MS Module 13: Two-factor ANOVA, one observation per cell – practice problems

(The attached PDF file has better formatting.)

Exercise 13.1: Two-factor ANOVA (one observation per cell)

A classification table has two rows, two columns, and one observation in each cell:

	Column 1	Column 2
Row 1	80	50
Row 2	40	20

We test whether Row 1 differs from Row 2 and whether Column 1 differs from Column 2. The ANOVA table calls the rows the A dimension and the columns the B dimension, following the usage in the textbook.

A. What is the square of the sum of all the observations, or $x_{..2}$?
B. What is the sum of the squares of all the observations, or $\Sigma_i \Sigma_j x_{ij}^2$?
C. What is the sum of the squares of the row totals, or $\Sigma_j x_{i..2}$
D. What is the sum of the squares of the column totals, or $\Sigma_j x_{.j.2}$
E. What is SST, the total sum of squared deviations?
F. What is SSA, the sum of squared deviations for the *i* dimension?
G. What is SSB, the sum of squared deviations for the *j* dimension?
H. What is SSE, the error sum of squared deviations?
I. What are the degrees of freedom for the rows (SSA)?
J. What are the degrees of freedom for the columns (SSB)?
K. What are the degrees of freedom for the total sum of squares (SST)?
L. What are the degrees of freedom for the error sum of squares (SSE)?
M. What is MSA, the mean squared deviation for the rows?
N. What is MSB, the mean squared deviation for the columns?
O. What is MSE, the mean squared error?
P. What is f_A, the *f* value for testing significance of the row differences?
Q. What is f_B, the *f* value for testing significance of the column differences?

Part A: The sum of all the observations is

$$80 + 50 + 40 + 20 = 190$$

The square of this sum is $190^2 = 36{,}100$

This squared sum, which does not differentiate by row or column, is used for the total sum of squares SST.

Part B: The sum of the squares of all the observations is

$$80^2 + 50^2 + 40^2 + 20^2 = 10{,}900$$

Part C: The row totals are 80 + 50 = 130 for Row 1 and 40 + 20 = 60 for Row 2. The sum of squares is

$$130^2 + 60^2 = 20{,}500$$

This sum of squares differentiates by row but not by column, so it is used for SSA.

	Column 1	Column 2	Total	Squared
Row 1	80	50	130	16,900
Row 2	40	20	60	3,600
Total	120	70	190	20,500
Squared	14,400	4,900	19,300	

Part D: The column totals are 80 + 40 = 120 for Column 1 and 50 + 20 = 70 for Column 2. The sum of squares is

$$120^2 + 70^2 = 19{,}300$$

This sum of squares differentiates by column but not by row, so it is used for SSB.

Part E: SST = $\Sigma_i \Sigma_j x_{ij}^2 - x_{..2}$ / N = 10,900 – 36,100 / 4 = 1,875.00

Part F: SSA =

the sum of squares of the row totals ÷ the number of columns
– the square of the sum of all the observations ÷ the number of observations =

$$½ \times 20{,}500 - 36{,}100 / 4 = 1{,}225.00$$

Part G: SSB =

the sum of squares of the columns totals ÷ the number of rows
– the square of the sum of all the observations ÷ the number of observations =

$$½ \times 19{,}300 - 36{,}100 / 4 = 625.00$$

Part H: SSE, the error sum of squares, = SST – (SSA + SSB) = 1,875 – (1,225 + 625) = 25

Part I: The degrees of freedom for the rows = (the number of rows – 1) = 2 – 1 = 1.

Part J: The degrees of freedom for the columns = (the number of columns) – 1 = 2 – 1 = 1.

Part K: The degrees of freedom for the total sum of squares = (the number of observations – 1) = 4 – 1 = 3.

Part L: The degrees of freedom for the total sum of squares = the sum of the degrees of freedom for SSA, SSB, and SSE ⇛ the degrees of freedom for SSE = 3 – 1 – 1 = 1.

Part M: MSA, the mean squared deviation for the rows, is SSA / degrees of freedom = 1,225 / 1 = 1,225.

Part N: MSB, the mean squared deviation for the columns, is SSB / degrees of freedom = 625 / 1 = 625.

Part O: MSE, the mean squared error, is SSE / degrees of freedom = 25 / 1 = 25.

Part P: The f_A (f value for testing significance of the row differences) is MSA / MSE = 1,225 / 25 = 49.00

The p value is $F_{49,1,1}$ = 0.090334.

Part Q: The f_B (f value for testing significance of the column differences) is MSB / MSE = 625 / 25 = 25.00

The p value is $F_{25,1,1}$ = 0.125666.