MS Module 16 Regression summary statistics practice exam questions

(The attached PDF file has better formatting.)

A regression analysis on 11 data points has summary statistics

- $\Sigma x_i = 8$
- $\Sigma y_i = 15$
- $\Sigma x_i^2 = 41$
- $\Sigma y_i^2 = 55$
- $\Sigma x_i y_i = 41$

Question 16.1: $\overline{x}$

What is $\overline{x}$, the average X value?

Answer 16.1: 8 / 11 = 0.727273

(average = total / number of observations)

Question 16.2: $\overline{y}$

What is $\overline{y}$, the average Y value?

Answer 16.2: 15 / 11 = 1.363636

(average = total / number of observations)

Question 16.3: S_{xx}

What is S_{xx}, the sum of squares of the X values?

Answer 16.3: $41 - 0.727273^2 \times 11 = 35.182$

(S_{xx}, the sum of squared deviations of the X values, is $\Sigma x_i^2 - N \times \overline{x}^2$)

Question 16.4: S_{yy}

What is S_{yy}, the sum of squares of the Y values?

Answer 16.4: $55 - 1.363636^2 \times 11 = 34.545$

(S_{yy}, the sum of squares of the Y values, is $\Sigma y_i^2 - N \times \overline{y}^2$)

Question 16.5: S_{xy}

What is S_{xy}, the cross sum of squares of the X and Y values?

Answer 16.5: $41 - 8 \times 15 / 11 = 30.091$

(S_{xy}, the cross sum of squares of the X and Y values, is $\Sigma x_i y_i - N \times \overline{x} \times \overline{y} = \Sigma x_i y_i - \Sigma x_i \times \Sigma y_i / N$)

Question 16.6: Least squares estimate for β_1

What is the least squares estimate for β_1?

Answer 16.6: $30.091 / 35.182 = 0.855$

(least squares estimate for $\beta_1 = S_{xy} / S_{xx}$)

Question 16.7: Least squares estimate for β_0

What is the least squares estimate for β_0?

Answer 16.7: $1.364 - 0.727 \times 0.855 = 0.742$

(least squares estimate for $\beta_0 = \overline{y} - \overline{x} \times \beta_1$)

Question 16.8: Error sum of squares

What is the error sum of squares?

Answer 16.8: $55 - 0.742 \times 15 - 0.855 \times 41 = 8.815$; with more significant digits for β_0 and β_1, ESS = 8.809

(error sum of squares SSE is $\Sigma y_i^2 - \beta_0 \times \Sigma y_i - \beta_1 \times \Sigma x_i y_i$)

Question 16.9: Least squares estimate for σ^2

What is s^2, the least squares estimate for σ^2?

Answer 16.9: $8.809 / (11 - 2) = 0.979$

(least squares estimate for σ^2 = error sum of squares / (number of observations – 2))

Question 16.10: Least squares estimate for σ

What is s, the least squares estimate for σ?

Answer 16.10: $0.979^{0.5} = 0.989$

(standard deviation = square root of variance)

Question 16.11: Standard deviation of least squares estimate for β_1

What is the standard deviation of the least squares estimate for β_1?

Answer 16.11: $0.989 / 35.182^{0.5} = 0.167$

(the standard deviation of the least squares estimate for $\beta_1 = \sigma / S_{xx}^{0.5}$)

Question 16.12: R^2

What is the least squares estimate for R^2?

Answer 16.12: $1 – 8.809 / 34.545 = 0.745$

(the least squares estimate for R^2 = 1 – error sum of squares / S_{yy})

Question 16.13: Correlation

What is the estimated correlation ρ between X and Y?

Answer 16.13: $30.091 / (35.182 \times 34.545)^{0.5} = 0.863$

(the estimated correlation ρ between X and Y = $S_{xy} / (S_{xx} \times S_{yy})^{0.5}$