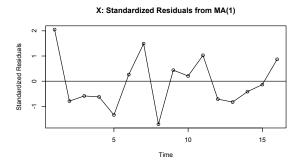
Residuals analysis

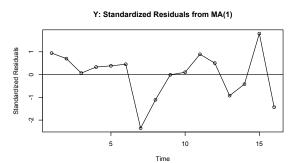
Since the Moving Average Model is selected for this data, the Durbin Watson and Box-Pierce Q test for the white noise of the residuals are unnecessary.

$$DWS = \frac{Summation \ of \ Residuals \ Difference \ Squared}{Summation \ of \ Residuals \ Squared} = \frac{\sum (\varepsilon_t - \varepsilon_{t-1})^2}{\sum \varepsilon_t^2}$$

$$Box - PierceQ = T * \sum\nolimits_{k=1}^{K} \widehat{r_k}^2$$

Looking at the graphs of the standardized residuals of the MA(1) models below, there is no apparent trends and hence the plots generally support the models. The residuals have the near properties of a stochastic white noise process.





Forecasting and Conclusion

Autoregressive models are best for time series hinting towards long-term dependencies. Since the annual percentage of Final Jeopardy questions asked within a certain category is completely arbitrary (subject to the show's question developers), it is reasonable that no autoregressive model fits this data well. These conclusions make me wish that reliable historical data was readily available for the nightly Jeopardy payouts (contestant-winnings/show-losses) as discussed in the introduction. This data could lead to a more interesting ARIMA model.

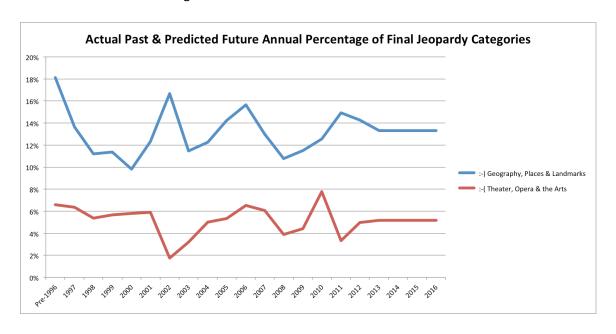
Although the percentages seem to be arbitrary from year to year, I will still predict them for the next 5 years from the final data in 2011. To compute I used the following MA(1) formulas:

$$X_t = e_t - 0.5483e_{t-1} + 0.1330$$
 and $Y_t = e_t - 0.1002e_{t-1} + 0.0518$

The predicted future percentages are italicized in the chart below and can also be found in the Appendix: R output.

Year	X: Geography, Places & Landmarks	Y: Theater, Opera & the Arts
Pre-1996	18.12%	6.59%
1997	13.63%	6.36%
2011	14.94%	3.32%
2012	14.28%	4.97%
2013	13.30%	5.18%
2014	13.30%	5.18%
2015	13.30%	5.18%
2016	13.30%	5.18%

Below is a repeat graph now containing these predictions. Unfortunately it does not look like Jeopardy will eliminate questions regarding the 'Theater, Opera & the Arts'. On the bright side, there will continue to be a higher percentage of questions from the 'Geography, Places & Landmarks' associated categories.



Appendix: R output

```
> library(TSA)
> getwd()
[1] "/Users/Owner"
> setwd("~/Desktop/VEE Time Series/Projects")
> data <- read.table("JEOPboth%.csv", header=TRUE, sep=",")</pre>
> data
        X
1 0.1812 0.0659
2 0.1364 0.0636
3 0.1121 0.0538
4 0.1135 0.0568
5 0.0982 0.0580
6 0.1232 0.0591
  0.1667 0.0175
8 0.1146 0.0318
9 0.1227 0.0500
10 0.1422 0.0533
11 0.1565 0.0652
12 0.1299 0.0606
13 0.1078 0.0388
14 0.1150 0.0442
15 0.1255 0.0779
16 0.1494 0.0332
> attach(data)
> dev.new(width=3, height=3, pointsize=8)
> plot(acf(X,lag.max= 15), xlab="Lag", ylab=NULL, main =
"Autocorrelation Function X")
> dev.new(width=3, height=3, pointsize=8)
> plot(acf(Y,lag.max= 15), xlab="Lag", ylab=NULL, main =
"Autocorrelation Function Y")
> dev.new(width=3,height=3,pointsize=8)
> plot(y=X, x=zlag(X,1), ylab=expression(X[t]), xlab=expression(X[t-
1]), type='p', main="Exploring Lag 1 Correlation of X")
> dev.new(width=3,height=3,pointsize=8)
> plot(y=X, x=zlag(X,2), ylab=expression(X[t]), xlab=expression(X[t-
2]), type='p', main="Exploring Lag 2 Correlation of X")
> dev.new(width=3,height=3,pointsize=8)
> plot(y=X, x=zlag(X,3), ylab=expression(X[t]), xlab=expression(X[t-
3]), type='p', main="Exploring Lag 3 Correlation of X")
> dev.new(width=3,height=3,pointsize=8)
> plot(y=Y, x=zlag(Y,1), ylab=expression(Y[t]), xlab=expression(Y[t-
1]), type='p', main="Exploring Lag 1 Correlation of Y")
> dev.new(width=3,height=3,pointsize=8)
> plot(y=Y, x=zlag(Y,2), ylab=expression(Y[t]), xlab=expression(Y[t-
2]), type='p', main="Exploring Lag 2 Correlation of Y")
> dev.new(width=3,height=3,pointsize=8)
> plot(y=Y, x=zlag(Y,3), ylab=expression(Y[t]), xlab=expression(Y[t-
3]), type='p', main="Exploring Lag 3 Correlation of Y")
```

```
> arima(X, order=c(1,0,0))
arima(x = X, order = c(1, 0, 0))
Coefficients:
        arl intercept
      0.2179
             0.1321
s.e. 0.2992
                0.0071
sigma^2 estimated as 0.0004767: log likelihood = 38.46, aic = -72.92
>
> arima(X, order=c(2,0,0))
Call:
arima(x = X, order = c(2, 0, 0))
Coefficients:
        ar1
                 ar2 intercept
      0.3053 -0.5283
                         0.1301
s.e. 0.2746 0.2411
                         0.0042
sigma^2 estimated as 0.0003621: log likelihood = 40.34, aic = -74.68
>
> arima(X, order=c(3,0,0))
Call:
arima(x = X, order = c(3, 0, 0))
Coefficients:
        ar1
                 ar2
                          ar3 intercept
      0.2050 - 0.4759 - 0.1944
                                  0.1297
s.e. 0.3036
             0.2472
                      0.2734
                                  0.0035
sigma^2 estimated as 0.0003482: log likelihood = 40.58, aic = -73.16
>
>
> arima(X, order=c(0,0,1))
arima(x = X, order = c(0, 0, 1))
Coefficients:
        mal intercept
      0.5483
             0.1330
                0.0079
s.e. 0.2483
sigma^2 estimated as 0.0004258: log likelihood = 39.21, aic = -74.42
>
>
>
```

```
> arima(Y,order=c(1,0,0))
arima(x = Y, order = c(1, 0, 0))
Coefficients:
        arl intercept
      0.0568
             0.0518
s.e. 0.2618
                0.0039
sigma^2 estimated as 0.0002213: log likelihood = 44.62, aic = -85.25
>
>
> arima(Y,order=c(2,0,0))
Call:
arima(x = Y, order = c(2, 0, 0))
Coefficients:
        ar1
                 ar2 intercept
      0.1028 -0.2702
                         0.0513
s.e. 0.2620 0.2866
                         0.0032
sigma^2 estimated as 0.0002079: log likelihood = 45.05, aic = -84.09
>
> arima(Y,order=c(3,0,0))
Call:
arima(x = Y, order = c(3, 0, 0))
Coefficients:
                         ar3 intercept
        ar1
                 ar2
      0.1244 -0.2898 0.0594
                                 0.0514
             0.3029 0.3055
s.e. 0.2849
                                 0.0034
sigma^2 estimated as 0.0002071: log likelihood = 45.06, aic = -82.13
>
>
> arima(Y, order=c(0,0,1))
arima(x = Y, order = c(0, 0, 1))
Coefficients:
        mal intercept
      0.1002
             0.0518
s.e. 0.3591
                0.0041
sigma^2 estimated as 0.0002207: log likelihood = 44.64, aic = -85.28
>
>
>
>
```

```
> modelXAR1 <- lm(X ~ zlag(X,1) +1)
> summary(modelXAR1)
Call:
lm(formula = X \sim zlag(X) + 1)
Residuals:
      Min
                 1Q
                       Median
                                     3Q
                                              Max
-0.027052 -0.013997 -0.001531 0.008523 0.040054
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
                                  3.646 0.00296 **
                       0.02988
(Intercept) 0.10894
                                  0.633 0.53772
                        0.22703
zlaq(X)
             0.14371
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.01971 on 13 degrees of freedom
  (1 observation deleted due to missingness)
Multiple R-squared: 0.0299, Adjusted R-squared: -0.04472
F-statistic: 0.4007 on 1 and 13 DF, p-value: 0.5377
>
>
>
> modelXAR2 <- lm(X \sim zlag(X,1) + zlag(X,2) +1)
> summary(modelXAR2)
Call:
lm(formula = X \sim zlag(X, 1) + zlag(X, 2) + 1)
Residuals:
      Min
                 10
                       Median
                                     30
-0.033249 -0.007999 -0.001594 0.012569 0.027273
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
                                        0.0034 **
(Intercept) 0.15152 0.04077
                                3.717
                                 0.833
                                          0.4228
zlaq(X, 1)
             0.23089
                        0.27734
zlag(X, 2) -0.41285
                        0.21918 - 1.884
                                          0.0863 .
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.01863 on 11 degrees of freedom
  (2 observations deleted due to missingness)
Multiple R-squared: 0.2551, Adjusted R-squared: 0.1196
F-statistic: 1.883 on 2 and 11 DF, p-value: 0.198
>
>
>
```

```
> modelXAR3 <- lm(X \sim zlag(X,1) + zlag(X,2) + zlag(X,3) +1)
> summary(modelXAR3)
lm(formula = X \sim zlag(X, 1) + zlag(X, 2) + zlag(X, 3) + 1)
Residuals:
      Min
                 1Q
                       Median
                                     3Q
-0.033121 -0.008847 -0.000265 0.012861 0.024652
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
                     0.06654
                                  2.780
(Intercept) 0.18496
                                         0.0214 *
zlaq(X, 1)
            0.12887
                       0.33880
                                 0.380
                                          0.7125
zlag(X, 2)
           -0.40303
                       0.30874 -1.305
                                          0.2241
zlag(X, 3) -0.16924
                       0.27199 - 0.622
                                          0.5492
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.02009 on 9 degrees of freedom
  (3 observations deleted due to missingness)
Multiple R-squared: 0.2562, Adjusted R-squared: 0.008278
F-statistic: 1.033 on 3 and 9 DF, p-value: 0.4232
>
>
> modelYAR1 <- lm(Y \sim zlag(Y,1) +1)
> summary(modelYAR1)
Call:
lm(formula = Y \sim zlag(Y, 1) + 1)
Residuals:
      Min
                 10
                      Median
                                     30
-0.033799 -0.009205 0.002576 0.008393 0.027542
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.04757 0.01562
                                  3.045
                                          0.0094 **
                                  0.223
zlag(Y, 1)
             0.06317
                        0.28374
                                          0.8273
___
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.016 on 13 degrees of freedom
  (1 observation deleted due to missingness)
Multiple R-squared: 0.003799, Adjusted R-squared: -0.07283
F-statistic: 0.04957 on 1 and 13 DF, p-value: 0.8273
```

```
> modelYAR2 < lm(Y \sim zlag(Y,1) + zlag(Y,2) +1)
> summary(modelYAR2)
Call:
lm(formula = Y \sim zlaq(Y, 1) + zlaq(Y, 2) + 1)
Residuals:
                 1Q
                      Median
-0.031153 -0.007961 0.002519 0.010164 0.024974
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.06056
                     0.02111
                                2.869
                                         0.0153 *
zlag(Y, 1)
           0.07108
                       0.30993
                                0.229
                                         0.8228
zlag(Y, 2) -0.27774
                       0.33834 -0.821
                                         0.4291
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.01648 on 11 degrees of freedom
  (2 observations deleted due to missingness)
Multiple R-squared: 0.0578, Adjusted R-squared: -0.1135
F-statistic: 0.3374 on 2 and 11 DF, p-value: 0.7208
>
>
> modelYAR3 <- lm(Y \sim zlag(Y,1) + zlag(Y,2) + zlag(Y,3) +1)
> summary(modelYAR3)
Call:
lm(formula = Y \sim zlag(Y, 1) + zlag(Y, 2) + zlag(Y, 3) + 1)
Residuals:
      Min
                  10
                         Median
-0.0305192 -0.0108302 0.0004322 0.0110347
      Max
0.0236959
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
                                         0.0929 .
                                1.880
(Intercept) 0.05855 0.03115
                       0.37316
                                0.226
zlag(Y, 1)
            0.08418
                                         0.8266
zlag(Y, 2) -0.36727
                       0.42829 - 0.858
                                         0.4134
zlag(Y, 3)
           0.10198
                       0.42396
                                0.241
                                         0.8153
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.01797 on 9 degrees of freedom
  (3 observations deleted due to missingness)
Multiple R-squared: 0.07808, Adjusted R-squared: -0.2292
F-statistic: 0.2541 on 3 and 9 DF, p-value: 0.8566
>
```

```
> modelXMA1 = arima(X,order=c(0,0,1))
> dev.new(width=5,height=3,pointsize=8)
> plot(rstandard(modelXMA1), ylab='Standardized Residuals', type='o',
main="X: Standardized Residuals from MA(1)"); abline(h=0)
> modelYMA1 = arima(Y,order=c(0,0,1))
> dev.new(width=5,height=3,pointsize=8)
> plot(rstandard(modelYMA1), ylab='Standardized Residuals', type='o',
main="Y: Standardized Residuals from MA(1)"); abline(h=0)
>
>
> predict(arima(X,order=c(0,0,1)),n.ahead=5)
$pred
Time Series:
Start = 17
End = 21
Frequency = 1
[1] 0.1428137 0.1329763 0.1329763 0.1329763
[5] 0.1329763
$se
Time Series:
Start = 17
End = 21
Frequency = 1
[1] 0.02063584 0.02353448 0.02353448 0.02353448
[5] 0.02353448
> predict(arima(Y,order=c(0,0,1)),n.ahead=5)
$pred
Time Series:
Start = 17
End = 21
Frequency = 1
[1] 0.04967142 0.05180394 0.05180394 0.05180394
[5] 0.05180394
$se
Time Series:
Start = 17
End = 21
Frequency = 1
[1] 0.01485645 0.01493085 0.01493085 0.01493085
[5] 0.01493085
```