Time Series Project

Session: *Winter 2016*

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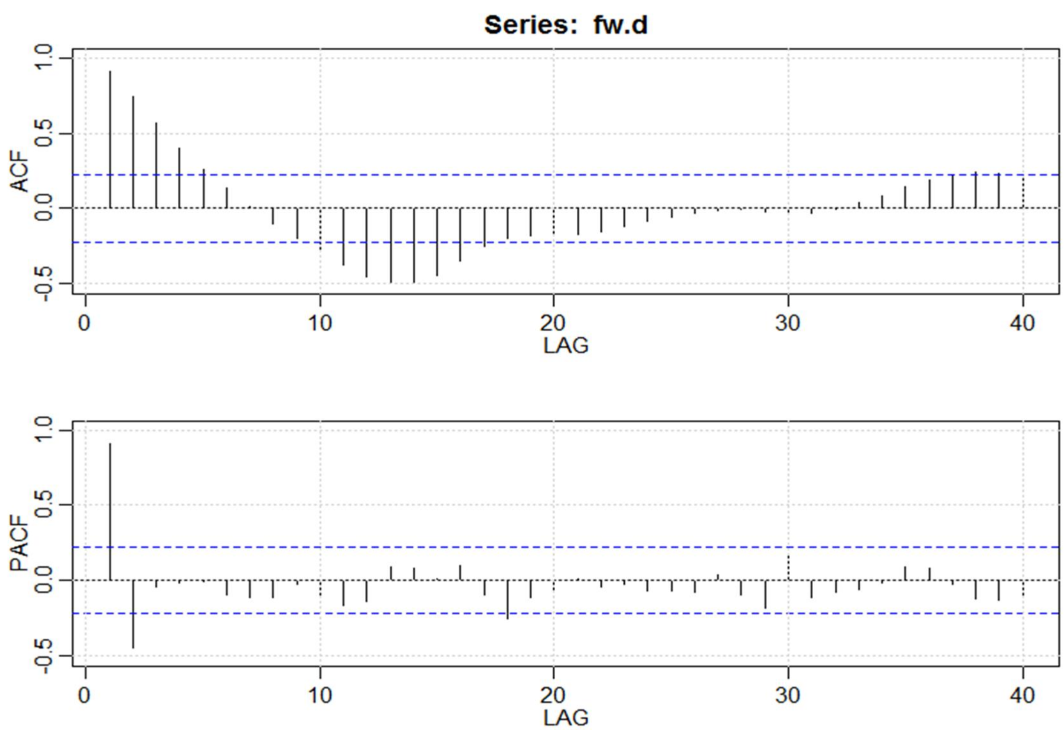
Farm Wages Series with Supply and Price of Hog

The time series of Farm Wages is denoted by *fw*, Hog Price by *hp*, and Hog Supply by *hs*.

Preliminaries

1.The ACF and PACF of detrended Farm Wages, i.e. fw=fw.d, are



We can see that there are only two large spikes in the PACF. Hence the PACF appears to cut off after lag 2, while ACF appears to be dying down, indicating a tentative AR(2) model.

The coefficients of and AR(2) model fitted to *fw.d* are

```
Coefficients:
         ar1      ar2   xmean
      1.4483  -0.5597  0.0085
s.e.  0.0903   0.0918  0.0294
sigma^2 estimated as 0.000914
```

We use these coefficients to pre-whiten *fw* to get *fw.pw*.

The diagnostics all check out:

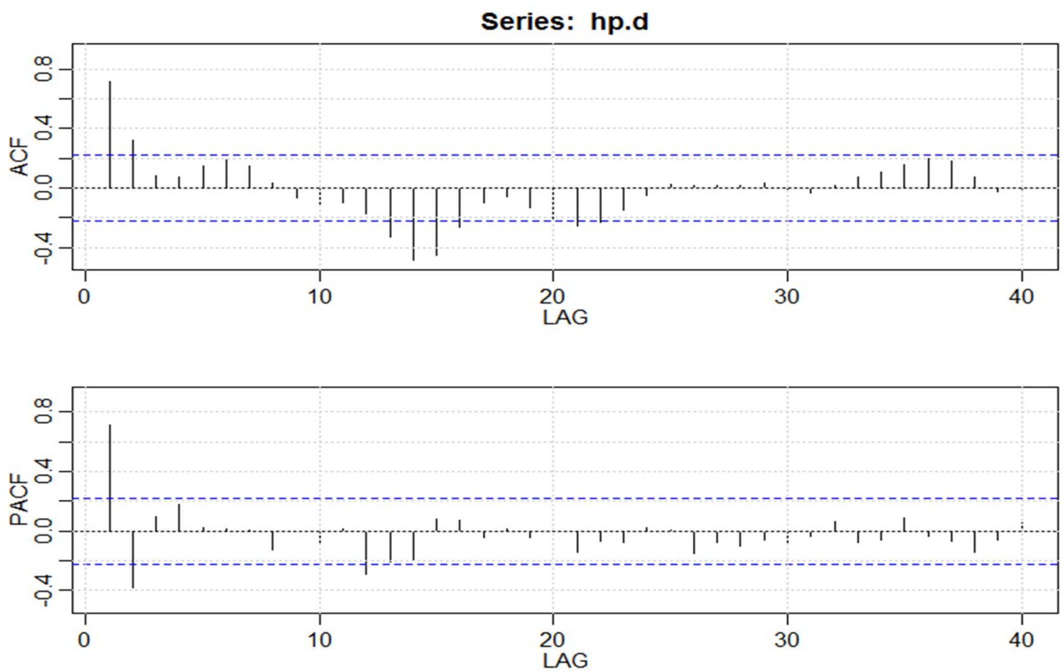
1. The standardized residuals are mostly within 2 standard deviations with only a few outliers;

2. The ACF of the residuals appear to be uncorrelated;

3. The QQ-plot shows that the normality assumption is approximately satisfied except for some outliers.

4. The p-values for Ljung-Box statistic are highly non-significant in all the available lags.

2.The ACF and PACF of detrended Hog Price, i.e. hp=hp.d, are



It appears that the PACF cuts off after lag 2, while the ACF is dying down, indicating a tentative AR(2) model. Or it is also possible that the ACF cuts off after lag 2, while the PACF is dying down, indicating a tentative MA(2) model.

The AIC for AR(2) fit is -3.487953, while the AIC for MA(2) is -3.494924. And the coefficients for MA(2) have smaller standard error. Also the p-values of the Ljung-Box statistic are larger in MA(2). So we conclude MA(2) is a more appropriate model.

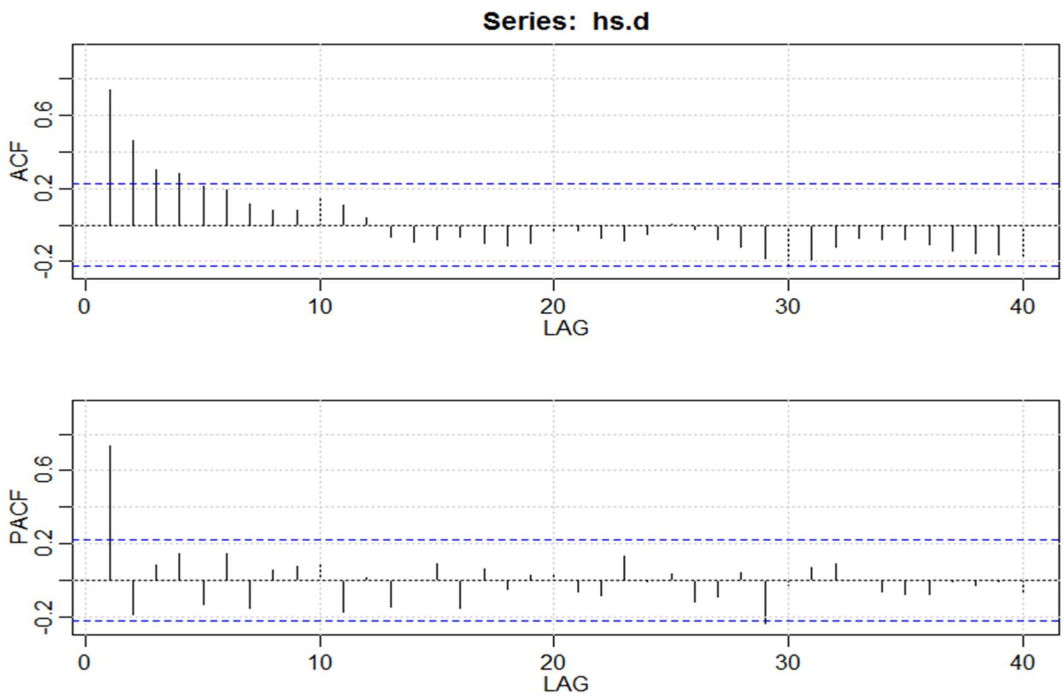
The coefficients of and MA(2) model fitted to *hp.d* are

```
Coefficients:
         ma1     ma2    xmean
      1.0128  0.5623  -0.0001
s.e.  0.0866  0.0933   0.0288
sigma^2 estimated as 0.01037
```

We use these coefficients to pre-whiten *hp* to get *hp.pw*.

The diagnostics are similar to *fw.d*, except that we only have non-significant p-values before lag 15 in the Ljung-Box statistics.

<u>3.The ACF and PACF of detrended Hog Supply, i.e. hs=hs.d, are</u>



The PACF appears to cut off after lag 1, and the ACF appears to die down, indicating a tentative AR(1) model.

The coefficients of and AR(1) model fitted to *hs.d* are

```
Coefficients:
         ar1    xmean
      0.7508  -0.0060
s.e.  0.0731   0.0222
sigma^2 estimated as 0.002634
```

We use these coefficients to pre-whiten *hs* to get *hs.pw*.

The diagnostics are similar to the case in *fw.d*, except that the p-values before lag 15 in the Ljung-Box statistics are not as large, but still non-significant in most of the available lags.

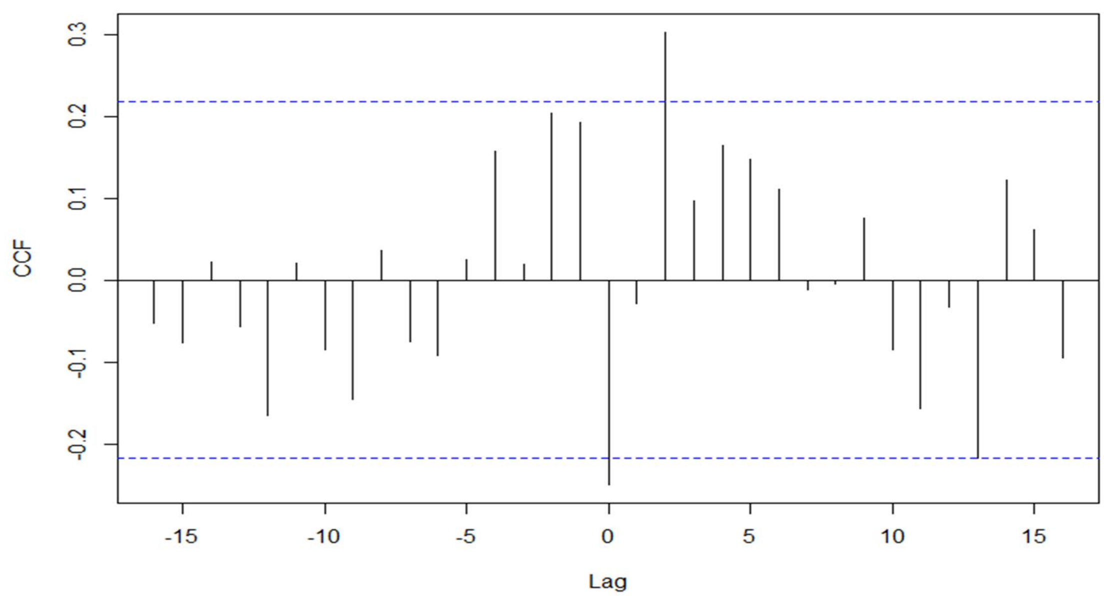
Further Analysis

1.Hog Supply vs Hog Price

We use the parameters of the MA(2) fitting to *hp.d* to filter *hs.d*. Hence

$$hs.d.fil_{hp} = \frac{1}{1 + ma1_{hp}B + ma2_{hp}B^2} hs.d$$

The CCF between the Hog Supply series filtered by Hog Price, i.e. *hs.d.fil_hp* and the prewhitened Hog Price series, i.e. *hp.pw*, is



We can see that the largest spike is at lag 2 and the CCF appears to cut off, indicating the following form of the transfer function:

$$\alpha(B) = (\delta_0 + \delta_1 B)B^2$$

Hence the transfer function model can be written as

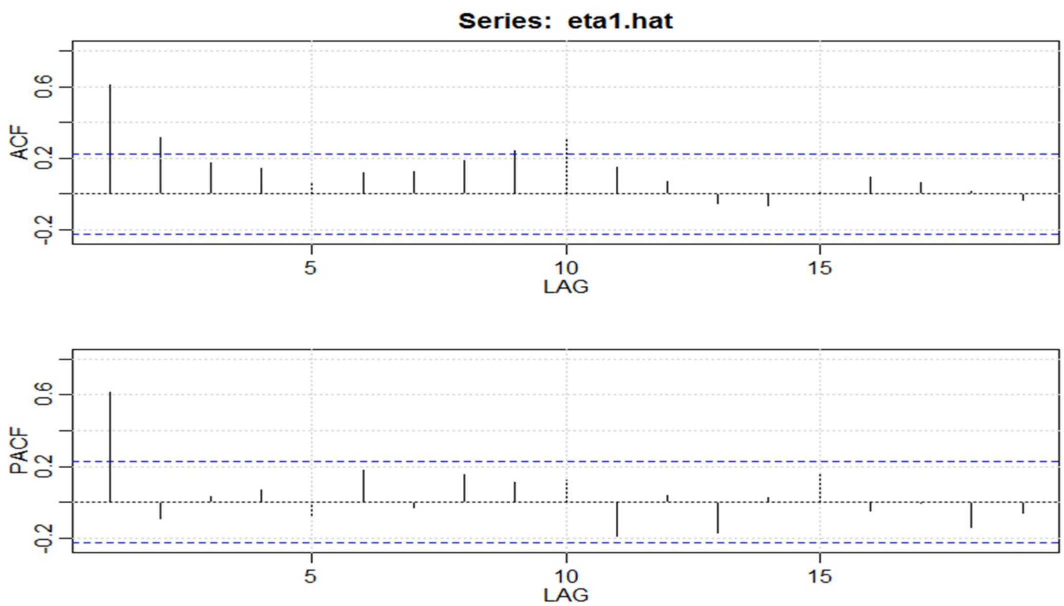
$$hs.d_t = (\delta_0\, B^2 + \delta_1 B^3)hp.d_t + \eta_t$$

Running the regression using this model, we get

```
Coefficients:
       Estimate Std. Error t value Pr(>|t|)
hp.d2 -0.001146   0.066538  -0.017  0.98630
hp.d3  0.221045   0.066351   3.331  0.00134 **
```

We notice that δ_0 is highly insiginificant, but $\delta_1 = 0.2210$.

The ACF and the PACF of the regression residuals η_t are



We can see that the PACF appears to cut off after lag 1, while the ACF appears to die down, indicating an AR(1) model.

The coefficients of and AR(1) model fitted to *eta1.hat* are

```
Coefficients:
         ar1   xmean
      0.6416  0.0023
s.e.  0.0890  0.0157
sigma^2 estimated as 0.002568
```

The diagnostics are good: All the standardized residuals seem uncorrelated and mostly satisfy the normal assumption, and the p-values for the Ljung-Box statistic are highly non-significant in almost all the available lags.

We can also rule out possible MA(1) and MA(2) model for *eta1.hat* based on Ljung-Box statistic, because all the p-values are significant in the case of MA(1) and MA(2).

Therefore, the final transfer function model for Hog Supply vs Hog Price is

$$hs.d_t = (0.2210\ B^3)\ hp.d_t + \frac{1}{1 - 0.6416\ B} z_t$$

where z_t is white noise with variance 0.002568.

The forecasts for the next 4 values using SAS are

Forecasts for variable hogsupply

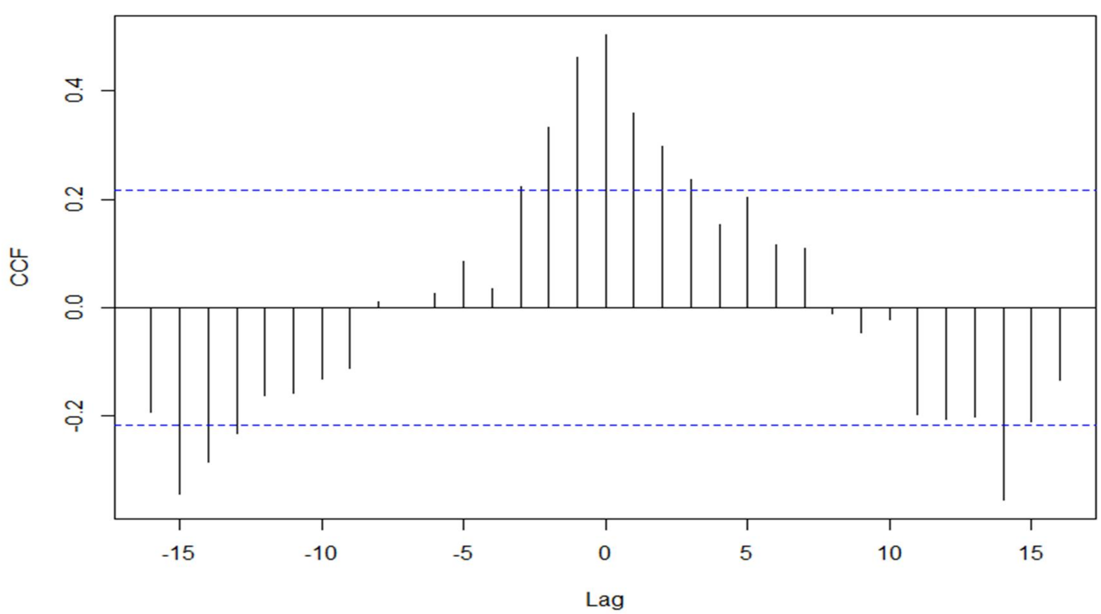
Obs	Forecast	Std Error	95% Confidence Limits	
82	6.6263	0.0546	6.5193	6.7332
83	6.6358	0.0764	6.4861	6.7854
84	6.6367	0.0925	6.4554	6.8180
85	6.6245	0.1057	6.4174	6.8316

2.Farm Wages vs Hog Price

We use the parameters of the MA(2) fitting to *hp.d* to filter *fw.d*. Hence

$$fw.d.fil_{hp} = \frac{1}{1 + ma1_{hp}B + ma2_{hp}B^2} fw.d$$

The CCF between the Farm Wages series filtered by Hog Price, i.e. *fw.d.fil_hp* and the prewhitened Hog Price series, i.e. *hp.pw*, is



We can see that the largest spike is at lag 0 and the CCF appears to die down, indicating the following form of the transfer function:

$$\alpha(B) = \frac{1}{1 - \omega_1 B}$$

Hence the transfer function model can be written as

$$(1-\omega_1 B)\, fw.d_t = \delta_0\, hp.d_t + (1-\omega_1 B)\,\eta_t$$

We will denote $(1-\omega_1 B)\eta_t$ as u_t.

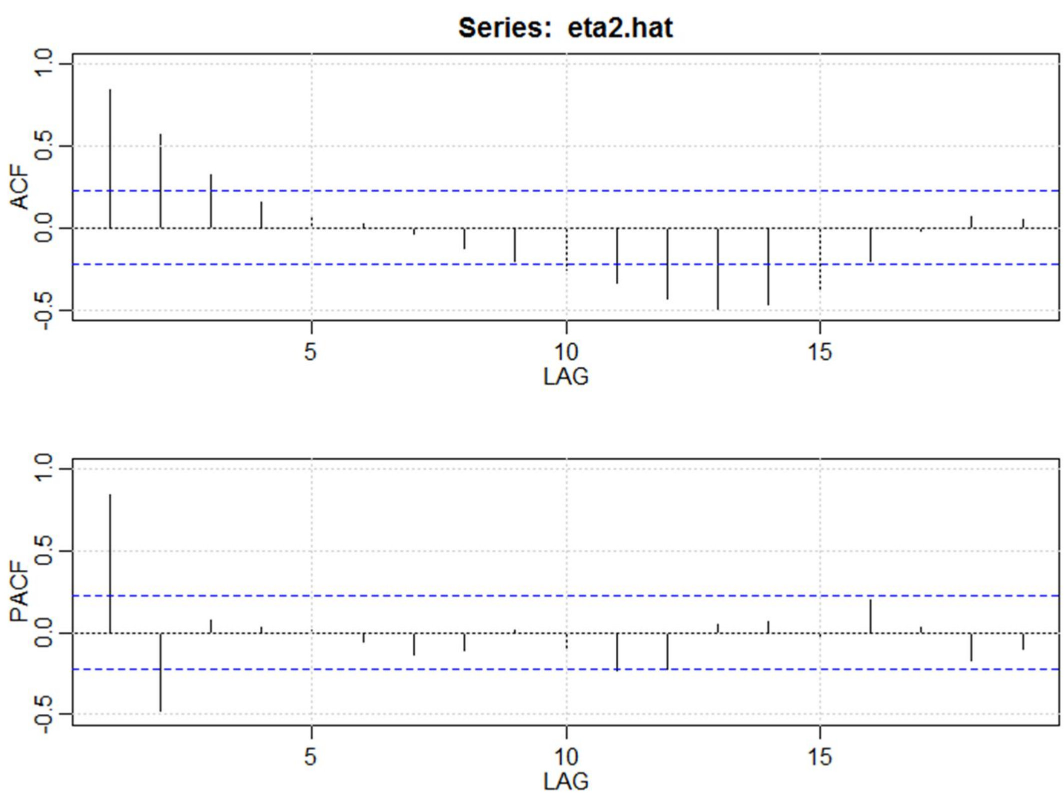
Running the regression using this model, we get

```
Coefficients:
      Estimate Std. Error t value Pr(>|t|)
fw.d1  0.78545    0.06150  12.772   <2e-16 ***
hp.d   0.12115    0.03572   3.391   0.0011 **
```

So $\omega_1 = 0.7855$ and $\delta_0 = 0.1212$.

And we use $\eta_t = \frac{1}{1-\omega_1 B} u_t$ in order to find η_t, where u_t is the regression residuals.

The ACF and the PACF of the regression residuals η_t are



We can see that the PACF appears to cut off after lag 2, while the ACF appears to die down, indicating an AR(2) model.

The coefficients of and AR(1) model fitted to *eta2.hat* are

```
Coefficients:
         ar1      ar2    xmean
      1.2764  -0.4959  -0.0002
s.e.  0.0958   0.0980   0.0148
sigma^2 estimated as 0.0008669
```

The diagnostics are okay: All the standardized residuals seem uncorrelated and mostly satisfy the normal assumption except for some outliers, and the p-values for the Ljung-Box statistic are highly non-significant, except for after lag 13, but none of the p-values are below 0.05.

Therefore, the final transfer function model for Farm Wages vs Hog Price is

$$fw.d_t = \frac{0.1212}{1 - 0.7855\,B}\,hp.d_t + \frac{1}{1 - 1.2764\,B + 0.4959\,B^2}\,z_t$$

where z_t is white noise with variance 0.0008669.

The forecasts for the next 4 values using SAS are

Forecasts for variable farmwages

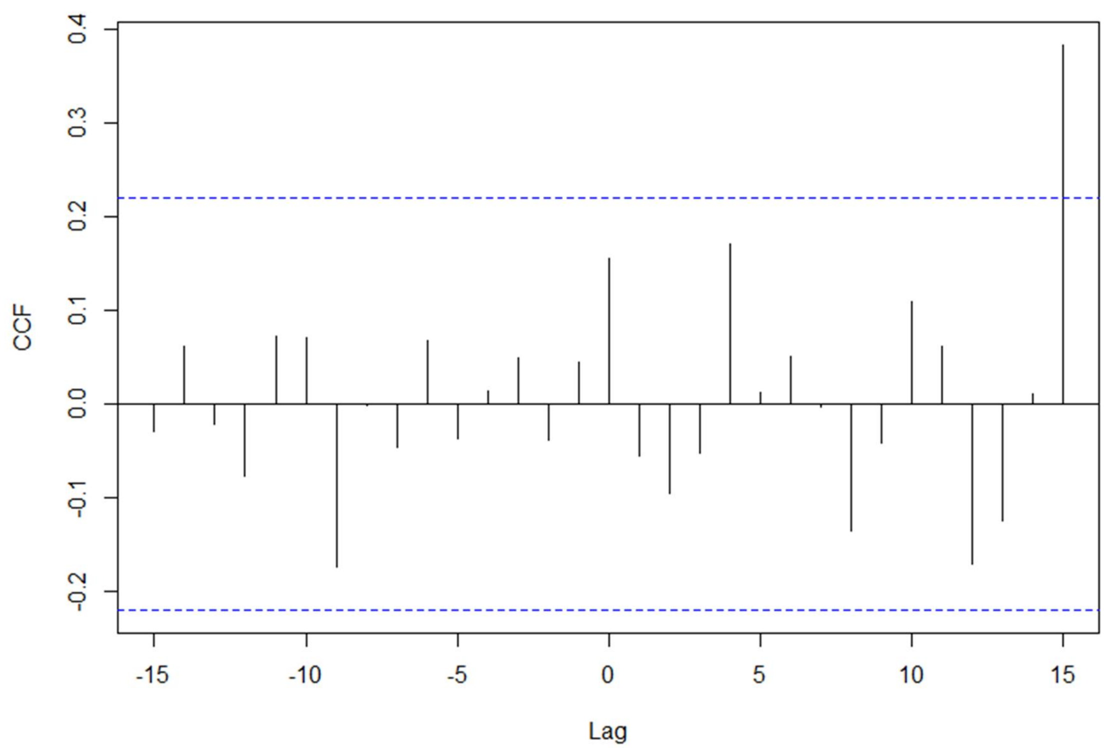
Obs	Forecast	Std Error	95% Confidence Limits	
82	7.3942	0.0321	7.3312	7.4572
83	7.3850	0.0554	7.2764	7.4935
84	7.3803	0.0748	7.2337	7.5269
85	7.3808	0.0913	7.2018	7.5599

3.Hog Supply vs Farm Wages

We use the parameters of the AR(2) fitting to *fw.d* to filter *hs.d*. Hence

$$hs.d.fil_{fw} = (1 - ar1_{fw}B - ar2_{fw}B^2)hs.d$$

The CCF between the Hog Supply series filtered by Farm Wages, i.e. *hs.d.fil_fw* and the prewhitened Farm Wages series, i.e. *fw.pw*, is



There is no distinguishable feature on the CCF except for the spike at lag 15. But there are not enough data (approximately 80) to perform statistical analysis with a lag of 15.