Time Series Student Project
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Course: Fall 2015 TS

Title: Percent of Men with full beards, 1866 – 1911

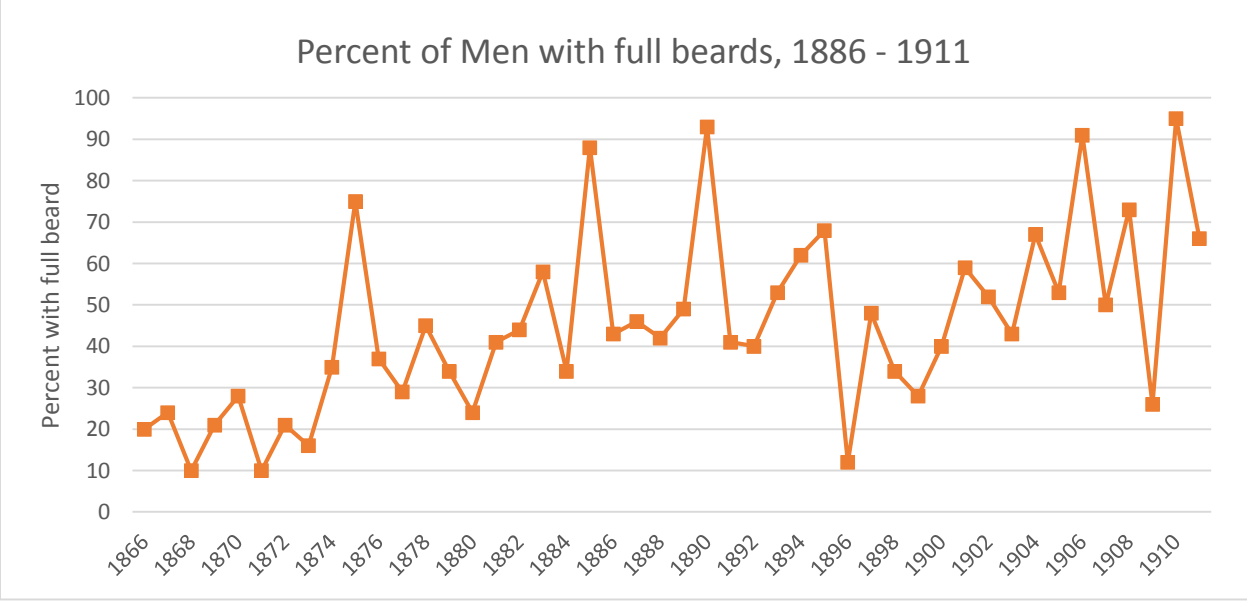
Introduction

Some may say a man wearing a full beard is back in style, and has been increasing in popularity among hipster identifying cultures. This project looks at beard style trends not from today, but over a century ago, and aims to fit an ARIMA model to the time series using methods learned in the time series on-line course.

Data Source

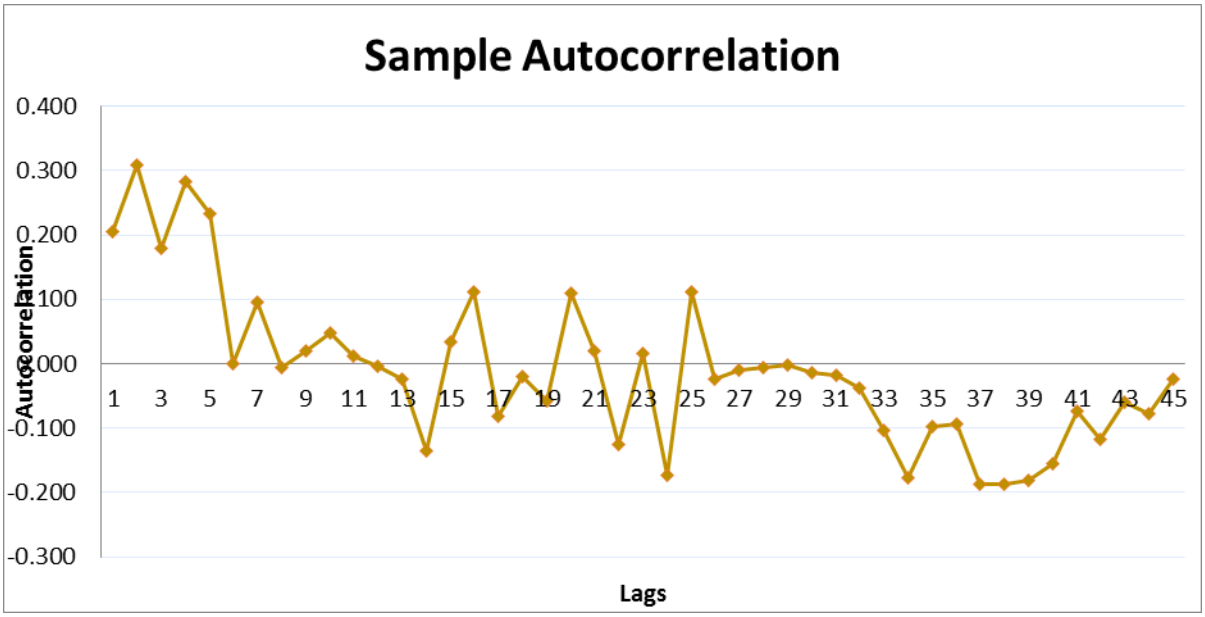
Data is sourced from datamarket.com and is presented below.

Year	Percent with Full Beard	Year	Percent with Full Beard	Year	Percent with Full Beard	Year	Percent with Full Beard
1866	20	1878	45	1890	93	1902	52
1867	24	1879	34	1891	41	1903	43
1868	10	1880	24	1892	40	1904	67
1869	21	1881	41	1893	53	1905	53
1870	28	1882	44	1894	62	1906	91
1871	10	1883	58	1895	68	1907	50
1872	21	1884	34	1896	12	1908	73
1873	16	1885	88	1897	48	1909	26
1874	35	1886	43	1898	34	1910	95
1875	75	1887	46	1899	28	1911	66
1876	37	1888	42	1900	40		
1877	29	1889	49	1901	59		



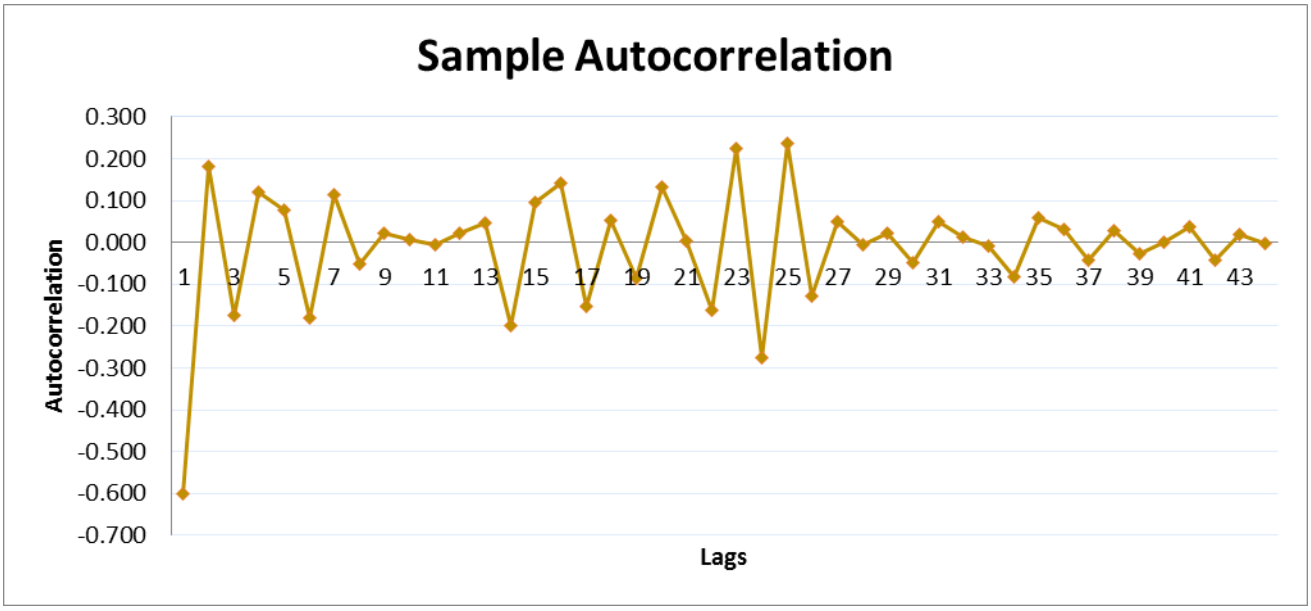
Stationary

The plot of the time series would seem to indicate an upward trend over the period. I hypothesized that the data is not stationary. To test this hypothesis, I graphed the sample autocorrelation over all lags.

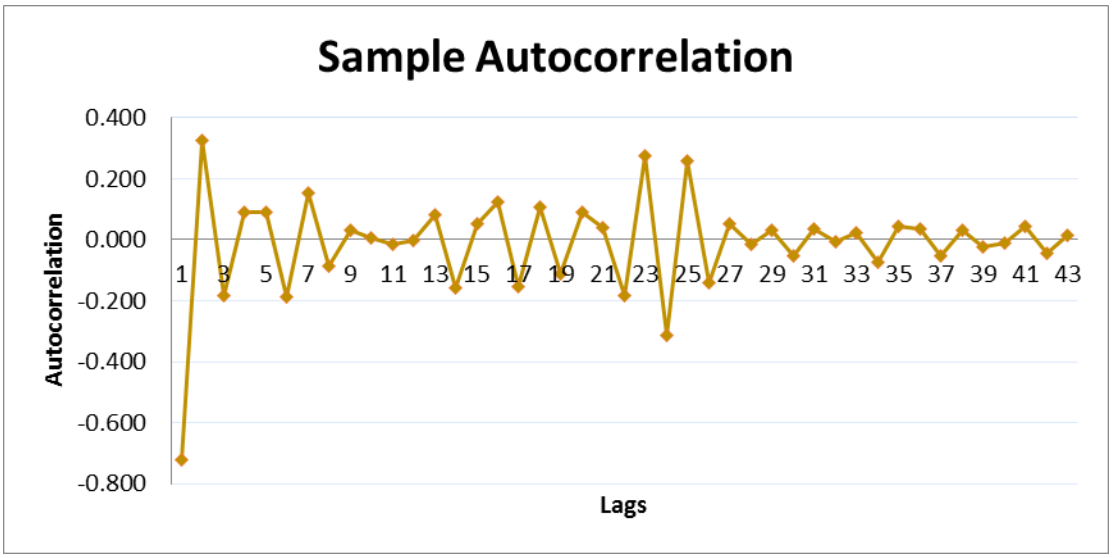


This confirms the time series is not stationary since the values do not drop quickly to zero, and they do not return to zero at subsequent lags with minimum fluctuations.

The correlogram of the first difference of Y(t) can be seen below.



The correlogram of the second difference of Y(t) can be seen below.



Both first and second differences drop quickly to zero and oscillate around zero. The oscillations in both graphs fluctuate minimally towards the end of the lags. Both suggest stationarity and since second differences do not present any significant improvement over first differences, I will choose first differences to make the process stationary in order to avoid over differencing.

Autoregressive Process

We will now use ARI(1,1), ARI(2,1) and ARI(3,1) processes to fit a model to the time series.

The ARI(1,1) model is:

$Yt - Yt - 1 = \varphi 1*(Yt - 1 - Yt - 2) + \theta$

The ARI(2,1) model is:

$Y_{t} - Y_{t-1} = \varphi_1*(Y_{t-1} - Y_{t-2}) + \varphi_2*(Y_{t-2} - Y_{t-3}) + \theta$

The ARI(3,1) model is:

$Y_{t} - Y_{t-1} = \varphi_1*(Y_{t-1} - Y_{t-2}) + \varphi_2*(Y_{t-2} - Y_{t-3}) + \varphi_3*(Y_{t-3} - Y_{t-4}) + \theta$

Using regression analysis in Excel, I obtained the following results:

ARI(1,1): $Yt = -0.6181*(Yt - 1 - Yt - 2) + Yt - 1 + 2.0524$

ARI(2,1): $Y_{t} = -0.7738*(Y_{t-1} - Y_{t-2}) + -0.2810*(Y_{t-2} - Y_{t-3}) + Y_{t-1} + 2.3494$

ARI(3,1): $Y_{t} = -0.8634*(Y_{t-1} - Y_{t-2}) + -0.5416*(Y_{t-2} - Y_{t-3}) + -0.3825*(Y_{t-3} - Y_{t-4}) + Y_{t-1} + 2.9838$

Please see appendix A for details of the regression analysis.

Regression Summary

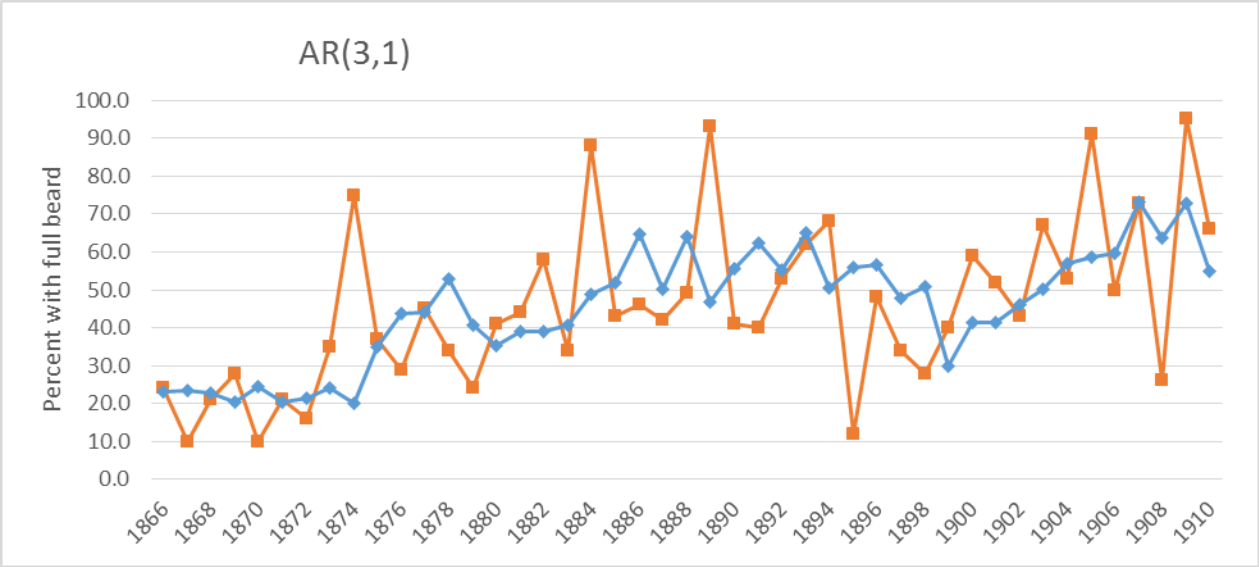
	R^2	Adj. R^2	Std. Error
ARI(1,1)	0.37149	0.35687	22.06164
ARI(2,1)	0.41370	0.38578	21.56016
ARI(3,1)	0.48636	0.44878	20.42453

Residual Diagnostic

I will use the Box Pierce Q Statistic to determine if the residuals from the ARIMA model are a random walk. See appendix A for details of the analysis. Using a critical Chi2 value for a 10% significance level, the BPQS calculated is below the critical value after a few lags. We do not reject the null hypothesis that the residuals are a white noise process.

Conclusion

The time series of historical percent of men with full beards is not a stationary series. In order to make it stationary it was transformed by taking first differences. Autoregressive models were then considered of orders 1, 2 and 3. Regression analysis produced three ARIMA models and further residual analysis confirmed residuals of each model may follow a white noise process. Based on these results I choose the ARI(3,1) model to fit the time series due to it having the largest R^2 values and lowest standard error.



As shown, the fitted line in blue follows the time series on its general course but has some obvious deviations from actual. Due to the dates in the series and unavailable information on how the data was collected, some of the collection values may be subject. I notice at least 5 or 6 outlying observations and wonder how accurate those data points are. Nonetheless, if you are or know somebody thinking about growing a beard, know you are in good company as many men over the centuries have embraced the fashion as their own.

Appendix A

Time series, differences and fitted models:

Percent of Men with full beards, 1866 – 1911

Year	Percent with Full Beard	Percent with Full Beard Y(t) - Y(t-1)	Percent with Full Beard Y(t-1) - Y(t-2)	Percent with Full Beard Y(t-2) - Y(t-3)	Percent with Full Beard Y(t-3) - Y(t-4)
1866	24.0	4.0	0.0	0.0	0.0
1867	10.0	-14.0	4.0	0.0	0.0
1868	21.0	11.0	-14.0	4.0	0.0
1869	28.0	7.0	11.0	-14.0	4.0
1870	10.0	-18.0	7.0	11.0	-14.0
1871	21.0	11.0	-18.0	7.0	11.0
1872	16.0	-5.0	11.0	-18.0	7.0
1873	35.0	19.0	-5.0	11.0	-18.0
1874	75.0	40.0	19.0	-5.0	11.0
1875	37.0	-38.0	40.0	19.0	-5.0
1876	29.0	-8.0	-38.0	40.0	19.0
1877	45.0	16.0	-8.0	-38.0	40.0
1878	34.0	-11.0	16.0	-8.0	-38.0
1879	24.0	-10.0	-11.0	16.0	-8.0
1880	41.0	17.0	-10.0	-11.0	16.0
1881	44.0	3.0	17.0	-10.0	-11.0
1882	58.0	14.0	3.0	17.0	-10.0
1883	34.0	-24.0	14.0	3.0	17.0
1884	88.0	54.0	-24.0	14.0	3.0
1885	43.0	-45.0	54.0	-24.0	14.0
1886	46.0	3.0	-45.0	54.0	-24.0
1887	42.0	-4.0	3.0	-45.0	54.0
1888	49.0	7.0	-4.0	3.0	-45.0
1889	93.0	44.0	7.0	-4.0	3.0
1890	41.0	-52.0	44.0	7.0	-4.0
1891	40.0	-1.0	-52.0	44.0	7.0
1892	53.0	13.0	-1.0	-52.0	44.0
1893	62.0	9.0	13.0	-1.0	-52.0
1894	68.0	6.0	9.0	13.0	-1.0
1895	12.0	-56.0	6.0	9.0	13.0
1896	48.0	36.0	-56.0	6.0	9.0
1897	34.0	-14.0	36.0	-56.0	6.0
1898	28.0	-6.0	-14.0	36.0	-56.0
1899	40.0	12.0	-6.0	-14.0	36.0
1900	59.0	19.0	12.0	-6.0	-14.0
1901	52.0	-7.0	19.0	12.0	-6.0
1902	43.0	-9.0	-7.0	19.0	12.0
1903	67.0	24.0	-9.0	-7.0	19.0
1904	53.0	-14.0	24.0	-9.0	-7.0
1905	91.0	38.0	-14.0	24.0	-9.0
1906	50.0	-41.0	38.0	-14.0	24.0
1907	73.0	23.0	-41.0	38.0	-14.0
1908	26.0	-47.0	23.0	-41.0	38.0
1909	95.0	69.0	-47.0	23.0	-41.0
1910	66.0	-29.0	69.0	-47.0	23.0

ARI(1,1)														
22.05		SUMMARY OUTPUT - ARI(1,1)			ARI(1,1)									
23.58					Phi	**-0.618087**								
20.71		*Regression Statistics*			Theta0	**2.0523677**								
16.25		Multiple R	0.609498708											
25.73		R Square	0.371488675											
23.18		Adjusted R Square	0.356872132											
16.25		Standard Error	22.06163915											
21.14		Observations	45											
25.31														
52.33		ANOVA												
62.54			*df*	*SS*	*MS*	*F*	*ignificance F*							
36.00		Regression	1	12370.19313	12370.19	25.415633	8.838E-06							
37.16		Residual	43	20928.78465	486.7159									
42.85		Total	44	33298.97778										
32.23														
32.54			*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*Jpper 95.0%*				
44.20		Intercept	2.052367717	3.295096842	0.622855	0.5366693	-4.592828	8.697563814	-4.59283	8.6975638				
51.40		X Variable 1	-0.618087297	0.122602508	-5.04139	8.838E-06	-0.865339	-0.370835775	-0.86534	-0.370836				
50.89														
56.68														
72.87														
46.20		RESIDUAL OUTPUT					*Residual*							
46.52					*Residual*	*Residual*	*Difference*					*sum*41/*	*chi2*	*chi2*
46.73		*Observation*	*Predicted Y*	*Residuals*	*Squared*	*Difference*	*Squared*		*sumpro duct*	*squared*	*sums*	*sum-rsd-squared*	*inv*	*dist*
67.86		1	2.052367717	1.947632283	3.793272				-3087.26	9531174.4	9531174	0.9791989		
75.19		2	-0.41998147	-13.58001853	184.4169	180.62363	32624.896		-6829.86	46647021	5.6E+07	5.7715475	2.70554	0.01629
42.67		3	10.70558987	0.29441013	0.086677	-184.3302	33977.632		-2439.45	5950906.1	6.2E+07	6.3829224	4.60517	0.04111
47.02		4	-4.746592547	11.74659255	137.9824	137.89576	19015.24		3417.655	11680362	7.4E+07	7.5829212	6.25139	0.05547
58.49		5	-2.27424336	-15.72575664	247.2994	109.31699	11950.203		1847.569	3413510	7.7E+07	7.9336131	7.77944	0.09404
66.34		6	13.17793906	-2.177939057	4.743419	-242.556	58833.415		-3964.86	15720134	9.3E+07	9.5486436	9.23636	0.08908
48.67		7	-4.746592547	-0.253407453	0.064215	-4.679203	21.894943		334.8282	112109.9	9.3E+07	9.5601613	10.6446	0.14444
27.80		8	5.1428042	13.8571958	192.0219	191.95766	36847.743		655.3542	429489.12	9.3E+07	9.6042855	12.017	0.21213
44.71		9	-9.69129092	49.69129092	2469.224	2277.2025	5185651.3		-163.986	26891.431	9.4E+07	9.6070483	13.3616	0.2937
33.76		10	-22.67112415	-15.32887585	234.9744	-2234.25	4991872.9		727.6852	529525.68	9.4E+07	9.6614498	14.6837	0.37858
34.64		11	25.53968499	-33.53968499	1124.91	889.93603	791986.15		780.5005	609181.07	9.5E+07	9.7240349	15.9872	0.46503
49.31		12	6.99706609	9.00293391	81.05282	-1043.858	1089638.8		2007.057	4028279	9.9E+07	10.137886	17.275	0.51803
58.38		13	-7.83702903	-3.16297097	10.00439	-71.04843	5047.8799		-1814.99	3294189.3	1E+08	10.476319	18.5493	0.57425
50.62		14	8.85132798	-18.85132798	355.3726	345.36818	119279.18		-6163.11	37983985	1.4E+08	14.378658	19.8119	0.34773
54.22		15	8.233240684	8.766759316	76.85607	-278.5165	77571.439		3217.866	10354661	1.5E+08	15.442459	21.0641	0.34859
63.71		16	-8.455116327	11.45511633	131.2197	54.363621	2955.4033		4369.67	19094016	1.7E+08	17.404111	22.3071	0.29529
69.57		17	0.198105827	13.80189417	190.4923	59.272593	3513.2402		-2848.66	8114885.7	1.8E+08	18.237805	23.5418	0.31016
77.39		18	-6.600854437	-17.39914556	302.7303	112.23798	12597.365		-2251.51	5069295.5	1.8E+08	18.758606	24.769	0.34256
60.84		19	16.88646284	37.11353716	1377.415	1074.6844	1154946.5		122.9685	15121.256	1.8E+08	18.76016	25.9894	0.40673
57.10		20	-31.3243463	-13.6756537	187.0235	-1190.391	1417031.1		3800.646	14444912	2E+08	20.244179	27.2036	0.38002
54.40		21	29.86629607	-26.86629607	721.7979	534.77436	285983.62		-942.269	887871.19	2E+08	20.335395	28.412	0.43713
		22	0.198105827	-4.198105827	17.62409	-704.1738	495860.7		-1934.19	3741074.3	2E+08	20.71974	29.6151	0.47617
		23	4.524716903	2.475283097	6.127026	-11.49707	132.18253		1262.943	1595025.4	2E+08	20.883607	30.8133	0.52794
		24	-2.27424336	46.27424336	2141.306	2135.1786	4558987.5		-3333.74	11113830	2.1E+08	22.025403	32.0069	0.51872
		25	-25.14347334	-26.85652666	721.273	-1420.033	2016492.5		2017.833	4071648.9	2.2E+08	22.443709	33.1962	0.5528
		26	34.19290714	-35.19290714	1238.541	517.26769	267565.86		-340.851	116179.26	2.2E+08	22.455645	34.3816	0.60932
		27	2.670455013	10.32954499	106.6995	-1131.841	1281064.5		126.3531	15965.105	2.2E+08	22.457285	35.5632	0.66344
		28	-5.98276714	14.98276714	224.4833	117.78381	13873.026		814.0747	662717.69	2.2E+08	22.525371	36.7412	0.71023
		29	-3.510417953	9.510417953	90.44805	-134.0353	17965.451		220.7604	48735.164	2.2E+08	22.530377	37.9159	0.75617
		30	-1.656156063	-54.34384394	2953.253	2862.8053	8195654.3		-540.033	291636.16	2.2E+08	22.560339	39.0875	0.79627
		31	36.66525633	-0.665256331	0.442566	-2952.811	8719091.7		1648.597	2717870.6	2.2E+08	22.839563	40.256	0.8218
		32	-20.19877496	6.198774964	38.42481	37.982245	1442.6509		1074.271	1154058.5	2.2E+08	22.958127	41.4217	0.85065
		33	10.70558987	-16.70558987	279.0767	240.65192	57913.347		-1882.64	3544328.2	2.3E+08	23.322259	42.5847	0.86789
		34	5.760891497	6.239108503	38.92647	-240.1503	57672.146		-2598.28	6751048.4	2.3E+08	24.015838	43.7452	0.87323
		35	-5.364679844	24.36467984	593.6376	554.71115	307704.46		1041.796	1085338.9	2.3E+08	24.127341	44.9032	0.89522
		36	-9.69129092	2.69129092	7.243047	-586.3946	343858.6		1495.228	2235706.4	2.4E+08	24.35703	46.0588	0.9112
		37	6.378978793	-15.37897879	236.513	229.26994	52564.706		122.7183	15059.771	2.4E+08	24.358577	47.2122	0.9299
		38	7.615153387	16.38484661	268.4632	31.95021	1020.8159		31.94579	1020.5334	2.4E+08	24.358682	48.3634	0.9453
		39	-12.7817274	-1.218272596	1.484188	-266.979	71277.792		-712.871	508184.89	2.4E+08	24.410891	49.5126	0.95706
		40	10.70558987	27.29441013	744.9848	743.50064	552793.2		274.1246	75144.271	2.4E+08	24.418611	50.6598	0.9671
		41	-21.43494956	-19.56505044	382.7912	-362.1936	131184.22		611.8875	374406.29	2.4E+08	24.457076	51.8051	0.97478
		42	27.39394688	-4.393946881	19.30677	-363.4844	132120.93		-579.084	335337.92	2.4E+08	24.491528	52.9485	0.98087
		43	-12.16364011	-34.83635989	1213.572	1194.2652	1426269.4							
		44	31.10247066	37.89752934	1436.223	222.65076	49573.361							
		45	-40.59565575	11.59565575	134.4592	-1301.763	1694588.2	DWS						
					20928.78		45778017	2187.323258						

ARI(2,1)

22.35	SUMMARY OUTPUT				ARI(2,1)									
23.25					Phi1	**-0.7738**								
22.06	*Regression Statistics*				Phi2	**-0.281**								
18.77	Multiple R	0.643192482			Theta0	**2.34942**								
21.84	R Square	0.413696569												
24.31	Adjusted R Square	0.385777358												
19.90	Standard Error	21.56016217												
19.13	Observations	45												
24.05														
41.06	ANOVA													
57.51		*df*	*SS*	*MS*	*F*	*gnificance F*								
48.22	Regression	2	13775.673	6887.8364	14.8176	1.35E-05								
37.22	Residual	42	19523.305	464.84059										
40.37	Total	44	33298.978											
37.18														
33.00		*Coefficients*	*andard Erro*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*wer 95.0%*	*pper 95.0%*					
39.25	Intercept	2.349421766	3.2247252	0.728565	0.47031	-4.15834	8.857181	-4.1583	8.85718					
48.67	X Variable 1	-0.77383802	0.1495956	-5.172866	6.1E-06	-1.07573	-0.47194	-1.0757	-0.4719					
50.99	X Variable 2	-0.28102131	0.161614	-1.738843	0.08939	-0.60717	0.045129	-0.6072	0.04513					
55.31														
65.00														
58.67														
46.60	RESIDUAL OUTPUT					*Residual*								
47.06				*Residual*	*Residual*	*Difference*						*sum*41/*	*chi2*	*chi2*
59.33	*Observation*	*Predicted Y*	*Residuals*	*Squared*	*Differe nce*	*Squared*		*sumpro duct*	*square d*	*sums*	*sum-rsd-*	*inv*	*dist*	
71.22	1	2.349421766	1.6505782	2.7244085				-1774.2	3147802	3147802	0.37163			
57.74	2	-0.74593032	-13.25407	175.67036	172.946	29910.3		-4670.4	2.2E+07	2.5E+07	2.94689	2.7055	0.086	
45.57	3	12.05906882	-1.059069	1.1216268	-174.55	30467.26		-4536.5	2.1E+07	4.6E+07	5.37651	4.6052	0.068	
53.73	4	-2.2284981	9.2284981	85.165177	84.0436	7063.318		3102.34	9624510	5.5E+07	6.51279	6.2514	0.0892	
63.18	5	-6.15867882	-11.84132	140.21689	55.0517	3030.691		1541.17	2375215	5.8E+07	6.79321	7.7794	0.1472	
56.00	6	14.31135697	-3.311357	10.965085	-129.25	16706.03		-3309.7	1.1E+07	6.8E+07	8.08647	9.2364	0.1515	
38.23	7	-1.10441285	-3.895587	15.175599	4.21051	17.72843		-280.92	78916.5	6.9E+07	8.09578	10.645	0.2312	
37.07	8	3.12737744	15.872623	251.94015	236.765	56057.45		-103.72	10758	6.9E+07	8.09705	12.017	0.3241	
38.93	9	-10.9483941	50.948394	2595.7389	2343.8	5493392		818.971	670713	6.9E+07	8.17624	13.362	0.4164	
34.75	10	-33.943504	-4.056496	16.45516	-2579.3	6652704		611.902	374424	7E+07	8.22044	14.684	0.5121	
43.27	11	20.5144141	-28.51441	813.07181	796.617	634598.1		1450.55	2104087	7.2E+07	8.46885	15.987	0.5831	
54.43	12	19.2189358	-3.218936	10.361548	-802.71	644343.8		1006.53	1013107	7.3E+07	8.58846	17.275	0.6598	
54.28	13	-7.78381608	-3.216184	10.343839	-0.0177	0.000314		-2446.7	5986164	7.9E+07	9.29519	18.549	0.6775	
53.31	14	6.365299008	-16.3653	267.82301	257.479	66295.52		-5039	2.5E+07	1E+08	12.2929	19.812	0.5038	
59.44	15	13.17903641	3.8209636	14.599763	-253.22	64122.01		2371.29	5623019	1.1E+08	12.9568	21.064	0.5299	
67.88	16	-7.99561148	10.995611	120.90347	106.304	11300.48		4394.8	1.9E+07	1.3E+08	15.237	22.307	0.4345	
73.40	17	-4.7494546	18.749455	351.54205	230.639	53194.15		-2394.8	5735205	1.3E+08	15.9141	23.542	0.459	
69.07	18	-9.32737447	-14.67263	215.28594	-136.26	18565.73		-2226.1	4955679	1.4E+08	16.4992	24.769	0.4888	
58.26	19	16.98723591	37.012764	1369.9447	1154.66	1333237		106.205	11279.5	1.4E+08	16.5005	25.989	0.5577	
57.16	20	-32.6933199	-12.30668	151.45438	-1218.5	1484719		3555.67	1.3E+07	1.5E+08	17.9932	27.204	0.5229	
	21	21.99698188	-18.99698	360.88532	209.431	43861.32		203.974	41605.2	1.5E+08	17.9981	28.412	0.5875	
	22	12.67386675	-16.67387	278.01783	-82.867	6867.021		-2898.7	8402259	1.6E+08	18.9901	29.615	0.5858	
	23	4.601709916	2.3982901	5.7517953	-272.27	74128.79		579.709	336063	1.6E+08	19.0297	30.813	0.6435	
	24	-1.94335914	45.943359	2110.7922	2105.04	4431195		-3050.3	9304322	1.7E+08	20.1282	32.007	0.6342	
	25	-33.6666004	-18.3334	336.11354	-1774.7	3149485		2138.74	4574191	1.8E+08	20.6682	33.196	0.6582	
	26	30.22406115	-31.22406	974.94199	638.828	408101.8		-674.73	455258	1.8E+08	20.722	34.382	0.708	
	27	17.73636802	-4.736368	22.433182	-952.51	907273		683.645	467371	1.8E+08	20.7772	35.563	0.7534	
	28	-7.4294512	16.429451	269.92687	247.494	61253.12		561.657	315459	1.8E+08	20.8144	36.741	0.7949	
	29	-8.26839748	14.268397	203.58717	-66.34	4400.956		384.565	147890	1.8E+08	20.8319	37.916	0.8323	
	30	-4.82279817	-51.1772	2619.106	2415.52	5834731		-68.916	4749.42	1.8E+08	20.8324	39.087	0.8652	
	31	43.99822309	-7.998223	63.971573	-2555.1	6528712		1783.67	3181464	1.8E+08	21.208	40.256	0.8813	
	32	-9.77155353	-4.228446	17.87976	-46.092	2124.455		793.27	629277	1.8E+08	21.2823	41.422	0.9043	
	33	3.066386831	-9.066387	82.19937	64.3196	4137.012		-1912.4	3657178	1.8E+08	21.7141	42.585	0.9148	
	34	10.92674826	1.0732517	1.1518693	-81.048	6568.697		-2579.2	6652505	1.9E+08	22.4995	43.745	0.9159	
	35	-5.25050662	24.250507	588.08707	586.935	344492.9		989.08	978278	1.9E+08	22.615	44.903	0.932	
	36	-15.7257564	8.7257564	76.138825	-511.95	262091		1518.43	2305632	1.9E+08	22.8872	46.059	0.9426	
	37	2.426882986	-11.42688	130.57365	54.4348	2963.151		-28.879	833.97	1.9E+08	22.8873	47.212	0.956	
	38	11.28111314	12.718887	161.77008	31.1964	973.2171		-49.618	2461.94	1.9E+08	22.8876	48.363	0.9666	
	39	-13.6934989	-0.306501	0.0939429	-161.68	26139.17		-572.39	327626	1.9E+08	22.9263	49.513	0.9745	
	40	6.438642576	31.561357	996.11928	996.025	992066.5		255.826	65446.8	1.9E+08	22.934	50.66	0.981	
	41	-23.1221247	-17.87788	319.61843	-676.5	457653.4		612.878	375619	1.9E+08	22.9783	51.805	0.9858	
	42	23.39797078	-0.397971	0.1583807	-319.46	102054.7		-567.46	322009	1.9E+08	23.0164	52.949	0.9895	
	43	-3.92697893	-43.07302	1855.2851	1855.13	3441495								
	44	32.25631859	36.743681	1350.0981	-505.19	255213.9								
	45	-37.8374	8.8374	78.09964	-1272	1617980	DWS							
				19523.305		45565687	2333.913							

ARI(3,1)

22.98	SUMMARY OUTPUT			ARI(3,1)										
23.53				Phi1	**-0.8633759**									
22.90	*Regression Statistics*			Phi2	**-0.5415729**									
20.54	Multiple R	0.697397		Phi3	**-0.3824261**									
24.34	R Square	0.486362		Theta0	**2.9838047**									
20.53	Adjusted R Square	0.448779												
21.56	Standard Error	20.42453												
24.23	Observations	45												
20.08														
35.07	ANOVA													
43.86		*df*	*SS*	*MS*	*F*	*Significance F*								
44.17	Regression	3	16195.4	5398.455	12.940931	4.3492E-06								
53.03	Residual	41	17103.6	417.1613										
40.88	Total	44	33299											
35.46														
38.93		*Coefficients*	*Standard Er*	*t Stat*	*P-value*	*Lower 95%*	*pper 95%*	*ower 95.0%*	*pper 95.0%*					
39.01	Intercept	2.983805	3.0662	0.973126	0.336197	-3.208521	9.1761	-3.208521	9.17613					
40.77	X Variable 1	-0.86338	0.14651	-5.892894	6.144E-07	-1.1592615	-0.567	-1.159262	-0.56749					
48.98	X Variable 2	-0.54157	0.18747	-2.888895	0.0061501	-0.9201705	-0.163	-0.920171	-0.16298					
52.01	X Variable 3	-0.38243	0.15879	-2.408397	0.020603	-0.7031063	-0.062	-0.703106	-0.06175					
64.77														
50.11														
64.02														
46.96	RESIDUAL OUTPUT					*Residual*								
55.73				*Residual*	*Residual*	*Difference*						*sum*41/*	*chi2*	*chi2*
62.37	*Observation*	*Predicted Y*	*Residuals*	*Squared*	*Difference*	*Squared*		*sumproduct*	*squared*	*sums*		*sum-rsd-squared*	*inv*	*dist*
55.18	1	2.983805	1.0162	1.032653				-1510.643	2282042	2282042		0.351043		
65.19	2	-0.4697	-13.53	183.0691	182.0364	33137.251		-617.3327	381099.7	2663141		0.409667	2.7055	0.5221
50.56	3	12.90477	-1.9048	3.628167	-179.44089	32199.0315		-2376.927	5649780	8312921		1.278764	4.6052	0.5276
55.96	4	-0.46101	7.46101	55.66672	52.038551	2708.01075		-1863.362	3472120	1.2E+07		1.812875	6.2514	0.6121
56.64	5	-3.66316	-14.337	205.5449	149.87819	22463.4717		1717.858	2951035	1.5E+07		2.266829	7.7794	0.6868
47.94	6	10.52687	0.47313	0.22385	-205.32106	42156.7367		-3218.729	10360218	2.5E+07		3.860525	9.2364	0.5697
50.99	7	0.558	-5.558	30.89137	30.667515	940.496476		141.486	20018.29	2.5E+07		3.863605	10.645	0.6951
29.98	8	8.227052	10.7729	116.0564	85.165039	7253.08385		-647.0532	418677.9	2.6E+07		3.928009	12.017	0.788
41.23	9	-14.9192	54.9192	3016.114	2900.0577	8410334.43		134.5356	18099.82	2.6E+07		3.930794	13.362	0.8633
41.38	10	-39.929	1.92898	3.720981	-3012.3931	9074512.09		2809.541	7893521	3.3E+07		5.145042	14.684	0.8215
46.15	11	6.863073	-14.863	220.9109	217.18997	47171.4821		-155.3427	24131.35	3.3E+07		5.148754	15.987	0.881
50.28	12	15.17354	0.82646	0.683041	-220.22791	48500.3314		-336.3576	113136.4	3.4E+07		5.166158	17.275	0.9229
56.81	13	8.034568	-19.035	362.3148	361.63173	130777.506		-1271.05	1615567	3.5E+07		5.414678	18.549	0.9427
58.52	14	6.875181	-16.875	284.7717	-77.543024	6012.92056		-4302.388	18510542	5.4E+07		8.262127	19.812	0.8261
59.58	15	11.45605	5.54395	30.73541	-254.03633	64534.4581		1650.329	2723585	5.6E+07		8.681092	21.064	0.8509
73.16	16	-2.07117	5.07117	25.71674	-5.0186674	25.187022		2199.61	4838283	6.1E+07		9.425358	22.307	0.8542
63.80	17	-4.9888	18.9888	360.5746	334.85783	112129.766		-1279.939	1638243	6.3E+07		9.677367	23.542	0.8829
72.79	18	-17.2294	-6.7706	45.84075	-314.73383	99057.3806		89.04385	7928.807	6.3E+07		9.678586	24.769	0.9165
55.07	19	14.97553	39.0245	1522.91	1477.0688	2181732.37		-1046.161	1094454	6.4E+07		9.846945	25.989	0.9368
	20	-35.9947	-9.0053	81.0953	-1441.8143	2078828.44		3048.556	9293692	7.3E+07		11.27658	27.204	0.9142
	21	21.76901	-18.769	352.2756	271.18033	73538.7698		8.82736	77.92229	7.3E+07		11.27659	28.412	0.9387
	22	4.113448	-8.1134	65.82803	-286.4476	82052.227		-2219.329	4925422	7.8E+07		12.03426	29.615	0.9387
	23	22.02177	-15.022	225.6534	159.82541	25544.1609		-31.6812	1003.699	7.8E+07		12.03442	30.813	0.9567
	24	-2.04081	46.0408	2119.756	1894.103	3587626.25		-3743.306	14012342	9.2E+07		14.18991	32.007	0.9213
	25	-37.266	-14.734	217.0896	-1902.6668	3620141.12		1922.064	3694331	9.6E+07		14.75821	33.196	0.9276
	26	21.37316	-22.373	500.5582	283.46855	80354.4177		118.8907	14135.01	9.6E+07		14.76038	34.382	0.9468
	27	15.18222	-2.1822	4.762097	-495.79607	245813.739		129.0041	16642.05	9.6E+07		14.76294	35.563	0.9615
	28	12.18765	-3.1877	10.16112	5.3990204	29.1494213		1113.522	1239931	9.7E+07		14.95368	36.741	0.9701
	29	-11.4446	17.4446	304.3141	294.15295	86525.9574		938.6528	881069.1	9.8E+07		15.08921	37.916	0.9775
	30	-12.0421	-43.958	1932.293	1627.9788	2650315.02		646.8435	418406.5	9.9E+07		15.15357	39.087	0.9838
	31	44.64158	-8.6416	74.6769	-1857.616	3450737.13		1251.934	1567339	1E+08		15.39468	40.256	0.9873
	32	-0.0642	-13.936	194.2066	119.52966	14287.3391		156.3722	24452.27	1E+08		15.39844	41.422	0.9913
	33	16.9903	-22.99	528.5541	334.34756	111788.292		-1201.94	1444661	1E+08		15.62067	42.585	0.9933
	34	1.97874	10.0213	100.4257	-428.12846	183293.982		-2068.813	4279986	1.1E+08		16.27905	43.745	0.9935
	35	1.226698	17.7733	315.8903	215.46461	46424.9984		357.5699	127856.2	1.1E+08		16.29872	44.903	0.9956
	36	-17.6247	10.6247	112.8833	-203.00697	41211.8319		1207.178	1457278	1.1E+08		16.52289	46.059	0.9966
	37	-5.85156	-3.1484	9.912652	-102.97064	10602.9524		-86.10119	7413.415	1.1E+08		16.52403	47.212	0.9978
	38	7.279101	16.7209	279.5885	269.6758	72725.0385		65.33999	4269.314	1.1E+08		16.52469	48.363	0.9985
	39	-10.1861	-3.8139	14.54601	-265.04244	70247.4956		-432.405	186974.1	1.1E+08		16.55345	49.513	0.999
	40	5.515152	32.4848	1055.265	1040.7194	1083096.8		73.40406	5388.156	1.1E+08		16.55428	50.66	0.9994
	41	-31.4207	-9.5793	91.7633	-963.50208	928336.262		550.5088	303059.9	1.1E+08		16.6009	51.805	0.9996
	42	23.15641	-0.1564	0.024464	-91.738832	8416.01327		-359.797	129453.9	1.1E+08		16.62081	52.949	0.9997
	43	-9.20154	-37.798	1428.723	1428.6989	2041180.51								
	44	46.78576	22.2142	493.4723	-935.25107	874694.565								
	45	-39.931	10.931	119.4868	-373.98546	139865.126	DWS							
				17103.61		41923323.6	2451.1							