MS Modules 9 and 10 Single-Factor ANOVA and Levene's test practice exam questions

(The attached PDF file has better formatting.)

A experiment has three groups and four observations in each group.

obsv →	1	2	3	4
group 1	14	22	16	14
group 2	12	15	14	25
group 3	7	17	24	32

The groups are normally distributed with the same variance.

The null hypothesis is that the means of the groups are the same: H_0: $\mu_1 = \mu_2 = \mu_3$

Question 10.1: Square of the sum of the observations

What is the square of the sum of all the observations, or $x_{..2}$?

Answer 10.1: $(14 + 22 + 16 + 14 + 12 + 15 + 14 + 25 + 7 + 17 + 24 + 32)^2 = 44{,}944$

Question 10.2: Sum of the squares of the observations

What is the sum of the squares of all the observations, or $\sum_i \sum_j x_{ij}^2$?

Answer 10.2: $14^2 + 22^2 + 16^2 + 14^2 + 12^2 + 15^2 + 14^2 + 25^2 + 7^2 + 17^2 + 24^2 + 32^2 = 4{,}260$

Question 10.3: Total sum of squares

What is SST, the total sum of squares?

Answer 10.3: 4,260 – 44,944 / 12 = 514.67

(the total sum of squares = the sum of the squares of all the observations – the square of the sum of all the observations / the number of observations)

Question 10.4: Sums of squares of group totals

What is the sum of squares of the group totals?

Answer 10.4: $(14 + 22 + 16 + 14)^2 + (12 + 15 + 14 + 25)^2 + (7 + 17 + 24 + 32)^2 = 15{,}112$

Question 10.5: Treatment sums of squares

What is SSTr, the treatment sum of squares?

Answer 10.5: 15,112 / 4 – 44,944 / 12 = 32.67

(treatment sums of squares = the sum of squares of the group totals / the number of observations per group – the square of the sum of all the observations / the total number of observations)

Question 10.6: Error sum of squares

What is SSE, the error sum of squares?

Answer 10.6: 514.67 – 32.67 = 482.00

(error sum of squares = total sum of squares – treatment sums of squares)

Question 10.7: Total degrees of freedom

What are the total degrees of freedom?

Answer 10.7: 12 – 1 = 11

(total degrees of freedom = number of observations – 1)

Question 10.8: Degrees of freedom for the groups

What are the degrees of freedom for the groups?

Answer 10.8: 3 – 1 = 2

(degrees of freedom for the groups = number of groups – 1)

Question 10.9: Degrees of freedom for the error sum of squares

What are the degrees of freedom for the error sum of squares?

Answer 10.9: 11 – 2 = 9

(degrees of freedom for the error sum of squares = total degrees of freedom – degrees of freedom for the groups)

Question 10.10: Mean squared deviation for the groups

What is MSTr, the mean squared deviation for the groups?

Answer 10.10: 32.667 / 2 = 16.33

(mean squared deviation for the groups = treatment sums of squares / degrees of freedom for the groups)

Question 10.11: Mean squared error

What is MSE, the mean squared error?

Answer 10.11: 482 / 9 = 53.556

(mean squared error = error sum of squares / degrees of freedom for the error sum of squares)

Question 10.12: *F* value

What is the *F* value for testing the null hypothesis?

Answer 10.12: 16.333 / 53.556 = 0.305

(*F* value = treatment mean square / mean squared error)

Levene's method

Levene's method tests whether the group variances are the same. The groups are normally distributed, and the null hypothesis is that the variances are the same: H_0: $\sigma^{21} = \sigma^{22} = \sigma^{23}$ [σ^{2j} is the variance of Group *j*].

Question 10.13: Absolute deviations

What are the absolute deviations of the observations in each group?

Answer 10.13: absolute deviation = absolute value of cell value – group mean

obsv →	1	2	3	4	Mean
group 1	14.0	22.0	16.0	14.0	16.5
group 2	12.0	15.0	14.0	25.0	16.5
group 3	7.0	17.0	24.0	32.0	20.0
absolute deviations					sample variance
group 1	2.5	5.5	0.5	2.5	14.333
group 2	4.5	1.5	2.5	8.5	33.667
group 3	13.0	3.0	4.0	12.0	112.667

Question 10.14: Sample variance

What is the sample variance in each group?

Answer 10.14: the sample variances indicate whether the variances appear to differ significantly; the group means do not differ much here, but the sample variances differ greatly

Question 10.15: Square of sum of absolute deviations

What is the square of the sum of the absolute deviations?

Answer 10.15: $(2.5 + 5.5 + 0.5 + 2.5 + 4.5 + 1.5 + 2.5 + 8.5 + 13 + 3 + 4 + 12)^2 = 3{,}600$

Question 10.16: Sum of squares of absolute deviations

What is the sum of the squares of the absolute deviations?

Answer 10.16: $(2.5^2 + 5.5^2 + 0.5^2 + 2.5^2 + 4.5^2 + 1.5^2 + 2.5^2 + 8.5^2 + 13^2 + 3^2 + 4^2 + 12^2) = 482$

Question 10.17: Total sum of squares (SST) for Levene's test

What is the total sum of squares (SST) for Levene's test?

Answer 10.17: 482 – 3,600 / 12 = 182

(the total sum of squares = the sum of the squares of all the observations – the square of the sum of all the observations / the number of observations)

Question 10.18: Sums of squares of group totals

What is the sum of squares of the group totals for Levene's test?

Answer 10.18: $(2.5 + 5.5 + 0.5 + 2.5)^2 + (4.5 + 1.5 + 2.5 + 8.5)^2 + (13 + 3 + 4 + 12)^2 = 1,434$

Question 10.19: Treatment sums of squares

What is SSTr, the treatment sum of squares for Levene's test?

Answer 10.19: 1,434 / 4 – 3,600 / 12 = 58.50

(treatment sums of squares = the sum of squares of the group totals / the number of observations per group – the square of the sum of all the observations / the total number of observations)

Question 10.20: Error sum of squares

What is SSE, the error sum of squares for Levene's test?

Answer 10.20: 182 – 58.50 = 123.50

(error sum of squares = total sum of squares – treatment sums of squares)

Question 10.21: Total degrees of freedom

What are the total degrees of freedom for Levene's test?

Answer 10.21: 12 – 1 = 11

(total degrees of freedom = number of observations – 1)

Question 10.22: Degrees of freedom for the groups

What are the degrees of freedom for the groups for Levene's test?

Answer 10.22: 3 – 1 = 2

(degrees of freedom for the groups = number of groups – 1)

Question 10.23: Degrees of freedom for the error sum of squares

What are the degrees of freedom for the error sum of squares for Levene's test?

Answer 10.23: 11 – 2 = 9

(degrees of freedom for the error sum of squares = total degrees of freedom – degrees of freedom for the groups)

Question 10.24: Mean squared deviation for the groups

What is MSTr, the mean squared deviation for the groups for Levene's test?

Answer 10.24: 58.50 / 2 = 29.25

(mean squared deviation for the groups = treatment sums of squares / degrees of freedom for the groups)

Question 10.25: Mean squared error

What is MSE, the mean squared error for Levene's test?

Answer 10.25: 123.50 / 9 = 13.722

(mean squared error = error sum of squares / degrees of freedom for the error sum of squares)

Question 10.26: *F* value

What is the *F* value for testing the null hypothesis for Levene's test?

Answer 10.26: 29.25 / 13.722 = 2.132

(*F* value = treatment mean square / mean squared error)