MS Module 15: Linear and logistic regression models – practice problems

(The attached PDF file has better formatting.)

Exercise 15.1: Logistic regression

A probability Y is related to the independent variable X by logistic regression:

$$Y = p(x) = \exp(\beta_0 + \beta_1 x) / (1 + \exp(\beta_0 + \beta_1 x))$$

- When X = 7, the probability Y is 20%.
- When X = 8, the probability Y is 25%.

A. At X = 7, what is the odds ratio of Y?
B. At X = 8, what is the odds ratio of Y?
C. At X = 11, what is the odds ratio of Y?
D. At X = 11, what is the probability of Y?

Part A: The odds ratio of Y at x = 7 is 20% / (1 – 20%) = 0.2500.

Part B: The odds ratio of Y at x = 8 is 25% / (1 – 25%) = 0.3333.

Part C: The slope parameter β_1 is the change in the *log odds* for a 1-unit increase in x, so the odds ratio itself changes by the multiplicative factor $\exp(\beta_1)$ when x increases by 1 unit. This factor is

$$0.333333 / 0.25 = 1.33333$$

11 is 3 units more than 8, so the odds ratio of Y at x = 11 is $0.333333 \times 1.333333^3 = 0.79012$

Part D: If Y = the probability and R = the odds ratio, $R = Y / (1–Y) \Rightarrow Y = R / (1+R)$.

The probability is the odds ratio / (1 + odds ratio), so the probability of Y at x = 11 is

$$0.79012 / 1.79012 = 44.14\%$$

Exercise 15.2: Logistic regression

A statistician uses a logistic regression model:

- The independent variable X is a quantitative predictor.
- The dependent variable Y is 1 if the observation is a success and 0 otherwise.

The estimate of β_1 is –0.20.

The odds of success at X = 1 are 50%.

A. What is the probability of success at X = 1?
B. What are the odds of success at X = 3?
C. What is the probability of success at X = 3?
D. What are the odds of success at X = 0?
E. What is the probability of success at X = 0?
F. What is β_0?

Part A: If the probability of success is P, the odds of success are P/(1-P).

Given that P/(1-P) = 50%, P = ½ – ½P ⇛ P = ⅓.

The formula is probability = odds / (1 + odds) = 50% / (1 + 50%) = 0.3333

Part B: For each one unit increase in X, the odds of success increase by a factor $\exp(\hat{\beta}_1) = e^{-0.20} = 0.81873$

3 is 2 units more than 1, so the odds of success at X = 3 are $50\% \times 0.81873^2 = 0.335159$

Part C: P/(1-P) = 0.33516 ⇛ 1.33516 P = 0.33516 ⇛ P = 0.33516 / 1.33516 = 0.25103

Part D: 0 is 1 unit less than 1, so the odds of success at X = 0 are 50% / 0.81873 = 0.610702

Part E: The probability of success at X = 0 is 0.610702 / 1.610702 = 0.379153

Part F: For logistic regression, $Y = \exp(\beta_0) / (1 + \exp(\beta_0 + \beta_1 \times X))$.

If Y = 0.379153 at X = 0, then $\exp(\beta_0) / (1 + \exp(\beta_0)) = 0.379153$ ⇛

$(1 – 0.379153) \times \exp(\beta_0) = 0.379153$ ⇛

$\exp(\beta_0) = 0.379153 / (1 – 0.379153) = 0.610703$

$\beta_0 = ln(0.610703) = –0.49314$