

## Module 2: Economic Growth and Growth Models

### *Practice problems and illustrative test questions for the final exam*

(The attached PDF file has better formatting.)

This posting gives sample final exam problems. Other topics from the textbook are asked as well; these problems are just examples. All final exam problems are multiple choice; practice problems are not multiple choice so that the solutions can be better explained.

**\*\* Exercise 2.1: capital per worker**

- $K$  = the capital stock
- $L$  = the labor force

Which of the following is false?

- A.  $\Delta K / K$  = the growth rate of capital
- B.  $\Delta L / L$  = the growth rate of labor
- C.  $K/L = k$  = capital per worker
- D.  $\Delta k / k = (\Delta K / K) / (\Delta L / L)$  = the growth rate of capital per worker
- E. If the labor force percentage is not changing,  $\Delta L / L \approx n$ , the population growth rate

Answer 2.1: D

*Statement D:* The growth rate of capital per worker is  $\Delta k / k = (\Delta K / K) - (\Delta L / L)$ .

*Illustration:* Suppose capital grows at 5% per annum and labor grows at 1% per annum. The growth rate of capital per worker is  $5\% - 1\% = 4\%$ , not  $5\% / 1\% = 5.00$ .

*Statement E:* The growth rate of the labor force is not easily measured, so we use the growth rate of the total population as a proxy.

*Question:* Why might the growth rates of the labor force and the population differ?

*Answer:* In some developing countries, women who once remained at home are now entering the labor force. The growth rate of the labor force is higher than the growth rate of the total population.

See Barro, *Macroeconomics*, Chapter 3, economic growth, page 42

Question 2.2: Units of measurement

- $K$  = the capital stock
- $L$  = the labor force
- $s$  = savings rate
- $\delta$  = depreciation rate of capital
- $n$  = population growth rate

Which of the following does not have units of 1/time?

- A.  $\Delta K / K$  = the growth rate of capital
- B.  $\Delta L / L$  = the growth rate of labor
- C.  $s$  = savings rate
- D.  $\delta$  = depreciation rate of capital
- E.  $n$  = population growth rate

Answer 2.2: C

*Statement C:* The savings rate is unit-less.

## \*\*Exercise 2.3: Units of measurement

Macroeconomics deals with disparate items with different units of measurement. Labor is number of workers, the wage rate is dollars per worker hour, capital is dollars of machinery, and income is dollars per year. Some items have no units. Savings is dollars saved per year, and income is dollars earned per year, so the savings rate is dollars saved each year per dollars earned each year, which is a percentage with no units.

Many formulas in the textbook relate different macroeconomic variables. Be sure that the units are the same for all terms in the formula. Some multiple choice questions on the final exam have choices that are clearly wrong because the units are different on two sides of a formula.

Barro illustrates the units of measurement with the growth rate of capital per worker (equation 3.15 on page 42, column 2, and equation 3.16 on page 43, column 1).

- Equation 3.15 says:  $\Delta k/k = s \times (Y/K) - s\delta - n$
- Equation 3.16 says:  $\Delta k/k = s \times (y/k) - s\delta - n$

Note that  $Y/K = \text{income} / \text{capital} = y/k = \text{income per worker} / \text{capital per worker}$ .

What are the units of measurement for each of the following?

- A.  $s \times (Y/K)$
- B.  $s\delta$
- C.  $n$
- D.  $\Delta k/k$

*Part A:* The savings rate is unit-less. A person with a savings rate of 10% saves \$1 from \$10 earned in an hour, €10 from €100 earned in a day, and £100 from £1,000 earned in a week. The savings rate does not depend on the currency.

$Y$  (income) and  $K$  (capital) have the same currency (if they are nominal figures) or the same goods (if they are real figures). The ratio of  $Y$  to  $K$  has no currency units or goods units. Income is the goods earned in a period of time, such as one year, and capital is a stock of goods at one date. The units of measurement are goods per time for income and goods for capital. The unit of measurement for  $Y/K$  is 1/time, such as 1/(1 year).

*Part B:* The depreciation rate  $\delta$  and population growth rate  $n$  have units of 1/time, such as 1/(1 year). The depreciation rate says how much capital depreciates each year. If capital 5% depreciates in one year, it depreciates 10% in two years for simple growth or  $1 - 95\% \times 95\% = 9.75\%$  for compound growth.

*Part C:* Similarly, if the population grows 2% in one year, it grows 4% in two years for simple growth or  $1.02 \times 1.02 - 1 = 4.04\%$  for compound growth.

*Part D:* Any expression  $\Delta z/z$  has units of 1/time.  $\Delta z$  is the change in  $z$  during one time period. The units of  $z$  cancel out of the fraction, leaving 1/time.

All terms in equations 3.15 and 3.16 have units of 1/time.

**\*\* Exercise 2.4: Average and marginal products of capital**

Much macroeconomic theory depends on patterns in the marginal products of capital and labor.

- A. As capital increases, how does the marginal product of capital MPK change?
- B. As capital increases, how does the average product of capital change?
- C. As labor increases, how does the marginal product of capital MPK change?
- D. As labor increases, how does the average product of capital change?

*Part A:* Decreasing marginal utility is a characteristic of all goods.

- The first loaf of bread has the greatest utility to a hungry consumer; subsequent loaves have less utility.
- The first unit of capital has the greatest use to a business; subsequent units have less utility.

The marginal product of capital is the utility of the last unit.

*Part B:* Average product follows the same pattern as marginal product. If a series is decreasing, the average of the first N terms decreases as N increases.

*Part C:* More workers to use the capital make each unit of capital more productive. As labor increases, the marginal product of capital increases.

*Part D:* If the marginal product of capital increases as labor increases, the average product of capital increases as well.

*Question:* Why does labor affect the marginal product of capital?

*Answer:* Think of labor as farm workers and capital as tractors.

Suppose the farm has 100 acres and 10 workers. The first tractor bought by the farm is used all day and all night; there is always land to plough and workers to drive the tractor. The first tractor has high utility.

The second tractor is not used as much, since there is less land left to plough and fewer workers to drive the second tractor. The second tractor has less utility. Once the farm has several tractors, an additional tractor is not worth much.

Now suppose the farm has 100 acres and two tractors. If the farm has only one worker, the marginal value of the last tractor is close to zero, since the farm worker can use only one tractor at a time. It is not exactly zero, since the first tractor may be worn out and needing repairs some days. If the farm has two workers, each one can use a tractor, so the marginal value of the second tractor is large.

