Module 5: Conditional convergence and long-run economic growth practice problems

(The attached PDF file has better formatting.)

This posting gives sample final exam problems. Other topics from the textbook are asked as well; these problems are just examples. All final exam problems are multiple choice; practice problems are not multiple choice so that the solutions can be better explained.

In the nineteenth century, Western European nations were more successful than Asian or African countries. Some Europeans presumed they were innately superior to Asians or Africans.

As third world countries developed, fewer people believed in the innate superiority of Europeans. Instead, they presumed that natural advantages of Europe made these nations successful: accessible coasts for shipping, fertile agricultural land, or even religious (Protestant) systems that fit well with capitalism.

In the first half of the twentieth century, European civilization produced two world wars. In the 1950's, 1960's, and 1970's, several countries in Asia, Europe, the Americas, and the Middle East developed rapidly. Differences were ascribed to historical happenstance. Low income countries would grow quickly by imitating the economies of advanced countries.

Some economists adopted a theory of absolute convergence. Countries were inherently similar, and eventually all would achieve the same economic level. The emergence of African nations after World War II and the rapid rise of Asian economies in the 1960's and 1970's seemed to support this theory. Japan's ability to duplicate or exceed the characteristics of Western economies seemed proof of the theory.

In recent years, some parts of the world (such as Asia) have caught up to developed countries and some parts (such as Africa and the Middle East) are falling further behind. Barro presumes that each country has a steady state level, but countries differ. He suggests that political, legal, and cultural attributes affect the steady state income of the economy. A culture that does not allow women to work may never reach the per capital income of Western economies.

Exam problems test absolute convergence, conditional convergence, and their attributes with scenarios. Focus on the following items:

- When absolute convergence occurs and when only conditional convergence occurs.
- How convergence relates to income inequality.
- What attributes cause faster or slower growth rates.

** Exercise 5.1: Solow growth model with technological progress

- The technology level is growing 3% a year.
- The economy has a Cobb-Douglas production function with $\alpha = 70\%$.
- The current capital per worker k is 50% of the steady state capital per worker k^*.
- The economy follows conditional convergence with a mean reversion of 20%.

A. What is the long-term annual growth rate of steady state capital per worker k^*?
B. What is the annual growth rate of capital per worker k during the transition phase?
C. What are the long-term annual growth rate of steady state income per worker y^* and the annual growth rate of income per worker y during the transition phase?

Part A: The growth rate of capital per worker and income per worker is $g / (1 - \alpha) = 3\% / (1 - 70\%) = 10\%$.

Question: Why are growth rates of capital per worker and income per worker greater than technological progress? If the technology level increases 3% per annum, shouldn't income increase 3% per annum?

Answer: When the technology level rises, the marginal product of capital rises. Businesses use more capital, until the marginal product of capital declines to the real rental price.

Part B: With conditional convergence, capital per worker approaches steady state capital per worker. Since capital per worker k is lower than steady state capital per worker k* in this exercise, capital per worker grows faster than steady state capital per worker. Suppose k = 1 and k* = 2. Next year's forecasts with no mean reversion are 1.1 and 2.2. With 20% mean reversion, next year's forecast for k is 80% × 1.1 + 20% × 2.2 = 1.320, or a 32% growth rate. The final exam for this course does not test the mathematics in Part B, but you must know the growth rate formula in Part A.

Question: Isn't 32% a high growth rate? No country grows that fast.

Answer: We generally assume technological progress of about 1% a year, leading to a growth rate of capital per worker of about 2% to 3% a year; this is the average for developed countries over the past century. If a country is now under-developed but has an open economy, good education, and legal systems that promote private business, it may grow twice as quickly. China, India, Brazil, and a few other developing countries have growth rates of 6% to 10% a year.

Part C: The growth rate of income per worker is the same as the growth rate of capital per worker.

See Barro, Macroeconomics, Chapter 5, conditional convergence, page 82

** Exercise 5.2: Exogenous vs endogenous growth theory

A. Why are exogenous or endogenous growth models needed to explain economic development?
B. How do exogenous growth models differ from endogenous growth models?
C. Why might the principle of decreasing marginal utility not apply to technological progress?

Part A: If the technology level does not change, real GDP soon reaches an equilibrium. With negative population growth in most of the developed world and weakening work ethics (declining ratios of the labor supply to population), economic growth should cease. But growth continues in most developed countries.

Part B: The continued growth of the world economy is clearly related to technological progress. Growth of the technology level can be modeled two ways:

- Exogenous growth in unrelated to policy choices. The automobile, ratio, and internet are invented; they are not modeled as effects of research and development.
- Endogenous growth is modeled as the result of policy choices. More education, private research and development, government subsidies for pure research, and patent protection for inventors lead to faster technological progress.

Part C: Contrast technological progress with capital growth. Suppose a farm has two workers and no capital. It can rent an electric plough to replace its mechanical wood plough. Capital has decreasing marginal utility.

- One electric plough can be used by one worker during the day and the other worker at night, but night-time work is not efficient.
- A second plough allows both workers to worker during the day.
- A third plough sits idle in the shed.

Technological progress is different. A small tractor replacing the electric plough makes workers much more efficient, and a large combine further raises efficiency. Technological progress makes each worker ever more efficient, and its marginal utility may not decline. In practice, technology improvements become ever more productive each year. The slide rule is an improvement over pencil and paper calculations, and a mechanical adding machine is better than a slide rule. Hand calculators, laptop computers, and spreadsheets are vast improvements over earlier computing technology.

** Exercise 5.3: Intellectual property

Firms get private gains from technological progress and research and development several ways.

A. What are patents?
B. What are copyrights?
C. How else do firms profit from technological innovation and from research and development?
D. Why are intellectual property rights a serious problem for international trade?

Parts A and B: Technological progress stems from research and development, which is generally expensive. Technological progress is a non-rival good and is not easily kept proprietary. Patents and copyrights allow inventors and authors to retain the benefit of their research or writing for a limited period.

- Patents give the inventor exclusive rights to the invention for 17 to 20 years.
- Copyrights give authors exclusive rights to literary works for the author's lifetime + 50 years.

Part B: Much research can not be patented, and some inventions, even if patented, and easily copied in slightly different form. Producers of food products keep their recipes secret, so that their products are hard to copy. For example, Coca Cola keeps its Cola mixture secret.

Sometimes secrecy doesn't work. Insurers can't keep policy forms secret, since the policy is filed with state regulators and given to insureds. So insurers try to be the first to use better policy forms or pricing methods.

Illustration: Generalized linear modeling has improved insurance risk classification over the past twenty years. GLMs are a statistical tool; they can not be patented or kept secret. But they have a steep learning curve, and hiring a statistician is expensive. The first insurers to master the techniques had competitive advantages.

Part D: The United States has strong intellectual property rights, so it has strong technological progress. Some other countries do not respect intellectual property rights, so they have little research and development of their own. Courts operate within a single country, so it is hard to protect intellectual property rights internationally.

See Barro, Macroeconomics, Chapter 5, conditional convergence, page 84

** Exercise 5.4: Rival vs nonrival goods

A. What is meant by rival goods?
B. What are examples of rival goods?
C. What is meant by non-rival goods?
D. What are examples of non-rival goods?

Part A: A rival good means that users of the good are rivals: if one person uses the good, others can not.

Part B: Labor and most physical capital are rival goods. If one firm uses a machine or a worker, other firms can not use the same machine or worker during that time.

Part C: A non-rival good means that users are not rivals: all firm can use the good at the same time.

Part D: Technology is a non-rival good. All production requires three elements: technology, capital, and labor.

Illustration: Growing crops requires capital (machines to plow fields and harvest crops), workers to operate the machines, and technology, or the knowledge of when and how to plant seeds, when and how to water them, how to add fertilizer, and so forth. This knowledge seems simple, but it is actually highly complex. The basic knowledge of growing crops enabled people to form large communities and spread over the earth. More sophisticated knowledge acquired in the past century about irrigation, fertilizer, and genetically modified crops multiplied the output of farms several-fold (the *green revolution*).

Much output of modern economies depends on technology: high-tech goods, modern medicine, life sciences, and goods produced by automated systems.

Question: Technology can be patented, allowing only one person to use the technology.

Answer: The patent is a rival good. The underlying ideas are non-rival goods.

Question: What is the difference in business strategy for rival vs non-rival goods?

Answer: If one firm owns all the raw materials, it doesn't want to allow other firms to use these materials. If one firm has a patent on engineering knowledge, it may license other firms to use the knowledge.

See Barro, Macroeconomics, Chapter 5, conditional convergence, page 84

** Exercise 5.5: AK model

A. What is human capital?
B. What is infrastructure capital?
C. How does the AK model differ from the Solow growth model?
D. What is the formula for the growth rate of capital per worker in the AK model?
E. What does the AK model imply about convergence?

Part A: Human capital is formal education, on-the-job training, and health. Workers who are well educated and healthy produce more.

Part B: Infrastructure capital (often owned by the government) provides services like transportation, electric power, and water.

Part C: The Solow growth model assumes decreasing marginal utility of capital: as the capital per worker increases, the additional income from each additional unit of capital decreases. The AK model assume the marginal utility of capital is constant.

Question: Is the AK model reasonable? If a country has much capital but no workers to use the capital, won't the marginal product of capital be low?

Answer: Human capital is included in the capital stock. If workers are illiterate, computers won't raise work efficiency. If human education is included with capital, teaching workers computer skills and buying laptop computers may raise production more than adding uneducated workers.

Question: High school education is a big advantage over grade school only; college education is a moderate advantage over high school only; graduate school is a slight advantage over college only. Doesn't this show the decreasing marginal utility of human capital (like education)?

Answer: Liberal arts colleges and universities in the United States do not always teach marketable skills. For a business job, a college student studying English literature and art history is just passing time. A PhD in history or sociology doesn't help much. Education teaching practical skills, such as computer programming, statistics, and engineering, has a high marginal product.

Part D: The Solow growth model assumes income per worker $y = A \times f(k)$, where k is capital per worker and $f(k)$ is the production function. The AK model assumes the function f is a constant, so $y = Ak$.

The growth rate of capital per worker in the Solow growth model is $\partial k/k = s \times (y/k) - s\delta - n$, where s is the savings rate, δ is the depreciation rate, and n is the population growth rate. For the AK model, the growth rate of capital per worker is $\partial k/k = sA - s\delta - n$.

Part E: The AK model assumes no convergence among different economies.

Question: Why shouldn't capital have decreasing marginal utility?

Answer: Private physical capital has decreasing marginal utility. Adding airplanes to a delivery fleet of trucks may have decreasing marginal utility for the economy. But adding airplanes with new airports and runways (infrastructure) and training workers as pilots (human capital) may have constant marginal utility.

Question: If the labor force is finite, one can't keep adding human capital indefinitely.

Answer: That statement is true. But the world has billions of uneducated workers, many of whom are illiterate. Better education (especially for girls in developing countries) and health care (especially of infectious diseases like malaria, tuberculosis, and measles) creates billions of skilled workers. Human education is part of capital in the AK model. The problem is not the number of workers; the problem is poor education.

Question: Many countries are still not educating their work forces well.

Answer: Barro rejects the AK model: even human capital and infrastructure capital have decreasing marginal utility. But the technology level may not have decreasing marginal utility: advances in technology seem to be more rapid as people become better educated and more proficient in high-tech and life sciences. Endogenous growth models assume the technology level improves, perhaps with constant marginal utility.

** Exercise 5.6: Conditional Convergence

Conditional convergence means a country's long-term expected economic growth rate g(y, y*) is a function of its *steady state* income per worker (y*) and its *current* income per worker (y).

A. What is the sign of $\partial g(y, y^*)/\partial y$, the partial derivative of the growth rate with respect to y?
B. What is the sign of $\partial g(y, y^*)/\partial y^*$, the partial derivative of the growth rate with respect to y*?

Part A: For a given steady state income per worker, as the current income per worker rises, the growth rate decreases. As the current income per worker approaches the steady state income per worker, the growth rate slows.

Part B: For a given current income per worker, as the steady state income per worker rises, the growth rate increases. As the current income per worker becomes further away from the steady state income per worker, the growth rate quickens.

Jacob: Can one also write the long-term economic growth rate in terms of capital per worker?

Rachel: The signs of the partial derivatives are the same if we use capital per worker instead of income per worker.

** Exercise 5.7: Conditional Convergence

Suppose the rate of conditional convergence is 10% per annum for all countries. (Barro assumes a rate of 2% to 2.5%; we use 10% to simplify the arithmetic.) The 10% if the mean reversion rate. If steady state output is y* and current output is y, the expected growth rate is 10% × (y* – y) / y.

Illustration: If steady state real GDP is $20,000 per worker and current real GDP is $16,000 per worker, the expected growth rate is 10% × ($20,000 – $16,000) / $16,000 = 2.50%.

Countries W, Y, and Z have the following current and steady state real GDP per worker at 1/1/20X2. (Output per worker = real GDP per worker).

Country	*Current Output*	*Steady State Output*
W	20,000	28,000
Y	30,000	40,000
Z	30,500	31,000

A. What is Country W's growth rate in 20X2 and its real GDP per worker at 12/31/20X2?
B. What is Country Y's growth rate in 20X2 and its real GDP per worker at 12/31/20X2?
C. What is Country Z's growth rate in 20X2 and its real GDP per worker at 12/31/20X2?

Part A: Country W's growth rate in 20X2 is 10% × ($28,000 – $20,000) / $20,000 = 4.00%. Its real GDP per worker at 12/31/20X2 is $20,000 × 1.040 = $20,800.

Part B: Country Y's growth rate in 20X2 is 10% × ($40,000 – $30,000) / $30,000 = 3.33%. Its real GDP per worker at 12/31/20X2 is $30,000 × 1.0333 = $20,667.

Part C: Country Z's growth rate in 20X2 is 10% × ($31,000 – $30,500) / $30,500 = 0.16%. Its real GDP per worker at 12/31/20X2 is $30,500 × 1.00167 = $30,551.

** Exercise 5.8: Convergence and Fertility Rates

An economist examining convergence among developing countries finds that economic growth stimulates greater education of women and lower birth rates: an increase in GDP per capita leads to a decline in the fertility rate. All countries benefit from modern medicine. Infant mortality, infectious diseases, and mortality rates decline in all the countries, whether or not the fertility rate declines.

A. How does decreasing mortality with no change in the fertility rate affect capital per worker?
B. How does the effect of economic development on fertility rates affect convergence?
C. How does the effect of economic development on fertility rates affect observed patterns in Africa and the Middle East?

Part A: New capital must support new workers, and the capital per worker ratio may decline.

Part B: If development leads to lower fertility rates, conditional convergence may not occur, since countries that grow rapidly and have lower birth rates may grow even more rapidly in later years.

Part C: Endogenous population growth may help explain why Africa and the Middle East have lagged behind and not converged with North America, western Europe, and Asia. Economic development leads to better education and work opportunities for women and lower fertility rates, which leads to faster development.

** Exercise 5.9: Convergence

An economist finds convergence among the states of the U.S. but not among the countries of Africa.

How do each of the following affect convergence in the U.S. vs in Africa?

A. Educational and legal systems.
B. Currency and taxes.
C. Labor mobility.

Part A: U.S. states have similar public education, laws, and court systems; African countries do not. The similarity among U.S. states leads to similar steady state income per worker, which leads to convergence.

Part B: U.S. states have the same currency and federal taxes; African countries do not. Different currencies and tax rates may cause different steady state income per worker, preventing convergence.

Part C: Labor moves easily across U.S. states but not between African countries. If job opportunities are better in one state, workers move there. Workers and businesses are now leaving California and Illinois and moving to Texas and Florida. States don't want to love their workers, so this labor mobility helps deter excessive state regulation that prevents convergence.

** Exercise 5.10: Income Inequality

The removal of trade barriers, reduction in transportation costs, and better education enable workers in developing countries, such as China and India, to perform jobs formerly done in North America and Western Europe. Western countries now out-source skilled and unskilled jobs to China, India, and other Asian nations.

China and India were the world's most technologically advanced nations in the fifteenth through seventeenth centuries. They have highly intelligent populations, strong support for civil law, economic freedoms, and widespread education. Their steady state income per worker is the same as that of the Western world.

Countries focus on industries and professions where they have comparative advantages. Educated young people, skilled workers, and urban residents in China and India get high paying jobs. Some farm workers retain the same work; other farm workers get only partial benefit from global trade.

In Western countries, some firms benefit from lower costs, and consumers benefit from lower prices. Some worker who lose their jobs switch to better paying jobs; other workers who lose their jobs remain unemployed.

What are the effects of globalization on each of the following?

A. Absolute poverty levels in China and India.
B. Income inequality in China and India.
C. Income inequality in North America and Western Europe
D. Income inequality between nations of the world: that is, between (i) North America and Western Europe and (ii) China and India.
E. Income inequality among the entire world's population

Part A: International trade helps all parties, raising income and reducing poverty.

Part B: Educated workers in China and India benefit much; rural workers benefit least. Income inequality may increase or decrease.

Part C: Some firms and workers benefit; others lose. Income inequality in North America and Western Europe may increase or decrease.

Part D: North America, Western Europe, China, and Inida have similar steady state income levels, so absolute convergence occurs and income inequality between nations decreases.

Part E: The average income inequality among the world's population probably also decreases, but we can not say for sure.