

## Macro module 18: Taxes: practice problems

(The attached PDF file has better formatting.)

This posting gives sample final exam problems. Other topics from the textbook are asked as well; these problems are just examples. All final exam problems are multiple choice; practice problems are not multiple choice so that the solutions can be better explained.

### \*\* Exercise 18.1: Taxes on labor income

The government raises the tax rate on labor but government spending  $G$  does not change; transfers  $V$  rise by the amount of the new tax. What are the effects on

- A. The supply curve for labor?
- B. The demand curve for labor?
- C. The pre-tax real wage rate?
- D. The after-tax real wage rate?
- E. The (equilibrium) amount of labor?
- F. The marginal product of capital?
- G. The demand curve for capital services?
- H. The supply curve for capital services?
- I. The market clearing real rental price?
- J. The amount of capital services supplied?
- K. Real GDP?

*Part A:* The labor supply curve is affected by income and substitution effects.

Substitution effect: the higher tax on labor reduces the effective real wage rate, making labor less valuable (relative to leisure) than before. The substitution effect reduces the supply of labor for any pre-tax real wage rate, so it shifts the supply curve left. The supply curve is upward sloping, so a shift left is a shift upward.

Income effect: If the higher tax increased government spending, personal disposable income would fall and the income effect would cause labor to rise. Since the higher tax in this scenario increases transfers, not government spending, personal disposable income does not change, and no income effect occurs.

*Question:* Which is stronger, the income effect or the substitution effect?

Workers earn less after tax. Some workers can't manage with the lower income; they need more money so they work more (income effect). Some workers have enough money; they work less because they are paid less for each hour of work (substitution effect). The overall effect on the labor supply is uncertain.

Empirically, the supply of labor and real GDP generally change little. The substitution effect is usually stronger than the income effect in the United States and other developed countries, so the supply of labor and real GDP would decrease, not increase, but the change is small.

See Barro, *Macroeconomics*, Chapter 13, "Taxes," page 247, column 1 and top of column 2

*Part B:* The demand curve is the amount of labor demanded for a given real wage rate. Workers pay the tax on labor income, not employers, so the demand curve for labor does not change.

*Part C:* The pre-tax real wage rate is at the intersection of the supply and demand curves. The supply curve shifts left and the demand curve does not change, so the intersection is at a higher pre-tax real wage rate.

*Part D:* The equilibrium supply of labor is at the intersection of the supply and demand curves. The supply curve shifts left and the demand curve does not change, so the intersection is at a lower supply of labor.

*Part E:* The after-tax real wage rate is shown by the supply curve for labor. The equilibrium supply of labor is lower, so the after-tax real wage rate must be lower.

*Part F:* The supply of labor decreases, so there is less labor for each unit of capital, so the marginal product of capital decreases.

*Part G:* The marginal product of capital decreases, so the demand for capital services decreases. The demand curve for capital services shifts down. The demand curve is downward sloping, so a shift down is a shift left. Any given amount of capital services has a lower marginal product of capital (because of the lower supply of labor).

*Part H:* The supply curve for capital services does not change. For any real rental price, the amount of capital services supplied by owners of capital does not change.

*Part I:* The demand curve for capital services shifts left (down) and the supply curve does not change, so the (equilibrium) market clearing real rental price (R/P) decreases.

*Part J:* The market clearing real rental price (R/P) decreases, so the amount of capital services decreases.

*Part K:* The technology level A does not change, but L and  $\kappa K$  decrease, so real GDP decreases.

*Question:* What are the implications for social policy?

*Answer:* Raising taxes on labor to provide government transfers (unemployment benefits, housing, health care, welfare, pensions) reduces real GDP. This doesn't mean that government transfers are bad: countries want to help the poor, the needy, and the ill. But society pays for the high government transfers by lower real GDP.

**\*\* Exercise 18.2: Taxes on asset income**

The government raises the tax rate on asset income but government spending  $G$  does not change; transfers  $V$  rise by the amount of the new tax. What are the effects on

- A. The demand curve for capital services?
- B. The short term supply curve for capital?
- C. The after-tax real interest rate?
- D. The short term supply curve for capital services?
- E. The equilibrium amount of capital services in the short term?
- F. Short term real GDP?
- G. Current consumption and saving
- H. Long-term (steady state) real GDP per worker?

*Part A:* The demand curve for capital services does not change, since the tax is on owners of capital, not users of capital. For a given marginal product of capital, the level of capital services demanded does not change.

*Part B:* The capital stock (the supply of capital) is fixed in the short term.

*Part C:* The after-tax real interest rate is multiplied by  $(1 - \tau)$ , where  $\tau$  is the tax rate on asset income. The real interest rate on bonds equals the net real return on assets

(equation 11.8): 
$$r = (R/P) \times \kappa - \delta(\kappa)$$

If the tax rate is the same for all types of asset income, the after-tax real interest rate on bonds is

(equation 13.3): 
$$(1 - \tau) \times r = (1 - \tau) \times [(R/P) \times \kappa - \delta(\kappa)]$$

*Part D:* The supply curve for capital services does not change in the short term. The capital stock  $K$  is fixed in the short term. Owners of capital choose the capital utilization rate  $\kappa$  to maximize net rental income, which is  $[(R/P) \times \kappa - \delta(\kappa)] \times K$ .

To maximize real rental income, set the partial derivative with respect to  $\kappa$  equal to zero:

$$[R/P - \delta'(\kappa)] \times K = 0 \Rightarrow R/P = \delta'(\kappa)$$

The depreciation rate function  $\delta(\kappa)$  is upward sloping and convex: higher  $\kappa$  causes higher  $\delta$ , and the partial derivative of  $\delta(\kappa)$  is also upward sloping. As  $\kappa$  increases,  $\delta'(\kappa)$  increases and  $R/P$  increases, so the supply curve for capital services is upward sloping.

The  $\kappa$  that maximizes the after-tax real rental price is the same as the  $\kappa$  that maximizes the pre-tax real rental price: the equation has an additional term of  $(1 - \tau)$ , which does not affect the maximization.

*Intuition:* Given the capital stock  $K$ , the  $\kappa$  that maximizes the net real rental price depends on the depreciation rate function, not on the real interest rate.

*Part E:* The capital stock  $K$  is fixed and the capital utilization rate  $\kappa$  does not change, so the supply curve for capital services does not change. The tax is on owners of capital, not users of capital, so the demand curve for capital services does not change, and the amount of capital services provided does not change.

*Part F:* The amount of capital services does not change, and nothing changes labor or the technology level, so real GDP does not change in the short run. The next parts of this exercise deal with long-run effects.

*Part G:* As explained above, the after-tax real interest rate on bonds is

(equation 13.3): 
$$(1 - \tau) \times r = (1 - \tau) \times [ (R/P) \times \kappa - \delta(\kappa) ]$$

As  $\tau$  increases, the after-tax real interest rate decreases. The lower real interest rate has an inter-temporal substitution effect, inducing households to increase current consumption and reduce future consumption.

- If the real interest rate is high, people save more from current income, invest at the high interest rate, and consume more in later periods.
- If the real interest rate is low, people consume more in the current period, since investing money doesn't have a high return.

*Part H:* Real GDP does not change and government spending does not change (by assumption). Private consumption increases in the current period, so investment decreases, since  $Y = C + G + \text{investment}$ . (Lower savings causes lower investment.)

*Part I:* The lower investment causes lower long-term (steady state) capital per worker.

*Question:* What is the implication of this analysis?

*Answer:* The quickest way to ruin an economy is to tax asset income. To raise steady state income per worker, a country should encourage high savings and high investment. By taxing asset income, the government reduces savings and investment, lowering steady state capital per worker and income per worker.

*Question:* The United States has a high tax rate on asset income; why is this?

*Answer:* Populist politicians want to tax asset income, because investors are wealthy, whereas workers are poorer. Economists say that taxes on asset income are shifted to workers and consumers, so they are often regressive. A true progressive tax system taxes only labor income (at tax rates that depend on income), not asset income, and this tax system does not reduce savings or investment.

**\*\* Exercise 18.3: Consumption tax**

The government changes the income tax (tax on labor income) to a sales tax (tax on consumption).

- A. If the taxes are permanent, what is the effect of this change?
- B. If the taxes are temporary (one year), what is the effect of this change?

*Part A:* Most persons work to earn money to consume goods. When the tax is levied is not relevant. If the tax is levied on all labor income and all consumption, it might be levied at four points: a tax on employers paying wages, a tax on workers earning wages, a tax on producers selling goods, or a tax on consumers buying goods. The economic effects of the tax are identical: an hour of labor buys less goods after the tax.

*Question:* This ignores household decisions about whether to consume or to save. A tax on labor income is paid whether the person consumes goods with the wage income or save the income. A consumption tax is paid only if the person consumes goods. Shouldn't the consumption tax induce people to save more and consume less?

*Answer:* If the tax is permanent, saving the wage income is no different from consuming. Eventually the person uses the income to buy goods, and the consumption tax has to be paid.

*Part B:* If the taxes are temporary (one year), each tax has an inter-temporal substitution effect.

- A temporary tax on labor income of this year only causes lower labor this year and more labor next year.
- A temporary tax on consumption of this year only causes lower consumption this year and more next year.

See Barro, *Macroeconomics*, Chapter 13, page 245.

**\*\* Exercise 18.4: Government programs**

The government raises the tax rate on labor income to fund one of two national programs.

Program #1 use the money to save endangered species of frogs and newts. The value to the public is zero.

Program #2 use the money to improve public hygiene, ensure that drinking water is safe from cholera. The value to the public is high: a dollar spent by the government on public hygiene is worth many dollars spent by private persons.

- A. What is the effect of Program #1 on the supply curve for labor? Consider both the substitution effect and the income effect.
- B. What is the effect of Program #2 on the supply curve for labor? Consider both the substitution effect and the income effect.

*Part A:* A higher tax rate on labor makes working less valuable (in comparison with other uses of time, such as leisure), so it shifts the supply curve for labor to the left. Less labor is supplied for a given pre-tax real wage rate, since the after-tax real wage rate declines. This is the substitution effect.

The higher government spending with no value to most households causes people to feel less wealthy. The income effect induces them to work more (supply more labor).

The net effect of the substitution effect plus the income effect is ambiguous: the supply of labor may increase or decrease.

*Part B:* The substitution effect is the same as in Part A. The income effect is the opposite. People are wealthier by the government programs to improve public hygiene, since they no longer have to spend their own money to ensure that the water is safe to drink. Both the substitution effect and the income effect induce people to work less.

**\*\* Exercise 18.5: Laffer Curve**

Suppose the government raises the marginal tax rate on labor income.

- A. What is the substitution effect on the supply curve for labor?
- B. What is the income effect on the supply curve for labor?
- C. At low to moderate marginal tax rates, which effect is stronger?
- D. At very high marginal tax rates, which effect is stronger?
- E. As the marginal tax rate rises, does overall tax revenue increase, decrease, or stay the same?

*Part A:* As the marginal tax rate increases, the after-tax value of labor decreases, so people work less and enjoy more leisure. The supply curve for labor shifts to the left.

*Part B:* As the marginal tax rate increases, people have less disposable income, so they must work more. The supply curve for labor shifts to the right.

*Part C:* At low to moderate marginal tax rates, the substitution effect and income effect about offset each other, and the supply of labor does not change much as the marginal tax rate changes.

*Part D:* At very high marginal tax rates, the substitution effect becomes very strong, and the supply of labor decreases as the marginal tax rate increases.

*Part E:* At low tax rates, overall tax revenue increases as the marginal tax rate increases. At a marginal tax rate of zero, overall tax revenue is zero, and tax revenue increases as the tax rate increases.

At high tax rates, overall tax revenue decreases as the marginal tax rate increases. At a tax rate of 100%, no one works, since no one keeps any labor income, so tax revenue is zero. At lower tax rates, people work, so overall tax revenue is positive.

At some intermediate marginal tax rate, overall tax revenue is highest. Barro says this tax rate is about 70%, which is higher than the average tax rate in all countries. However, the tax rate on high wage earners has been as high as 90% in the U.S. and European countries. This high tax rate is silly: it causes wealthy people to work less, reducing (not increasing) the taxes collected by the government.

*Question:* Is a 70% tax rate also the tax rate that maximizes real GDP?

*Answer:* Real GDP is maximized at a tax rate close to zero. Taxes raise real GDP only when government work is more efficient than private work. This condition is true for some work (public hygiene, national security, and perhaps some police work). Most government work can be done more efficiently by private enterprise (mail, airlines, public transportation), though countries have public policy reasons to use government enterprises.

**\*\* Exercise 18.6: Explicit and implicit marginal tax rates**

A country considers three government programs to help people buy food.

Program #1: All persons receive \$1,000 of food stamps which can be traded for food at grocery stores.

Program #2: Persons with income less than \$20,000 receive food stamps equal to  $10\% \times (\$20,000 - \text{income})$ .

Program #3: Persons receive food stamps equal to  $10\% \times \text{wage income}$ , up to a maximum of \$2,000.

- A. What are the income and substitution effects of Program #1?
- B. What are the income and substitution effects of Program #2?
- C. What are the income and substitution effects of Program #3?

*Part A:* Program #1 has no substitution effect, since people receive the same food stamps no matter how much or how little they work. The income effect induces people to work less. People work to earn money to consume goods. People need \$1,000 less to buy food, so they work less.

*Question:* Do we expect all persons to earn \$1,000 less?

*Answer:* We expect people to earn between \$0 less and \$1,000 less. Some people hate their jobs so much that they work only enough to eat; these people will earn \$1,000 less. Most people do not hate their jobs and work for money to consume many things. If they need \$1,000 less to buy food, they spend \$1,000 more on clothes and other goods. These people work a little less if they get food stamps, but not \$1,000 less.

*Part B:* A transfer program that pays more to people who earn less is like a tax on labor. The substitution effect induces people to work less so they get more food stamps. The income effect induces people to work less as well, with a stronger income effect on people who work little (and get more food stamps).

*Part C:* A transfer program that pays more to people who earn more is like a negative tax on labor. The more a person earns, the more food stamps the person gets, so the substitution effect induces people to work more. The income effect induces people to work less as well, with a stronger income effect on people who work more (and get more food stamps).



**\*\* Exercise 18.7: Budget constraint, consumption, labor, and taxes**

Suppose households have a two year budget constraint: they consider only years 1 and 2, not later years.

The government is preparing for elections in November 20X1 (year 1). It raises transfers for 20X1, and it raises taxes for 20X2. The present value of the taxes equals the present value of the transfers. The new policy is announced on January 1, 20X1, and it is believed by the public.

- A. All transfers and taxes are lump-sum; they do not depend on labor income.
- B. All transfers and taxes depend on labor income. Transfers are  $50\% \times (\$50,000 - \text{income})$  for persons with income below \$50,000, and taxes are  $50\% \times (\text{income} - \$50,000)$  for persons with income above \$50,000.

What are the effects on  $C_1$ ,  $C_2$ ,  $L_1$ ,  $L_2$  (consumption and labor in years 20X1 and 20X2)?

*Part A:* For the income effect, people consider the present value of all transfers and taxes. Consumption and labor each year depends on the present value of all income, not the net income in that year. The present value of all transfers and taxes does not change, so consumption and labor do not change. The transfers and taxes are lump-sum, not related to income, so there is no substitution effect.

*Part B:* The taxes raise the explicit marginal tax rate and the transfers raise the implicit marginal tax rate. Low income workers and unemployed persons (that is, all persons with income below \$50,000) work less in 20X1 and more in 20X2, so they get more transfers in 20X1 (when the transfer program is in effect). They don't earn enough to pay taxes in 20X2, so they make up for low earnings in 20X1 with higher earnings in 20X2.

*Question:* How do people shift earnings from one year to another?

*Answer:* Suppose a person with low earnings wants to spend a year at a vocational college to learn skills for a better job. The transfer program in 20X1 provides money for people not working, so the worker takes that year to go to school. Similarly, people move their vacations to 20X1.

High income workers (with income above \$50,000) work more in 20X1 and work less in 20X2. In 20X2, an hour of work (after the first \$50,000 of income) is worth half as much as the same hour of work in 20X1. A high income worker who wants to spend six months at an executive MBA program will take the courses in 20X2, not in 20X1.

The effect on total labor supply depends on the strength of the substitution effect and the income effect for low and high income workers.

\*\* Exercise 18.8: Explicit and implicit marginal tax rates

- A. What is the difference between the tax rate and the marginal tax rate?  
B. What is an implicit marginal tax rate?

*Part A:* The tax rate is the tax liability divided by taxable income. The marginal tax rate is the change in the tax liability divided by the change in taxable income.

*Illustration:* Suppose persons earn income between \$0 and \$100,000, and the tax liability for income of Y is

$$\text{tax liability} = 50\% \times Y^2 / \$100,000.$$

- The tax rate is  $50\% \times Y / \$100,000$ .
- The marginal tax rate is  $\partial (\text{tax liability}) / \partial Y = 100\% \times Y / \$100,000$ .

A person earning \$10,000 has a tax liability of  $50\% \times \$10,000^2 / \$100,000 = \$500$ , so the tax rate is  $\$500 / \$10,000 = 5\%$ . If the person earns \$1 more, the tax liability is  $50\% \times \$10,000^2 / \$100,000 = \$500.10$ , so the marginal tax rate is  $\$0.10 / \$1.00 = 10\%$ .

*Part B:* Government transfer payments are often related to the person's income. Suppose poor persons get food stamps worth  $10\% \times (\$10,000 - \text{income})$ . A person earning \$2,000 gets food stamps worth \$800 and a person earning \$9,000 gets food stamps worth \$100. Persons earning more than \$10,000 do not get food stamps.

The implicit tax rate is  $- [ 10\% \times (\$10,000 - \text{income}) ] / \text{income} = 10\% - \$1,000 / \text{income}$ . A person earning \$4,000 has an implicit tax rate of  $10\% - \$1,000 / \$4,000 = -15\%$ . That is, a person earning \$4,000 gets  $15\% \times \$4,000 = \$600$  of food stamps.

The implicit marginal tax rate is  $\partial [ -10\% \times (\$10,000 - \text{income}) ] / \partial (\text{income}) = +10\%$ .

A person earning \$4,000 gets food stamps of \$600. A person earning \$4,001 gets food stamps of

$$10\% \times (\$10,000 - \$4,001) = \$599.90.$$

The difference in food stamps divided by the difference in income is  $-\$0.10 / \$1.00 = -10\%$ . This is a receipt of value, so it is the opposite of a tax payment. The implicit marginal tax rate is 10%.