Macro modules 14 and 15: Inflation, Money growth, and Interest rates: practice problems

Practice problems and illustrative test questions for the final exam

(The attached PDF file has better formatting.)

This posting gives sample final exam problems. Other topics from the textbook are asked as well; these problems are just examples. All final exam problems are multiple choice; practice problems are not multiple choice so that the solutions can be better explained.

**Question 15.1: Real rates of return

- The real return on capital R/P is 20%.
- The utilization rate of capital (lowercase κ) is 80%.
- The depreciation rate is 5%.

What is the real rate of return on bonds?

Answer 15.1: See equation 11.8: $r = (R/P) \times \kappa - \delta(\kappa) = 20\% \times 80\% - 5\% = 11\%$

The real rate of return on bonds equals the real rate of return on capital. Bonds are used to fund new capital.

- If the real rate of return on bonds were higher than that on capital, businesses would be paying more for capital than they receive from it. They would buy less capital, and the real rate of return on capital would increase (by the principle of decreasing marginal utility).
- If the real rate of return on bonds were less than that on capital, businesses would be paying less for capital than they receive from it. They would buy more capital, and the real rate of return on capital would decrease (by the principle of decreasing marginal utility).

See Barro, *Macroeconomics*, Chapter 11, equation 11.8

** Exercise 15.2: Utilization rate of capital

The real return on capital R/P is 20%, and the depreciation function $\delta(\kappa)$ is $\kappa^2/5$.

A. What is the net return on capital as a function of the utilization rate of capital κ?
B. What is the optimal utilization rate of capital κ?

Part A: The net return on capital is the real return on capital × the capital utilization rate – the depreciation rate:

$$r = (R/P) \times \kappa - \delta(\kappa) = 20\% \times \kappa - 20\% \times \kappa^2$$

Part B: To find the optimal utilization rate of capital, maximize: $r = (R/P) \times \kappa - \delta(\kappa) = 20\% \times \kappa - 20\% \times \kappa^2$

$$\Rrightarrow \text{maximize } 20\% \times \kappa \times (1 - \kappa) = 20\% \times (\kappa - \kappa^2)$$

Set the partial derivative with respect to κ equal to zero:

$$\Rrightarrow 1 - 2\kappa = 0 \Rrightarrow \kappa = 50\%.$$

Question: Why is the depreciation rate a function of the capital utilization rate?

Answer: If capital is used only part of the time, it wears down less quickly, and it can be fixed or maintained as needed. During boom periods, capital is used continually (κ is high): it is not repaired or maintained well, and it wears out quickly.

Question: Why would maintenance be low during boom times? Don't firms know that poor maintenance will ruin the capital stock?

Answer: In the short run, the capital stock is fixed; adding capital (new factories) may take years. To meet high demand, firms must work overtime. The high depreciation of capital in boom times is a cost of doing business.

See Barro, *Macroeconomics*, Chapter 11

** Exercise 15.3: Change in money growth rate

- In 20X2, the money stock grows 5% a month.
- On January 1, 20X3, the monetary authorities announce that money will grow 15% a month.
- The public believes the new money growth rate announced by the monetary authorities.
- Actual monetary growth rates are as announced by the monetary authorities.

What is the growth rates of the price level

A. from November 30, 20X2, to December 31, 20X2?
B. from January 31, 20X3, to March 1, 20X3?
C. from December 31, 20X2, to January 31, 20X3?

(Assume each period is one month. The exercise is not testing the number of days in each period. Assume real GDP and the population do not change.)

Part A: In 20X2, the money stock grows 5% a month, so the price level grows 5% a month.

Part B: In 20X3, the money stock grows 15% a month, so the price level grows 15% a month.

Part C: In equilibrium, the real supply of money equals the real demand for money. When the government announces the higher money growth rate, people expect higher inflation and a higher nominal interest rate.

With a higher nominal interest rate, people want to hold less cash (currency and checking account deposits) and more interest-bearing bonds. That is, the real demand for money decreases. For the money markets to be in equilibrium, the money supply = the demand for money. The nominal money stock has not changed, so for the real supply of money to decrease, the price level must increase. The price level jumps on January 1, 20X3, to a higher level, and then proceeds to climb at the same rate as the money growth rate.

Barro shows the jump occurring overnight in his textbook. In fact, the rise in the price level may take a few weeks, as people adjust to the higher expected inflation rate. Expected inflation doesn't jump 10 percentage points overnight; it rises gradually and people learn of the new money growth rate.

** Exercise 15.4: Money growth rate

Suppose the money growth rate increases from 2% in 20X2 to 3% in 20X3. What are the effects on

A. Real GDP
B. The inflation rate
C. The real interest rate
D. The nominal interest rate
E. The real quantity of money demanded L(Y, i).
F. The real quantity of money supplied
G. The price level

Part A: Money is neutral, so an increase in the money supply does not change real GDP.

Part B: An increase in the money growth rate causes the same increase in the inflation rate, leaving no change in the real money supply.

Part C: The real interest rate is not affected by the money supply or the price level.

Part D: The nominal interest rate is the expected inflation rate + the real interest rate, so a 1 percentage point increase in the money growth rate causes a 1 percentage point increase in the nominal interest rate.

Part E: The real quantity of money demanded L(Y, i) depends on real income (real GDP) Y and the nominal interest rate *i*. If the nominal interest rate increases, the real return on currency decreases, so the real demand for money decreases.

This *cash management* effect is a one-time effect when the nominal interest rate changes. Barro distinguishes the continuing effects of the money growth rate on the inflation rate from the one time effect on the price level.

Since real GDP does not change, the real demand for money changes only from the cash management effect.

Part F: In equilibrium, real supply of money = real demand for money. The real quantity of money demanded changes for the one-time cash management effect, so the real quantity of money supplied changes at the same time.

Part G: The price level is the nominal money supply / the real money supply. As the nominal money supply increases, the price level increases. The price level rises 2% in 20X2 and 3% in 20X3. It also jumps by a small amount when the money growth rate changes.

Question: Is this jump in an instant of time? Do prices actually change suddenly?

Answer: For the exam, conceive of the jump as an instantaneous effect. In practice, the effect occurs gradually. As households learn of the new money growth rate, they adjust their cash management (how much currency to hold, how much money to put in checking accounts vs savings accounts), and business adjust their prices.

** Exercise 15.5: Printing money

- In 20X2, the money stock is 100 million.
- Real GDP is not changing.
- On January 1, 20X3, the government announces that it will print new money to increase the money stock 10% a month. Government expenditures do not change; the new money will pay off existing debt.

A. What is the monthly inflation rate in 20X3?
B. What happens to the real demand for money on January 1, 20X3?
C. What happens to the price level on January 1, 20X3?
D. What is the real revenue from printing money in January 20X3
E. What happens to the real value of bonds?
F. Who gains and who loses from the new money printed?

Part A: The monthly inflation rate in 20X3 is 10%, the same as the money growth rate.

Part B: The real demand for money decreases on January 1, 20X3, since the inflation rate becomes $1.10^{12} - 1 = 213.84\%$ per annum. If the real interest rate is 3% per annum, the nominal interest rate is 223.26% per annum; the size of the real interest rate doesn't matter for this exercise. People want to hold as little money in currency as possible, since currency rapidly loses value. Instead, they hold money in bonds (money market accounts), which earn interest. Since the nominal interest rate is so high, the real demand for money drop quickly.

Part C: The real money supply = the real demand for money, and the price level = the nominal money supply / the real money supply, so the price level jumps on January 1, 20X3.

Part D: The government prints 10 million in January 20X3. But the price level has jumped, so the real revenue is less than 10 million.

Part E: All bonds lose value. If a bond is worth 100,000 on December 31, 20X2, and it matures in 10 years, investors compute the present value of the future coupon payments and principle payments at the new nominal interest rates. The new interest rates are higher, so the present value is lower. If the nominal interest rate was close to the real interest rate before January 1, 20X3 (that is, if the money growth rate had been close to zero), most of the bond's cash flows are the principle repayment in 10 years, which is now worth very little.

Part F: Borrowers who owe fixed amounts to creditors are the ones who gain. These are generally large corporations and the government. Creditors, bondholders, and pensioners living on fixed incomes lose. The economy as a whole loses because of the costs of inflation.

Question: Is it possible for the real revenue from printing money to be negative?

Answer: Barro thinks it is possible, but only if the money growth rate exceeds about 100% a month.

** Exercise 15.6: Money growth rate and inflation rate

You are preparing forecasts of inflation based on a regression analysis of the money growth rate μ, real GDP growth rate $\partial Y/Y$, and the inflation rate π.

$$\pi = \alpha + \beta_1 \mu + \beta_2 \partial Y/Y.$$

The velocity of money does not change: no changes in the transaction costs for getting money from savings occur. What values do you expect for the parameters of the regression equation?

A. α
B. β_1
C. β_2

Part A: If the money supply does not change and real GDP does not change, $\mu = 0$ and $\partial Y/Y$. The real quantity of money demanded stays the same, so the real quantity of the money stock stays the same, the price level does not change, and inflation is zero: $\alpha = 0$.

Question: How does population growth affect the parameters? Should we use $\partial Y/Y$ (the growth rate of real GDP) or $\partial y/y$, the growth rate of real GDP per capita (or per worker)?

Answer: $\partial y/y$ is the growth rate of income per worker. If the population grows 2% per annum and $\partial y/y$ is zero, real GDP grows 2% per annum, and the real quantity of money demanded grows 2% per annum. If the nominal money supply does not change, the price level must decrease 2% to equate money supplied with money demanded.

If the final exam problems do not mention a population growth rate, assume it is zero. Except for very low income countries, the world's population is relatively stable now.

Part B: $\beta_1 = +1$. In the equilibrium business cycle model, money is neutral. A 1 percentage point rise in the money growth rate causes a 1 percentage point rise in the inflation rate, with no change in real variables.

Part C: $\beta_2 = -1$. This parameter value is less intuitive, but the relations are tested on the final exam. If the growth rate of real GDP is 1% per annum, the real demand for money grows 1% per annum and the real supply of money grows 1% per annum. If the nominal money supply does not change, the price level must decline 1% per annum, so the inflation rate is –1% per annum.

** Exercise 15.7: Money growth and inflation

On December 31, 20X2, Country W has a money stock of 200 million, a price level of 100, and debt of 120 million which comes due on December 31, 20X4.

In 20X2, the population stays the same, real GDP stays the same, and the money stock stays the same.

On January 1, 20X3, the government takes one of two actions:

A. Prints 120 million of new money and announces that money will not increase afterward.
B. Announces that money will grow 5% a month in 20X3 (for 12 months).

Assume the government announcements are credible (people believe them).

What are the effects on the inflation rate in 20X3 and the price level on January 2, 20X3?

Part A: Inflation rate = zero; Price level = 100 × 320 / 200 = 160.

Part B: Inflation rate = 5% a month; price level is more than 100, but we don't know the exact figure.

Question: Why is the price level > 100 before inflation begins? Why doesn't the price level increase 5% a month?

Answer: The price level increases 5% a month beginning on January 2, 20X3. On January 1, 20X3, the price level jumps from 100 to a higher amount.

Question: Why should the price level jump before the money stock changes?

Answer: People now expect inflation of 5% a month. The money stock of 200 million is held in currency and checking accounts, which earn no interest. People no longer want to hold cash, which is devaluing at 5% a month

** Exercise 15.8: Nominal and real interest rates

- On January 1, 20X1, nominal assets are 1,000 and the price level is 110.
- Nominal interest rates are 5% in 20X1 and 7% in 20X2.
- Inflation rates are 2% in 20X1 and 3% in 20X2.
- What are real assets on January 1, 20X3?

Solution 15.8: Real assets on January 1, 20X1, are 1,000 / 110 = $9.09.

Real assets on January 1, 20X3, are 1,000 / 110 × (1.05 / 1.02) × (1.07 / 1.03) = $9.72.

Question: Should we use 1.05 / 1.02 or 1.05 - 0.02?

Answer: The first is more accurate. For low rates, the second is an approximation, and it is used in the textbook. 1,000 / 110 × (1.05 – 0.02) × (1.07 – 0.03) = $9.74. The final exam problems have large enough distinctions among the multiple choice solutions that either method is fine.

Question: Do we use 1,000 / 110 = 9.09 or 1,000 / 1.10 = 909.09?

Answer: Both methods are used. Table 11.2 on page 197 uses the first method, but the second method is also used. The final exam problems do not test this convention.

** Question 15.9: Monetary authorities

Money, interest rates, inflation rates, and the price level are related.

Which of the following can the monetary authorities directly affect?

A. The price level
B. The nominal interest rate
C. The real interest rate
D. The money supply
E. The inflation rate

Answer 15.9: D

Question: The textbook says the Federal Reserve Board targets the federal funds rate, not the money supply.

Answer: The FED's target is the federal funds rate, but it affects the federal funds rate by changing the nominal money supply.

** Exercise 15.10: Real demand for money

Which of the following affect the real demand for money?

A. Real GDP
B. Real interest rate
C. Nominal interest rate
D. Inflation rate
E. Nominal money supply
F. Real money supply
G. Price level

Part A: The real demand for money = L(Y, i), where Y = real income and i = the nominal interest rate. The real demand for money is proportional to real income minus an offset for the value of cash management.

Part B: The real interest rate has no effect on the real demand for money except through the nominal interest rate, which equals the real interest rate + expected inflation.

Part C: A higher nominal interest rate raises the real cost of holding (non-interest bearing) currency and reduces the real demand for money.

Part D: The inflation rate has no effect on the real demand for money except through the nominal interest rate, which equals the real interest rate + expected inflation.

Part E: Money is neutral. The nominal money supply affects the price level, not the real demand for money.

Part F: The real money supply = the real demand for money. The causation is the reverse: the real demand for money comes first, and it causes the price level to adjust so that the real money supply equals the real demand for money.

Part G: As explained in the preceding paragraph, the real demand for money causes the price level to change, not the reverse.

** Question 15.11: Price level

Which relation determines the price level?

A. Nominal interest rate = real interest rate + expected inflation rate
B. Real demand for money = L(Y, i)
C. Real demand for money = real supply of money
D. Real wage rate = marginal product of labor
E. Real rental price = marginal product of capital

Answer 15.11: C

- The nominal money supply is set by the monetary authorities.
- The real demand for money is determined by real income (Y) and the nominal interest rate (i).
- The real money supply = the real demand for money.
- The price level is the nominal money supply divided by the real money supply.

** Question 15.12: Real money supply

Which of the following causes the growth rate of the real money supply to increase 1% per annum?

A. The growth rate of real GDP increases 1% per annum.
B. The growth rate of the nominal money supply increases 1% per annum.
C. The growth rate of the real interest rate increases 1% per annum.
D. The growth rate of the nominal interest rate increases 1% per annum.
E. The growth rate of the price level increases 1% per annum.

Question: This exercise seems poorly worded. Is the real money supply increasing 1% per annum or the growth rate of the real money supply increasing 1% per annum? The same question applies also to the other items: real GDP, nominal money supply, real interest rate, and price level.

Answer: Yes, the wording is confusing. Barro uses these terms to mean that the growth rate of the real money supply is Z% in year X and changes to (Z+1)% in subsequent years.

Answer 15.12: *A:* This exercise uses two relations:

- The real money supply = the real demand for money.
- The real demand for money is a function if real GDP and the nominal interest rate: L(Y, i).

** Exercise 15.13: Inflation and the money supply growth rate

A. If the monetary authorities want no inflation, at what rate should the money supply grow?
B. Suppose the population growth rate (*n*) is 0.5%, the growth rate of real income per worker ∂y/y is 1.5%, and the growth rate of the money supply (μ) is 3.5%. What is the expected inflation rate π?

Part A: The money supply must grow to support the growth rate of real GDP. If real GDP grows P% *per annum* and the money supply also grows P% *per annum*, the expected inflation rate is zero.

Part B: ∂Y/Y (the growth rate of real GDP) = ∂y/y (the growth rate of real GDP per capita) + n (the population growth rate) = 2% + 1% = 3%. The population growth rate (*n*) is 0.5%, and the growth rate of real income per worker ∂y/y is 1.5%, so the growth rate of real GDP is 0.5% + 1.5% = 2% *per annum*.

The growth rate of the money supply (μ) is 3.5%, so the expected inflation rate π is 3.5% – 2% = 1.5% *per annum*. Know the equation in the textbook: $\pi = \mu - \partial y/y - n$

Question: Should this be π (the actual inflation rate) or π^e (the expected inflation rate)?

Answer: Barro uses the inflation rate π; he uses the expected inflation rate π^e to mean households' predictions are the beginning of the year. This exercise means the expected inflation rate in a statistical sense: the actual rate is stochastic, but this is our best estimate.