MS Module 4 *t* values and confidence intervals practice problems

(The attached PDF file has better formatting.)

Exercise 4.1: *t* values and confidence intervals

A statistician estimates the population mean for a normal distribution from a sample of 8 points. The 99% confidence interval for the population mean is (0, 2.500)

A. What is the critical *t* value for a 99% confidence interval from a sample of 8 points?
B. What is the standard deviation of the sample?
C. What is the critical *t* value for a 95% confidence interval from a sample of 8 points?
D. What is the 95% confidence interval for the population mean?

Part A: A sample with 8 points has 7 degrees of freedom. The critical *t* value for a 99% confidence interval with 7 degrees of freedom is 3.499 (table lookup).

Part B: The confidence interval is the estimate ± $\sigma/\sqrt{N} \times t$ value, so

- the point estimate is (2.500 + 0) / 2 = 1.250
- the width of the confidence interval is $2 \times \sigma/\sqrt{N} \times t$ value = 2.500 – 0 = 2.500 ⇛
  - $\sigma = 8^{0.5} \times 2.500 / (2 \times 3.499) = 1.010$

Part C: The critical *t* value for a 95% confidence interval with 7 degrees of freedom is 2.365 (table lookup).

Part D: The confidence interval is $1.250 \pm (1.010 / 8^{0.5}) \times 2.365$:

- lower bound: $1.250 - (1.010 / 8^{0.5}) \times 2.365 = 0.405$
- upper bound: $1.250 + (1.010 / 8^{0.5}) \times 2.365 = 2.095$

Exercise 4.2: *t* values and two confidence intervals

A statistician estimates the population mean for a normal distribution from a sample of 8 points.

- The upper bound of the 95% confidence interval for the population mean is 5.
- The lower bound of the 90% confidence interval for the population mean is 1.

We use the following notation:

- μ = the estimated population mean
- σ = the standard deviation of the sample
- N = the number of observations

A. What is the critical *t* value for a two-sided 95% confidence interval for a sample of 8 points?
B. What is the critical *t* value for a two-sided 90% confidence interval for a sample of 8 points?
C. What is $\sigma/\sqrt{N}$?
D. What is σ?
E. What is μ?

Part A: A sample of 8 points has 8 – 1 = 7 degrees of freedom.

The critical *t* value for a two-sided 95% confidence interval for a sample of 8 points is 2.3646 (table look-up).

Part B: A sample of 8 points has 8 – 1 = 7 degrees of freedom.

The critical *t* value for a two-sided 90% confidence interval for a sample of 8 points is 1.8946 (table look-up).

Part C: Combine the upper half of the 95% confidence interval with the lower half of the 90% confidence interval to estimate $\sigma/\sqrt{N}$:

- $2.3646 \times \sigma/\sqrt{N} = 5 - \mu$
- $1.8946 \times \sigma/\sqrt{N} = \mu - 1$

adding: $(2.3646 + 1.8946) \times \sigma/\sqrt{N} = (5 - \mu) + (\mu - 1) = (5 - 1) \Rightarrow$

$\sigma/\sqrt{N} = (5 - 1) / (2.3646 + 1.8946) = 0.93914$

Part D: $\sigma = 0.93914 \times 8^{0.5} = 2.656$

Part E: Solve for μ by either of the confidence intervals:

- $2.3646 \times \sigma/\sqrt{N} = 5 - \mu \Rightarrow \mu = 5 - 2.3646 \times 0.93914 = 2.779$
- $1.8946 \times \sigma/\sqrt{N} = \mu - 1 \Rightarrow \mu = 1 + 1.8946 \times 0.93914 = 2.779$