MS Module 2: Confidence intervals – practice problems

(The attached PDF file has better formatting.)

Exercise 2.1: Confidence interval

A sample from a normal distribution has summary statistics:

- $n = 50$
- $\sum x_i = 991$
- $\sum x_i^2 = 20{,}635$

A. What is the estimated μ, the mean of the normal distribution?
B. What is the estimated σ, the standard deviation of the normal distribution?
C. What is the 95% confidence interval for μ?

Part A: The estimated $\mu = 991 / 50 = 19.82$.

Part B: The estimated $\sigma = ((20{,}635 – 991^2/50) / (50 – 1))^{0.5} = 4.5026$

Part C: The 95% confidence interval is

- Lower bound: $19.82 – 1.96 \times 4.5025 / 50^{0.5} = 18.5720$
- Upper bound: $19.82 + 1.96 \times 4.5025 / 50^{0.5} = 21.0680$

Exercise 2.2: Confidence intervals

A statistician forms confidence intervals for the mean of a normally distributed population from a sample of 80 observations.

- The upper bound of the 95% confidence interval is 5.
- The lower bound of the 90% confidence interval is 1.

A. What is the estimated standard deviation of the population?
B. What is the estimated mean of the population?

Part A: Let μ be the estimated mean and σ be the estimated standard deviation.

- The (1–z) confidence interval for the mean is $(\mu - z_{\alpha/2} \times \sigma, \mu + z_{\alpha/2} \times \sigma)$.
- The $z_{\alpha/2}$ values are 1.96 for a 95% confidence interval and 1.645 for a 90% confidence interval.

We have two equations in two unknowns:

$\mu + 1.96 \times \sigma/\sqrt{n} = 5$
$\mu - 1.645 \times \sigma/\sqrt{n} = 1$

The first equation minus the second equation gives

$\sigma/\sqrt{n} = (5 - 1) / (1.96 + 1.645) = 1.1096$, so $\sigma = 1.1096 \times 8^{0.5} = 3.1384$

Part B: $\mu = 5 - 1.96 \times 1.1096 = 2.8252$

Exercise 2.3: μ and σ

A statistician estimates confidence intervals from a sample of N observations for the mean (μ) of a normal distribution with a known variance σ^2.

- The upper bound of the 95% confidence interval is 5.
- The lower bound of the 90% confidence interval is 1.

A. What is the $z_{\alpha/2}$ for the 95% confidence interval?
B. What is the $z_{\alpha/2}$ for the 90% confidence interval?
C. What is $\sigma/\sqrt{N}$, the standard deviation of the sample mean?
D. What is the estimated mean ($\bar{x}$)?
E. If N = 8, what is σ, the standard deviation of the normal distribution?

Part A: For the 95% confidence interval, $z_{\alpha/2} = z_{0.025} = 1.959964$ (table look-up or spreadsheet function).

Part B: For the 90% confidence interval, $z_{\alpha/2} = z_{0.05} = 1.644854$ (table look-up or spreadsheet function).

Part C: We have two equations in two unknowns: $\bar{x}$ and $\sigma/\sqrt{N}$

- $(5 - \bar{x}) = 1.959964 \times \sigma / \sqrt{N}$
- $(\bar{x} - 1) = 1.644854 \times \sigma / \sqrt{N}$

Adding the two equations gives

$(5 - 1) = (1.959964 + 1.644854) \times \sigma / \sqrt{N} \Rrightarrow$

$\sigma / \sqrt{N} = (5 - 1) / (1.959964 + 1.644854) = 1.109626$

Part D: $\bar{x} = 1 + 1.644854 \times \sigma / \sqrt{N} = 1 + 1.644854 \times 1.109626 = 2.825173$

Alternatively, $\bar{x} = 5 - 1.959964 \times \sigma / \sqrt{N} = 5 - 1.959964 \times 1.109626 = 2.825173$

Part E: If N = 8, $\sigma = 1.109626 \times 8^{0.5} = 3.138496$