MS Module 23: Actuarial risk classification – practice problems

(The attached PDF file has better formatting.)

Reading on discussion forum: Actuarial risk classification

Exercise 23.1: Balance principle multiplicative model

The mean values and the number of observations in each cell of a 2 × 2 classification table are

Means	*Column 1*	*Column 2*	*Observations*	*Column 1*	*Column 2*
Row 1	50	30	*Row 1*	15	12
Row 2	20	8	*Row 2*	6	10

Illustration: The cell in row 1 column 1 has a mean of 50 from a sample of 15 observations.

An actuary is setting class relativities for insurance pricing using a multiplicative model balance principle, with

- a base rate of 10
- a starting relativity for column 1 of 1.8
- a starting relativity for column 2 of 1.0

We use the following notation:

B = base rate
r_1 = relativity for Row 1
r_2 = relativity for Row 2
c_1 = relativity for Column 1
c_2 = relativity for Column 2

A. What are the observed totals for each cell, row, and column?
B. What are the formulas for each cell, row, and column using base rates and relativities?
C. What is the equation to balance along Row 1?
D. What is the implied relativity for Row 1, given the starting relativities by column?
E. What is the equation to balance along Row 2?
F. What is the implied relativity for Row 2, given the starting relativities by column?
G. What is the equation to balance down Column 1?
H. What is the implied relativity for Column 1, given the computed relativities by row?
I. What is the equation to balance down Column 2?
J. What is the implied relativity for Column 2, given the computed relativities by row?

Part A: The observed totals by cell are

- Row 1, Column 1: 50 × 15 = 750
- Row 1, Column 2: 30 × 12 = 360
- Row 2, Column 1: 20 × 6 = 120
- Row 2, Column 2: 8 × 10 = 80

The table below shows the totals by row and by column:

	Column 1	Column 2	Total
Row 1	750	360	1,110
Row 2	120	80	200
Total	870	440	1,310

Part B: The formulas for the mean values by cell are

- Row 1, Column 1: $B \times r_1 \times c_1$
- Row 1, Column 2: $B \times r_1 \times c_2$
- Row 2, Column 1: $B \times r_2 \times c_1$
- Row 2, Column 2: $B \times r_2 \times c_2$

Using $obss_{j,k}$ as the number of observations in Row j and Column k, the totals by cell are

- Row 1, Column 1: $B \times r_1 \times c_1 \times obss_{1,1}$
- Row 1, Column 2: $B \times r_1 \times c_2 \times obss_{1,2}$
- Row 2, Column 1: $B \times r_2 \times c_1 \times obss_{2,1}$
- Row 2, Column 2: $B \times r_2 \times c_2 \times obss_{2,2}$

We add the expressions above for the totals by row and by column:

- Row 1: $B \times r_1 \times (c_1 \times obss_{1,1} + c_2 \times obss_{1,2})$
- Row 2: $B \times r_2 \times (c_1 \times obss_{2,1} + c_2 \times obss_{2,2})$
- Column 1: $B \times (r_1 \times obss_{1,1} + r_2 \times obss_{2,1}) \times c_1$
- Column 2: $B \times (r_1 \times obss_{1,2} + r_2 \times obss_{2,2}) \times c_2$

Part C: Using the formula for the Row 1 relativity, the base rate of 10, and the starting relativities of 1.8 for Column 1 and 1.0 for Column 2, we balance the observed and theoretical values to give

$$750 + 360 = 10 \times r_1 \times (1.80 \times 15 + 1.00 \times 12)$$

Part D: The implied relativity for Row 1 is

$$r_1 = (750 + 360) / (10 \times (1.80 \times 15 + 1.00 \times 12)) = 2.846154$$

Part E: Using the formula for the Row 2 relativity, the base rate of 10, and the starting relativities of 1.8 for Column 1 and 1.0 for Column 2, we balance the observed and theoretical values to give

$$120 + 80 = 10 \times r_2 \times (1.80 \times 6 + 1.00 \times 10)$$

Part F: The implied relativity for Row 2 is

$$r_2 = (120 + 80) / (10 \times (1.80 \times 6 + 1.00 \times 10)) = 0.961538$$

Part G: Using the formula for the Column 1 relativity, the base rate of 10, and the implied relativities of 2.846154 for Row 1 and 0.961538 for Row 2, we balance the observed and theoretical values to give

$$750 + 120 = 10 \times (2.846154 \times 15 + 0.961538 \times 6) \times c_1$$

Part H: The implied relativity for Column 1 is

$$c_1 = (750 + 120) / (10 \times (2.846154 \times 15 + 0.961538 \times 6)) = 1.795238$$

Part I: Using the formula for the Column 2 relativity, the base rate of 10, and the implied relativities of 2.846154 for Row 1 and 0.961538 for Row 2, we balance the observed and theoretical values to give

$360 + 80 = 10 \times (2.846154 \times 12 + 0.961538 \times 10) \times c_1$

Part J: The implied relativity for Column 1 is

$c_2 = (360 + 80) / (10 \times (2.846154 \times 12 + 0.961538 \times 10)) = 1.005272$