MS Module 17 Confidence interval and prediction interval practice exam questions

(The attached PDF file has better formatting.)

A linear regression analysis relates Y to X.

- The X values are {1, 2, …, 7}
- The least squares estimator for β_1 is a linear function of the Y values = $\Sigma\, c_i Y_i$
- The error sum of squares (SSE) is 25

Question 17.2: $\overline{x}$

What is $\overline{x}$, the average X value?

Answer 17.2: (1 + 7) / 2 = 4

Question 17.3: S_{xx}

What is S_{xx}, the sum of squared residuals for the X values?

Answer 17.3: $(1 – 4)^2 + (2 – 4)^2 + (3 – 4)^2 + (4 – 4)^2 + (5 – 4)^2 + (6 – 4)^2 + (7 – 4)^2 = 28$

Question 17.4: Least squares estimate of σ^2

What is s^2, the estimate of σ^2?

Answer 17.4: 25 / (7 – 2) = 5

(least squares estimate of σ^2 = SSE / (number of observations – 2))

Question 17.5: Least squares estimate of σ

What is *s*, the estimate of σ?

Answer 17.5: $5^{0.5} = 2.2361$

(standard deviation = square root of variance)

Question 17.6: Standard deviation of the least squares estimate of β_1

What is the standard deviation of the least squares estimate of β_1?

Answer 17.6: $2.2361 / 28^{0.5} = 0.4226$

(standard deviation = square root of variance)

Question 17.7: *t* value for 90% two-sided confidence interval

What is the *t* value for a 90% two-sided confidence interval?

Answer 17.7: 2.015

(Table look-up)

Question 17.8: Width of confidence interval

What is the width of the 90% confidence interval at x = 2?

Answer 17.8: $2 \times 2.2361 \times 2.015 \times (1 / 7 + (2 - 4)^2 / 28)^{0.5} = 4.8168$

(width of the confidence interval is $2 \times t_{\alpha/2,n-2} \times s \times (1/n + (x^* - \overline{x})^2 / S_{xx})^{½}$)

Question 17.9: Width of prediction interval

What is the width of the 90% prediction interval at x = 2?

Answer 17.9: $2 \times 2.2361 \times 2.015 \times (1 + 1 / 7 + (2 - 4)^2 / 28)^{0.5} = 10.2181$

(width of the confidence interval is $2 \times t_{\alpha/2,n-2} \times s \times (1 + 1/n + (x^* - \overline{x})^2 / S_{xx})^{½}$)