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Course : Time Series
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Introduction

Global temperature is the popular topic what government scientists concern. There are a lot of news told about the relationship of global temperature change and sea level rise. Since many country are a little sea island, if the sea level continue rising up, they may flood by rising sea levels. Due to this reason, we want to know how the global temperature changing.

Description of Data

Our data is from National Oceanic and Atmospheric Administration (NOAA), National Climatic Data Center. The first data is monthly global land temperature anomalies (degrees C). The second data is the monthly global ocean temperature anomalies (degrees C). "Temperature anomaly" means a departure from a reference value or long-term average. A positive anomaly indicates that the observed temperature was warmer than the reference value, while a negative anomaly indicates that the observed temperature was cooler than the reference value. Our data is during 1915 to 2014 year.

Analysis of Data

I. Analysis of Global Land Temperature Anomalies Data

We analyze land temperature data as follows. First, we observe time series plot, SACF, SPACF, EACF of the raw data:

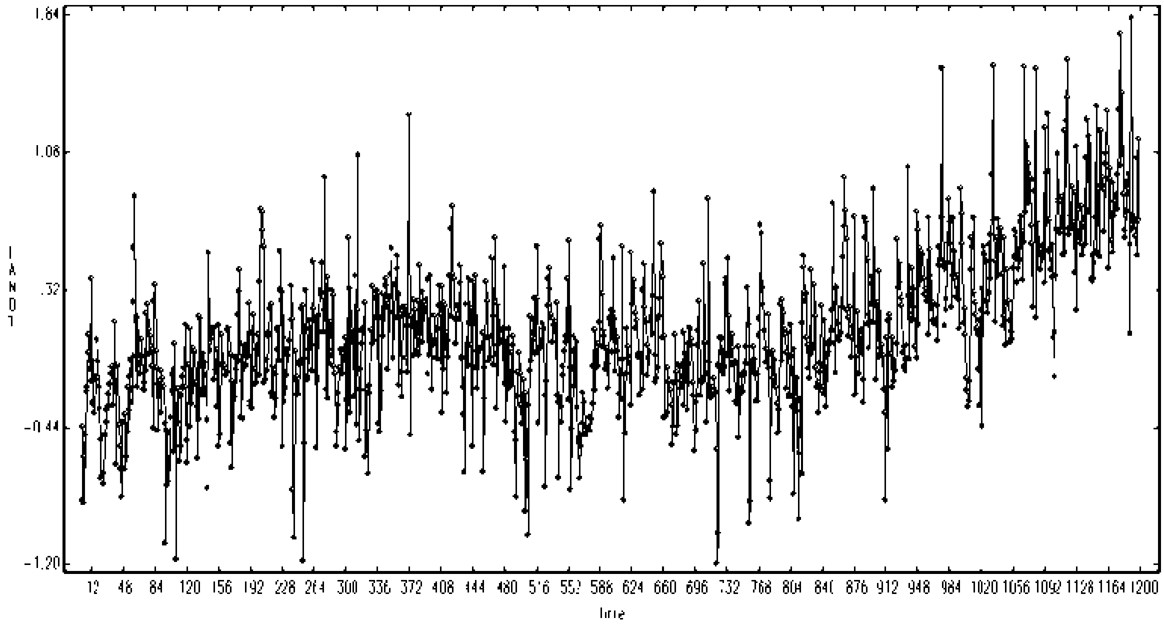

FIGURE 1-1：Land Temperature Data

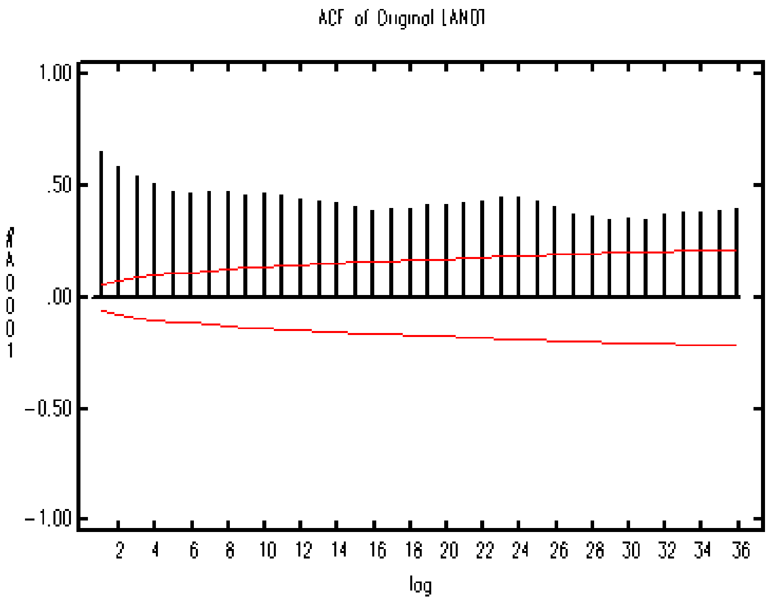

FIGURE 1-2：The SACF for The Original Values

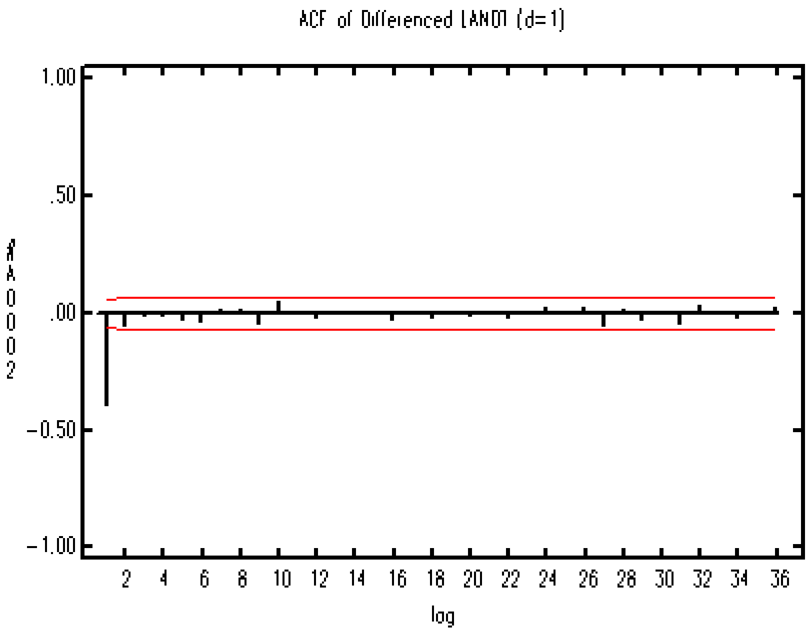

FIGURE 1-3：The SPACF for The Original Values

(Q-->)	0	1	2	3	4	5	6
(P= 0)	X	X	X	X	X	X	X
(P= 1)	X	0	0	0	0	0	0
(P= 2)	X	0	0	0	0	0	0
(P= 3)	X	0	0	0	0	0	0
(P= 4)	X	X	0	0	0	0	0
(P= 5)	X	X	0	0	X	0	0
(P= 6)	X	X	X	0	0	0	0

FIGURE 1-4：The EACF for The Original Values

From the time series plot, there is an upward trend at the late period. The SACF and SPACF at lag 1 is nonzero, but not very close to 1, so we consider two ways. First way is to observe the original values' EACF. The EACF shows that we can assume original values is ARMA(1,1) model.

$$L_t - 0.9514L_{t-1} = a_t - 0.6262\, a_{t-1} \qquad \text{model (1)}$$

Second way is to consider the first differences of the original values.

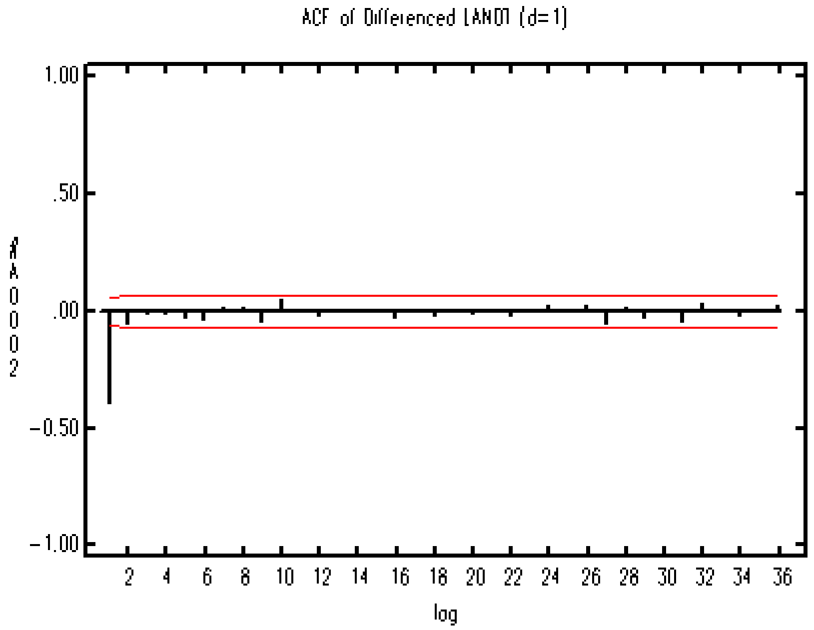

FIGURE 1-5：The SACF for The First Differences

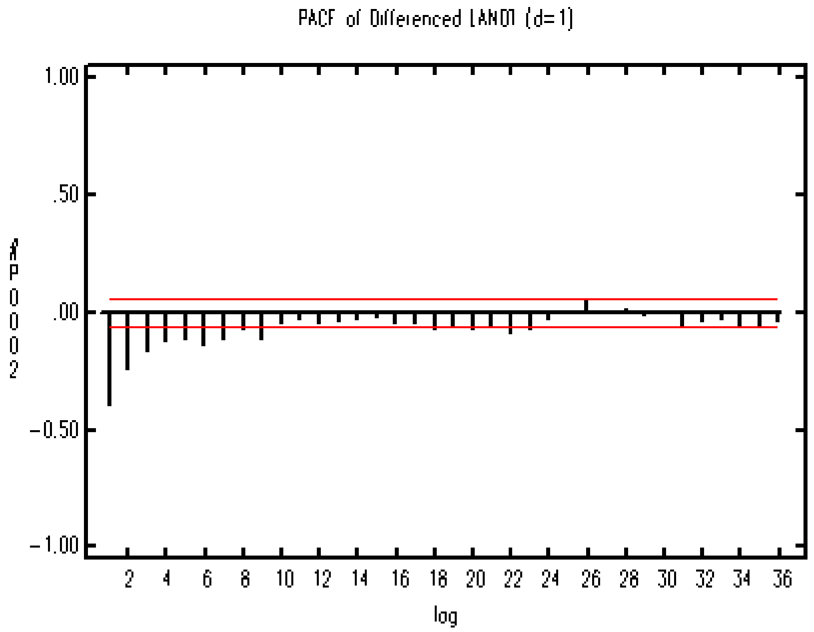

FIGURE 1-6：The SPACF for The First Differences

The SACF and SPACF for the series of first differences (1-B) is shown in Fig1-5 and Fig1-6. We find that the SACF cuts off after lag one, while the SPACF tails off. t L

This is indicative of an ARIMA(0,1,1) model.

$$\mathrm{L_t} - L_{t-1} = a_t - 0.706\, a_{t-1} \qquad \text{model (2)}$$

The series of first differences is stationary, but the results of the analysis are not as well as ARMA (1,1). The estimator of ϕ_1 in model(1) is close to 1, it is seem to the first differences, so we would tentatively to choose model(1).

```
            -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
              +----+----+----+----+----+----+----+----+----+----+
                                       I
  1   0.06                            +I+X
  2  -0.04                            XI+
  3  -0.05                            XI+
  4  -0.06                           X+I+
  5  -0.08                           X+I+
  6  -0.07                           X+I+
  7  -0.01                            +I+
  8   0.01                            +I+
  9  -0.01                            +I+
 10   0.06                            +IX
 11   0.03                            +IX
 12   0.00                            +I+
 13   0.01                            +I+
 14   0.00                            +I+
 15  -0.01                            +I+
 16  -0.04                            XI+
 17  -0.02                            XI+
 18  -0.02                            XI+
 19   0.01                            +I+
 20   0.01                            +I+
 21   0.03                            +IX
 22   0.03                            +IX
 23   0.07                            +I+X
 24   0.09                            +I+X
```

FIGURE 1-7：The SACF for Residuals of ARMA(1,1)

(Q-->)	0	1	2	3	4	5	6
(P= 0)	X	0	0	X	X	X	0
(P= 1)	X	0	0	0	0	0	0
(P= 2)	X	0	0	0	0	0	0
(P= 3)	X	X	0	0	0	0	0
(P= 4)	X	0	0	0	0	0	0
(P= 5)	X	X	0	0	0	0	0
(P= 6)	X	0	X	X	0	X	0

FIGURE 1-8：The EACF for Residuals of ARMA(1,1)

By the SACF of model(1)'s residuals does not like white noise, while from the EACF of residuals, it is obvious ARMA (1,1) model. Thus, the revised model is

$$(1-0.9696\mathrm{B})(1-0.3793\mathrm{B})\mathrm{L_t} = (1-0.9993B)(1-0.6925B)a_t \qquad \text{model (3)}$$

The residual autocorrelations for this revised model do not exceed twice their standard errors. Furthermore, the chi-square statistic applied to the first 24 autocorrelation is $\mathrm{Q} = 21.4 < \chi^2_{24-4,0.05} = 32.67$, we cannot reject the hypothesis that the residuals are a white noise series.

From model(3), the time series plot of residuals and outliers detection, there is 5 additive outliers can be found as Table 1.1.

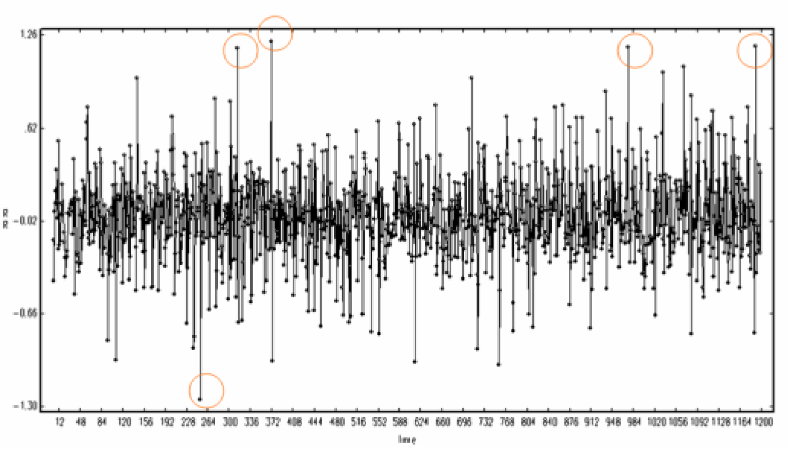

FIGURE 1-9：The Residuals of ARMA(1,1)

TIME	ESTIMAT	T VALUE	TYPE
372	1.28	4.61	AO
1191	1.16	4.2	AO
314	1.13	4.13	AO
252	-1.12	-4.13	AO
975	1.12	4.13	AO

TABLE 1-1：The Outliers of Model(3)

Consider intervention analysis

$$L_t = \frac{(1-\theta_1 B)(1-\theta_2 B)}{(1-\phi_1 B)(1-\phi_2 B)} a_t + w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4 + w_5 x_5 \qquad \text{model(4)}$$

$$x_1 = \begin{cases} 1, & t = 372 \\ 0, & o.w. \end{cases} ; x_2 = \begin{cases} 1, & t = 1191 \\ 0, & o.w. \end{cases} ; x_3 = \begin{cases} 1, & t = 314 \\ 0, & o.w. \end{cases} ; x_4 = \begin{cases} 1, & t = 252 \\ 0, & o.w. \end{cases} ; x_5 = \begin{cases} 1, & t = 975 \\ 0, & o.w. \end{cases}$$

But the performance of the residuals of model (4) does not like white noise and $Q(24) = 48.6 < \chi^2_{24-9,0.05} = 24.99$, we finally consider model (3) as the land temperature time series model.

	PARAMETER	VALUE	STD. ERROR	T VALUE	RESIDUAL STD. ERROR
Model(1)	θ_1	0.6262	0.0288	21.77	0.3005
	ϕ_1	0.9514	0.0117	81.59	
Model(2)	θ_1	0.7061	0.0210	33.58	0.3031
Model(3)	θ_1	0.9606	0.0149	64.49	0.2930
	θ_2	0.3793	0.0680	5.58	
	ϕ_1	0.9993	0.0035	282.44	
	ϕ_2	0.6925	0.0614	11.28	

TABLE 1-2 ：Summary for Models

Compare the actual data of January 2015 to November and 95% prediction confidence intervals. The actual data are included in the prediction confidence intervals. It means that the forecasting results are good, so we conclude the model (3) adequately describes the land temperature time series.

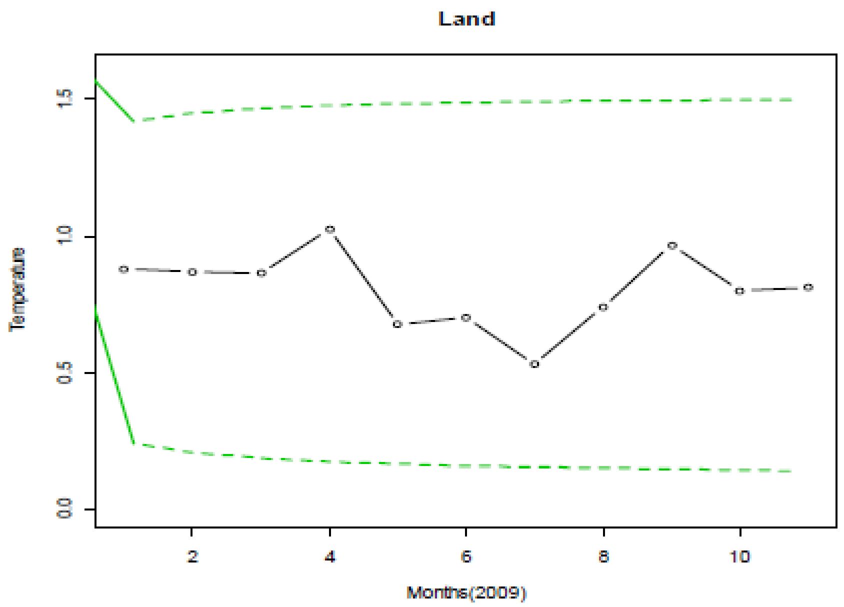

FIGURE 1-10：Forecasting Results

TIME	FORECAST	STD. ERROR	ACTUAL DATA
Jan-15	0.8295	0.293	0.8786
Feb-15	0.8275	0.3106	0.869
Mar-15	0.8259	0.3204	0.8629
Apr-15	0.8247	0.3262	1.0249
May-15	0.8236	0.33	0.6754
Jun-15	0.8227	0.3325	0.7006
Jul-15	0.8219	0.3344	0.529
Aug-15	0.8212	0.3359	0.7385
Sep-15	0.8205	0.3372	0.9657
Oct-15	0.8199	0.3383	0.7986
Nov-15	0.8193	0.3392	0.8122

TABLE 1-3：Forecasts

II. Analysis of Global Ocean Temperature Anomalies Data

We analyze ocean temperature data as follows. First, we observe time series plot, SACF, SPACF of the raw data:

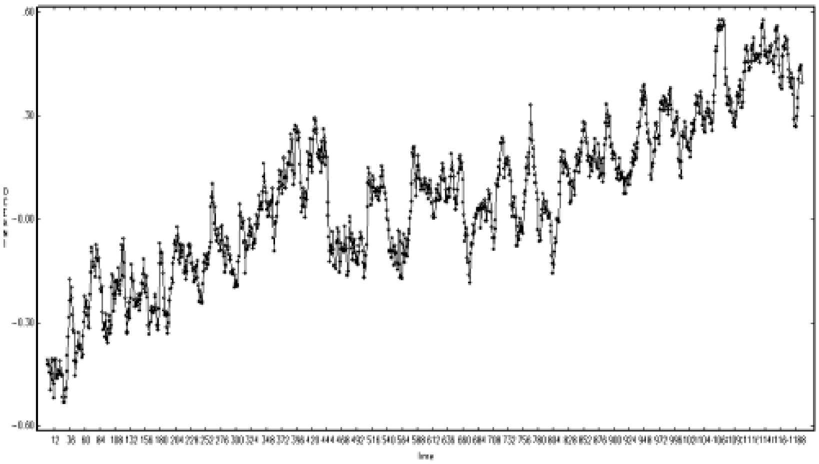

FIGURE 2-1：Ocean Temperature Data

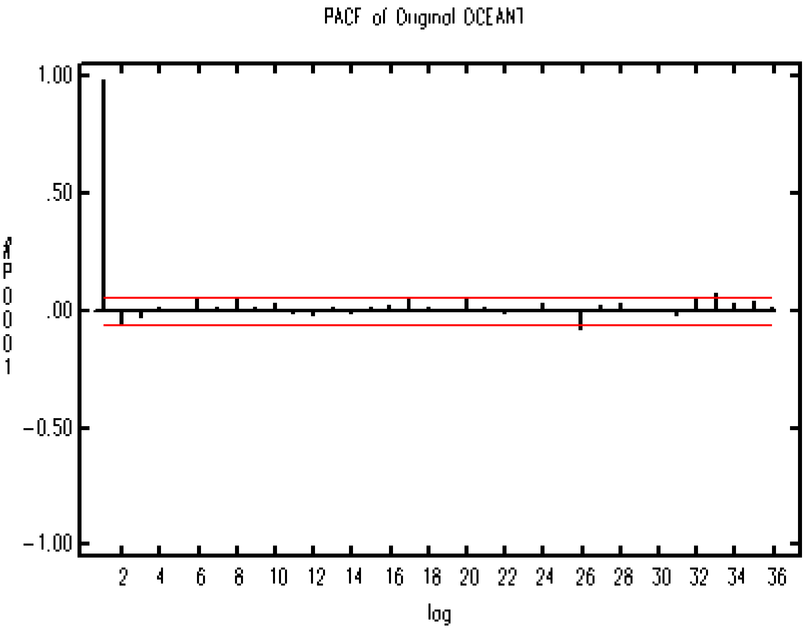

FIGURE 2-2：The SACF for The Original Values

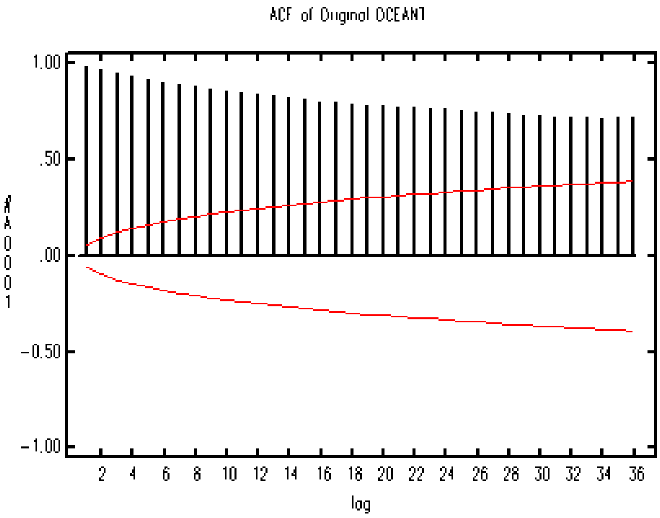

FIGURE 2-3：The SPACF for The Original Values

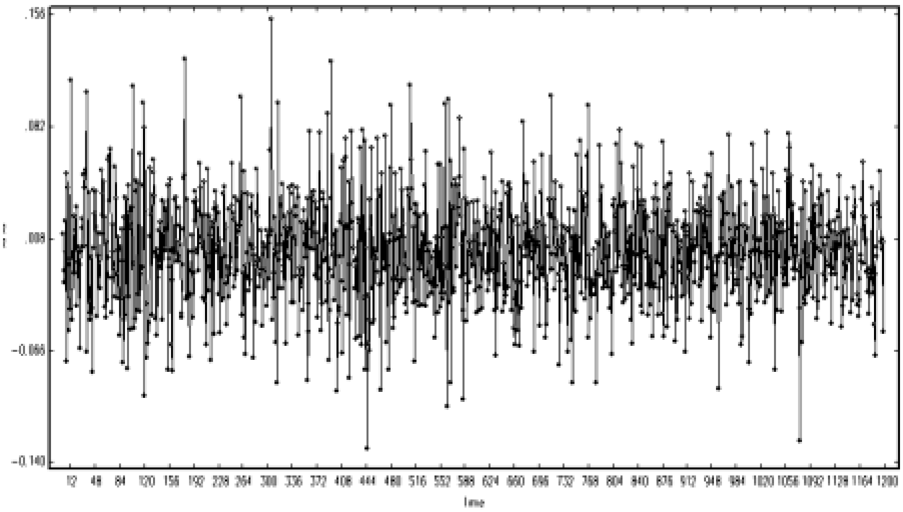

FIGURE 2-4：The First Differences

(Q-->)	0	1	2	3	4	5	6
(P= 0)	X	O	O	O	X	O	O
(P= 1)	O	O	O	O	X	O	O
(P= 2)	X	X	O	O	O	O	O
(P= 3)	X	X	O	O	O	O	O
(P= 4)	X	X	X	X	X	O	O
(P= 5)	X	X	X	X	X	O	O
(P= 6)	X	X	X	X	X	X	O

FIGURE 2-5：The EACF for The First Differences

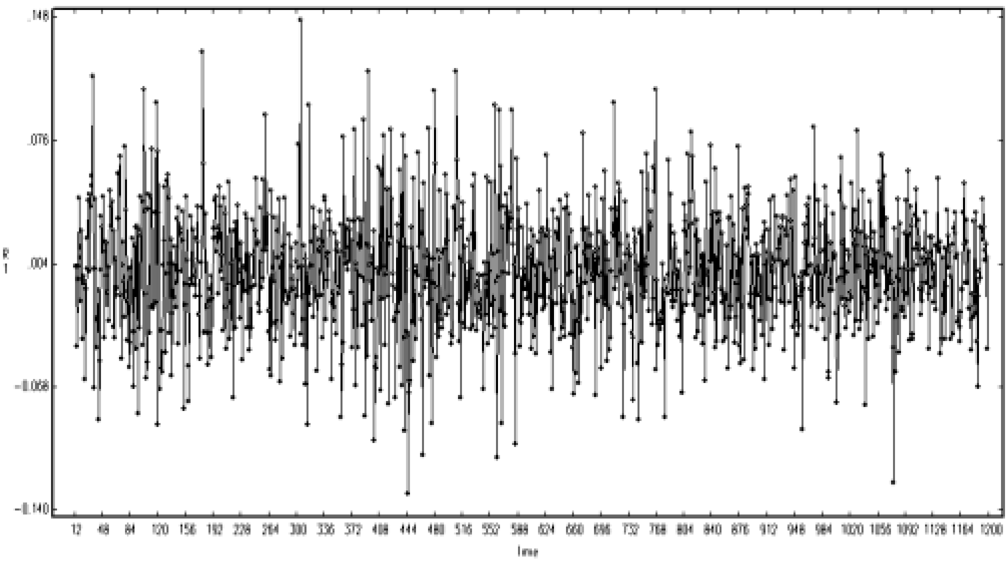

FIGURE 2-6：The Differences(1,12)

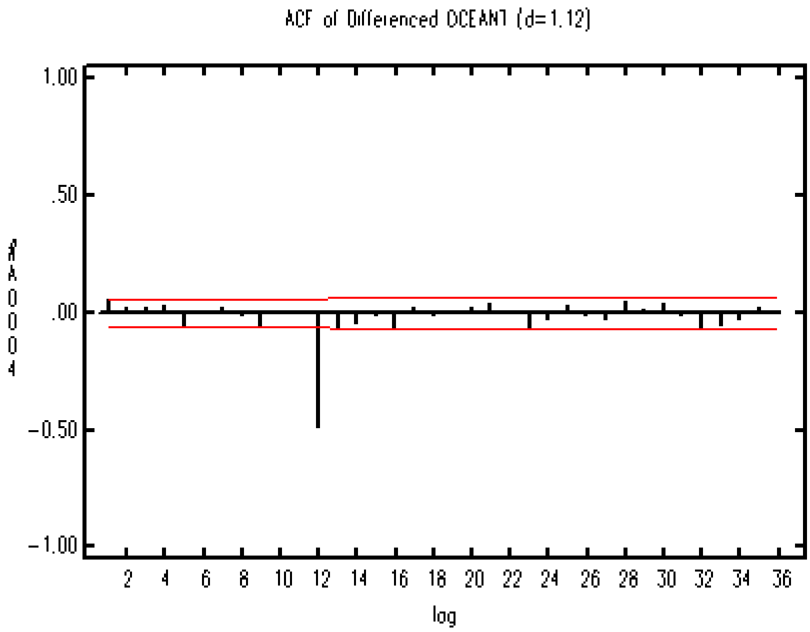

FIGURE 2-7：The SACF for The Differences(1,12)

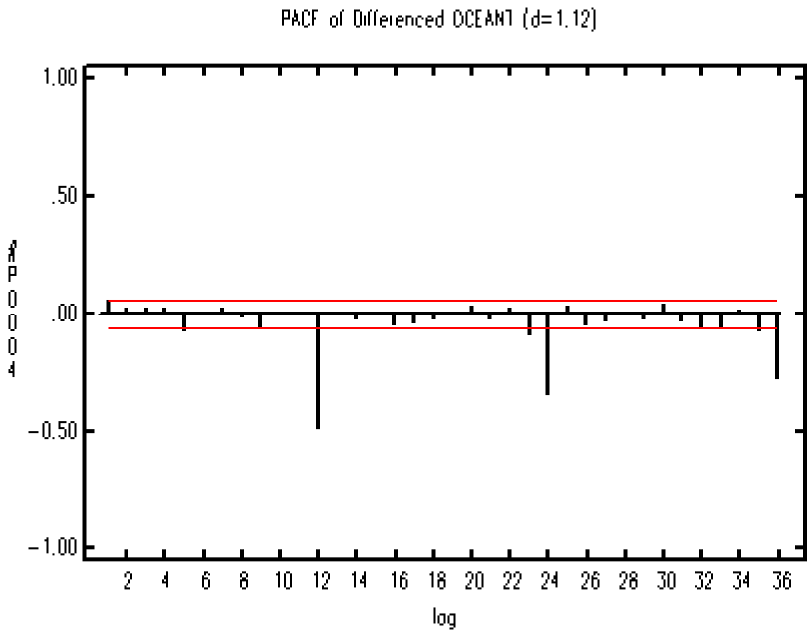

FIGURE 2-8：The SPACF for The Differences(1,12)

According as the time series plot、SACF and SPACF plot of original data, we found it is not stationary and cut off after lag1. We difference the original data. Hence, from the time series plot、SACF、SAPCF and EACF of the difference global temperature monthly series, we can consider two models.

According to EACF(Fig 2-7), the ARIMA(0,1,1) model is entertained, it would be

$$(1-\mathrm{B})\mathrm{O}_{\mathrm{t}} = (1+0.069B)a_t \qquad \text{model(1)}$$

Observe SACF plot of the difference data, the overall impression is that the autocorrelations are those of a white noise process, although the autocorrelations at lag 1 and 5 are relatively large. We would suggested an alternative model

$$(1-\mathrm{B})\mathrm{O_t} = (1+0.0675B-0.0952B^2)a_t \qquad \text{model(2)}$$

We also try seasonal difference. SACF shows the only sample autocorrelations which exceed twice their standard errors are $\hat{\rho}^{12}$, and thus a tentative model might be

$$(1-\mathrm{B})(1-B^2)\mathrm{O_t} = (1-0.9589B^2)a_t \qquad \text{model(3)}$$

Compare the models in Table 2-1:

	PARAMETER	VALUE	STD. ERROR	T VALUE	RESIDUAL STD. ERROR	
Model(1)	θ_1	-0.0697	0.0288	-2.42		
	θ_2					
					0.03576	37.1
Model(2)	θ_1	-0.0675	0.0287	-2.35		
	θ_2	0.0952	0.0286	3.33		
					0.03560	28.5
Model(3)	Θ_{12}	0.9589	0.0086	111.74		
					0.0359	41.1

TABLE 2-1：Summary for Models

From Table 2-1，we can know the residual error of model(2) is 0.0356 and has the smallest Q-value, Q(24)=28.5, we think model(2) is more appropriate.

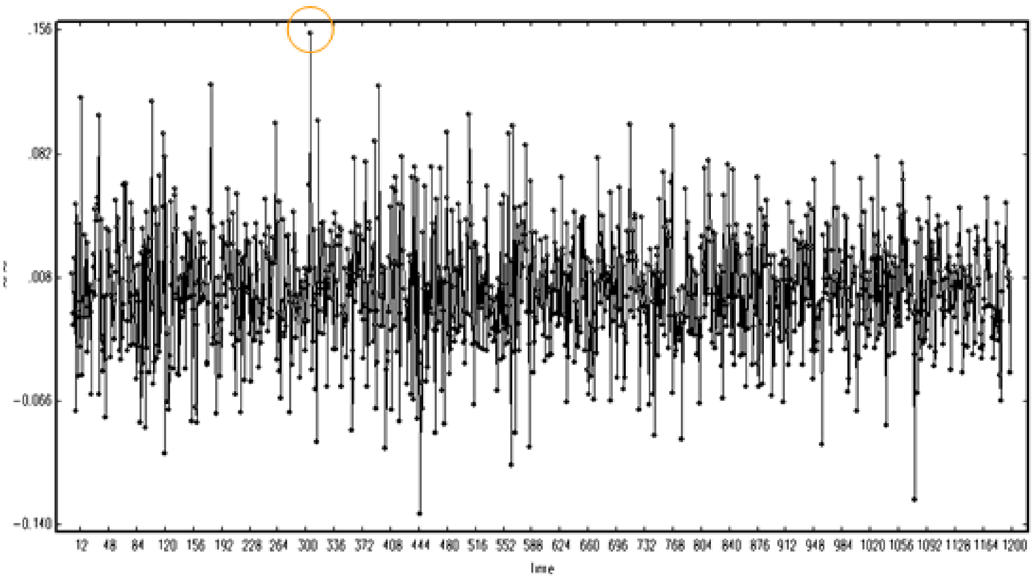

FIGURE 2-9：The Residuals if Model(2)

From Fig 2-11and outlier detection, we find out one outlier. So we consider the Intervention Model. The following is model(4):

$$O_t = \frac{1 + 0.0717B - 0.1023B^5}{1 - B} a_t + 0.1597x_1 \quad , x_1 = \begin{cases} 1 \ , & t = 306 \\ 0 \ , & o.w. \end{cases}$$

Outlier is the ocean temperature in 1934/06. Thermohaline circulation in the Atlantic turns stronger in 1934. So the temperature is higher than other years.

```
            -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
              +----+----+----+----+----+----+----+----+----+----+
                                       I
  1   0.00                            +I+
  2   0.00                            +I+
  3  -0.03                            XI+
  4   0.00                            +I+
  5  -0.10                           X+I+
  6  -0.01                            +I+
  7  -0.04                            XI+
  8  -0.02                            +I+
  9  -0.05                            XI+
 10   0.00                            +I+
 11   0.01                            +I+
 12   0.03                            +IX
 13   0.02                            +IX
 14  -0.05                            XI+
 15  -0.05                            XI+
 16  -0.08                           X+I+
 17  -0.02                            +I+
 18   0.00                            +I+
 19  -0.04                            XI+
 20  -0.02                            XI+
 21   0.00                            +I+
 22   0.00                            +I+
 23  -0.04                            XI+
 24   0.01                            +I+
```

FIGURE 2-10：The SACF for Residuals of Model(4)

(Q-->)	0	1	2	3	4	5	6
(P= 0)	0	0	X	0	0	X	0
(P= 1)	X	0	X	0	0	X	0
(P= 2)	X	X	X	0	0	X	X
(P= 3)	0	X	X	0	0	X	0
(P= 4)	X	0	X	X	0	X	0
(P= 5)	X	0	X	X	X	X	0
(P= 6)	X	X	X	0	0	X	0

FIGURE 2-11：The EACF for Residuals Square of Model(4)

Observe Fig 2-10.the pattern is like white noise and $Q(24) = 26.8 < \chi^2_{24-3,0.05} = 32.67$,so model(4) is appropriate. Then, we consider whether the variance of residual from the Model(4) is equal. From Fig 2-11, the EACF pattern of residual square shows that it is white noise, so we do not consider GARCH model. The confidence interval of forecast contains all the data that we observed during 2015. It indicates the result of forecast is great.

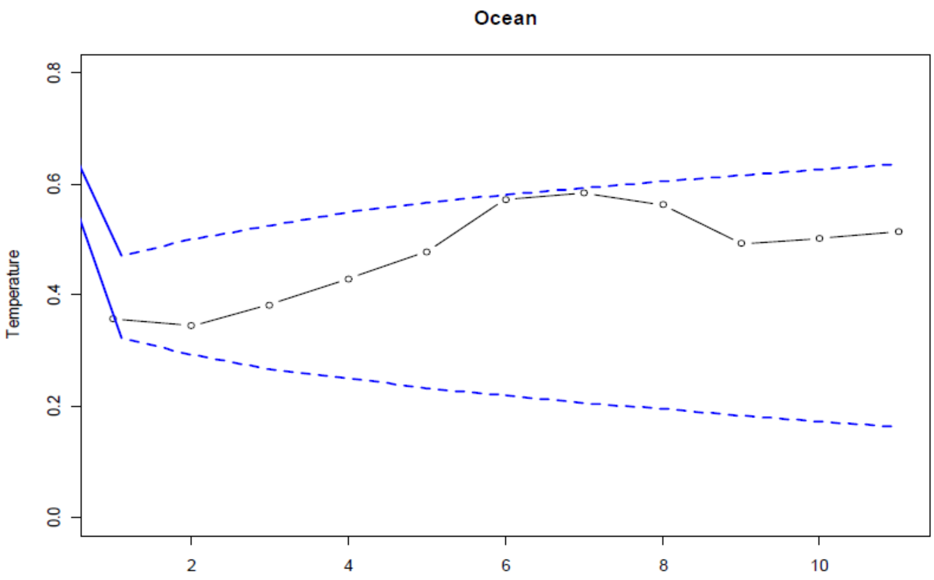

FIGURE 2-12：Forecasting Results

TIME	FORECAST	STD. ERROR	ACTUAL DATA
Jan-15	0.3967	0.0353	0.358
Feb-15	0.3965	0.0517	0.3443
Mar-15	0.3954	0.0641	0.3821
Apr-15	0.4003	0.0744	0.4292
May-15	0.3995	0.0835	0.4778
Jun-15	0.3995	0.0902	0.5721
Jul-15	0.3995	0.0965	0.5834
Aug-15	0.3995	0.1024	0.5619
Sep-15	0.3995	0.1079	0.4926
Oct-15	0.3995	0.1132	0.5009
Nov-15	0.3995	0.1183	0.5135

TABLE 2-2：Forecasts

III. Analysis of Vector ARMA models

We are interested in the structure of the relationship among the land and ocean temperature series, so we consider vector ARMA models as follows:

The sample cross correlation matrices (CCM) for the land and ocean temperature is show in Fig 3-1. The persistence of large sample auto- and cross-correlations indicates that the data are not likely to have come from a low-order MA model.

```
CROSS CORRELATION MATRICES IN TERMS OF +,-,.

LAGS  1 THROUGH  6

     + +      + +      + +      + +      + +      + +
     + +      + +      + +      + +      + +      + +

LAGS  7 THROUGH 12

     + +      + +      + +      + +      + +      + +
     + +      + +      + +      + +      + +      + +

LAGS 13 THROUGH 18

     + +      + +      + +      + +      + +      + +
     + +      + +      + +      + +      + +      + +

LAGS 19 THROUGH 24

     + +      + +      + +      + +      + +      + +
     + +      + +      + +      + +      + +      + +
```

FIGURE 3-1. Sample Cross-Correlation Matrices for Data

The pattern of the partial auto-regression and related statistics are given in Table 3-1. But it's still hard to tentatively select low-order auto-regression models.

LAG	RESIDUAL VARIANCES	EIGENVAL. OF SIGMA	CHI-SQ TEST	AIC	SIGNIFICANCE OF PARTIAL AR COEFF
1	8.91E-02	1.26E-03	4623.12	-9.089	+ +
	1.27E-03	8.92E-02			. +
2	8.67E-02	1.25E-03	42.21	-9.118	+ .
	1.26E-03	8.67E-02			. -
3	8.60E-02	1.24E-03	16.22	-9.125	+ .
	1.26E-03	8.60E-02			- .
4	8.56E-02	1.24E-03	7.67	-9.125	. .
	1.25E-03	8.56E-02			. .
5	8.54E-02	1.24E-03	5.15	-9.123	. .
	1.25E-03	8.54E-02			. .
6	8.52E-02	1.22E-03	14.51	-9.129	. .
	1.24E-03	8.52E-02			- +

TABLE 3-1. Pattern of Partial Autoregression and Related Statistics for Data

So we consider the method of Extended Cross Correlation Matrices (ECCM) and Smallest Canonical Correlation Analysis (SCAN). The pattern of Fig 3-2 and Fig 3-3 suggest it is possibility an ARMA(1,1) model.

```
***** THE SIMPLIFIED ECCM TABLE *****

 (Q-->)      0    1    2    3    4    5    6
-------------------------------------------
          I
 (P= 0)I  ++   ++   ++   ++   ++   ++   ++
          I  ++   ++   ++   ++   ++   ++   ++
          I
 (P= 1)I  -.   ..   ..   ..   ..   .+   ..
          I  -+   +.   -.   +.   .-   ..   ..
          I
 (P= 2)I  --   -.   ..   ..   ..   ..   ..
          I  ++   ..   ..   ..   .-   ..   ..
          I
 (P= 3)I  ++   ++   +.   ..   ..   ..   ..
          I  ++   ++   -.   ..   ..   ..   ..
          I
 (P= 4)I  -.   --   ++   ..   ..   ..   ..
          I  -+   .-   ++   ..   ..   ..   ..
          I
 (P= 5)I  --   -.   .+   --   ..   ..   ..
          I  --   -.   ..   +.   ..   ..   ..
          I
 (P= 6)I  -+   -.   -+   ..   ..   -.   ..
          I  +-   .-   +-   -+   +-   +.   ..
```

FIGURE 3-2. ECCM

```
SIMPLIFIED SCAN TABLE (1% LEVEL):

Q:     0    1    2    3    4    5    6
------------------------------------
 0:    X    X    X    X    X    X    X
 1:    X    0    0    0    X    0    0
 2:    X    0    0    0    0    0    0
 3:    0    0    0    0    0    0    0
 4:    0    0    0    0    0    0    0
 5:    X    0    0    0    0    0    0
 6:    0    0    0    0    0    0    0
```

FIGURE 3-3. SCAN

For this model, $(\mathrm{I}-\phi \mathrm{B})\mathrm{Z}_t = \mathrm{C} + (\mathrm{I}-\theta \mathrm{B})\mathrm{a}_t$ were fitted using the conditional likelihood method. The estimation results are

$$\mathrm{C} = \begin{bmatrix} 0.005 \\ 0.001 \end{bmatrix} \;,\; \phi = \begin{bmatrix} 0.771 & 0.270 \\ 0.008 & 0.976 \end{bmatrix} \;, \theta = \begin{bmatrix} 0.042 & -0.012 \\ 0.005 & -0.083 \end{bmatrix}$$

Then we set zero to those coefficients whose estimates were small compared to their standard errors. The restricted model’s estimation results are

$$\mathrm{C} = \begin{bmatrix} 0.005 \\ 0.002 \end{bmatrix} \;,\; \phi = \begin{bmatrix} 0.773 & 0.268 \\ 0 & 0.985 \end{bmatrix} \;, \theta = \begin{bmatrix} 0.449 & 0 \\ 0 & -0.077 \end{bmatrix}$$

Table 3-2 suggests that the restricted ARMA(1,1) model provides an adequate representation of the data.

```
              1              2

    1    .........+..   ............
    1    ..........++   ..-........+

    2    .+.....-.-..   ....-.......
    2    ............   ............
```

Table 3-2. Pattern of Residual Cross-Correlations after Restricted ARMA(1,1) Model

	1	2
1	0.085813	
2	0.001159	0.001268

Table 3-3. Covariance Metric of Residual

The final model implies that the temperature is approximately

$$(1-0.773\mathrm{B})\mathrm{Z}_{1,t}-(0.268B)Z_{2,t}=0.005+(1-0.449B)a_{1t}$$
$$(1-0.958\mathrm{B})\mathrm{Z}_{2,t}=0.002+(1+0.077B)a_{2t}$$

We also consider using first difference of data, but ARMA(1,1) model fit better, and produced a marginally better representation. Fig 3-5 shows the predict confidence interval of forecast contains all the data that we observed during 2015. It indicates the result of forecast is great.

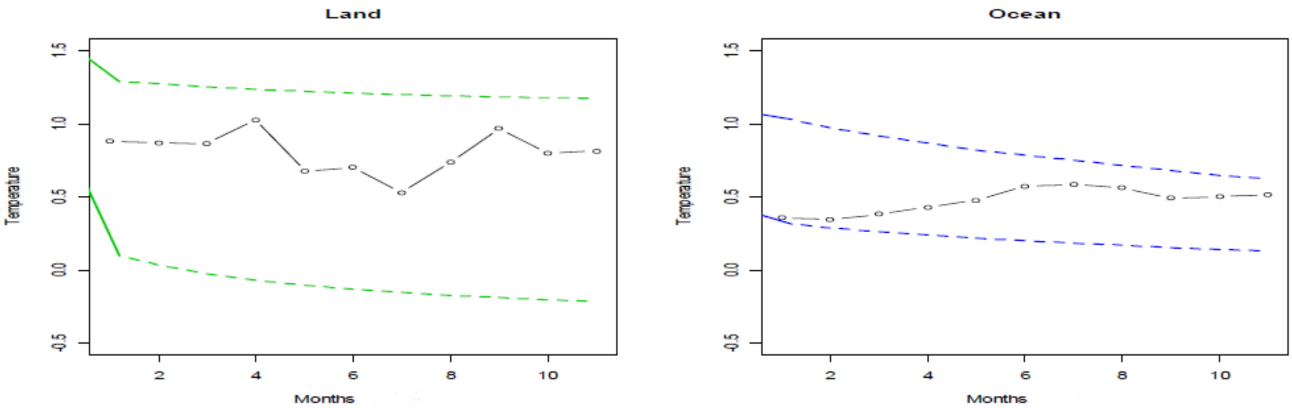

FIGURE 3-5. Actual Data and Predict Confidence Interval

Ⅳ.Conclusion:

1. The model for global land temperature anomalies series is

$$(1-0.9606\mathrm{B})(1-0.03793\mathrm{B})\mathrm{L_t} = (1-0.993\mathrm{B})(1-0.6925\mathrm{B})\mathrm{a_t}$$

   The residual of this model is consistent, so we don't consider using GARCH model.

2. The model for global ocean land temperature anomalies series is

$$\mathrm{O_t} = \frac{1+0.0717B-0.1023B^5}{1-B}a_t + 0.1597x_1 \quad , \mathrm{x}_1 = \begin{cases} 1\ , & t = 306 \\ 0\ , & o.w. \end{cases}$$

   The residual of this model is consistent, so we don't consider using GARCH model.

3. For the global land temperature anomalies series Z_{1t} , we have that

$$(1-0.773\mathrm{B})\mathrm{Z}_{1,t} - (0.268B)Z_{2,t} = 0.005 + (1-0.449B)a_{1t}$$

   For the global ocean temperature anomalies series Z_{2t} , we have that

$$(1-0.958\mathrm{B})\mathrm{Z}_{2,t} = 0.002 + (1+0.077B)a_{2t}$$

   We see that ocean temperature will effect land temperature. About 70% of the Earth's surface is sea water and the ocean currents have a major influence on climate and weather. For example, on a larger scale the sea acts as a reservoir of heat from the summer, keeping coastal regions milder in the autumn than regions inland. So the model reflects the phenomenon of nature.