

MS Module 11 Single-Factor ANOVA Tukey's procedure practice exam questions

(The attached PDF file has better formatting.)

Five groups, each with 7 observations, have sample means of

$$\bar{x}_1 = 3.2, \bar{x}_2 = 3.1, \bar{x}_3 = 3.5, \bar{x}_4 = 3.9, \bar{x}_5 = 3.7$$

The total sum of squares (SST) is 20, and the treatment sum of squares (SSTr) is 11.

The groups have normal distributions with equal variances. We test the null hypothesis $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$ at a 5% confidence level. If we reject the null hypothesis, we test which groups means differ significantly.

Question 11.1: Error sum of squares

What is the error sum of squares SSE?

Answer 11.1: $20 - 11 = 9$

(error sum of squares = total sum of squares – treatment sums of squares)

Question 11.2: Treatment mean square

What is the mean square for the groups (treatment mean square)?

Answer 11.2: $11 / (5 - 1) = 2.75$

(treatment mean square = treatment sums of squares / (number of groups – 1))

Question 11.3: Mean squared error

What is the mean squared error MSE?

Answer 11.3: $9 / (5 \times 7 - 5) = 0.30$

(degrees of freedom for mean squared error = (number of groups – 1) × observations per group; mean squared error = error sum of squares / its degrees of freedom)

Question 11.4: F value

What is the F value to test the null hypothesis?

Answer 11.4: $2.75 / 0.30 = 9.167$

Question 11.5: Critical F value

What is the critical F value for $\alpha = 5\%$?

Answer 11.5: 2.69

(Table look-up: $\alpha = 5\%$; degrees of freedom = 4, 30)

Question 11.6: Critical Q value

What is the critical Q value for $\alpha = 5\%$?

Answer 11.6: 4.10

(Table look-up: $\alpha = 5\%$; degrees of freedom = 5, 30)

Question 11.7: W (the width of the difference) for Tukey's honestly statistical difference?

What is w (the width of the difference) for Tukey's honestly statistical difference?

Answer 11.7: $4.1 \times (0.30 / 7)^{0.5} = 0.849$

(Tukey's $W = \text{critical Q value} \times (\text{mean squared error} / \text{observations per group})^{0.5}$)