Microeconomics, Module 5, Practice Problems: Behavior of Firms (Chapter 5)

Practice Problems

(The attached PDF file has better formatting.)

For questions 5.1 through 5.8, the demand curve is P = 40 – 5Q, and the firm's cost curve is Total Cost = $3Q^2$ + Q.

Marginal revenue and marginal cost can be determined on a discrete or continuous basis.

- *Discrete:* The marginal revenue for the second item is the revenue for two items minus the revenue for one item: (40 – 5 × 2) × 2 – (40 – 5 × 1) × 1 = \$25. The marginal cost for the second item is the cost for two items minus the cost for one item: (3 × 2^2 + 2) – (3 × 1^2 + 1) = \$10.
- *Continuous:* The marginal revenue is the partial derivative of the total revenue with respect to quantity. Total revenue = Q × (40 – 5Q) = 40Q – $5Q^2$ and ∂TR/∂Q = 40 – 10Q. At Q = 2, the marginal revenue is 40 – 10 × 2 = \$20. The marginal cost is the partial derivative of the total cost with respect to quantity: $\partial(3Q^2 + Q)/\partial Q = 6Q + 1$. For Q = 2, this is 6 × 2 + 1 = \$13.

For most of his illustrations, Landsburg uses the discrete approach; he avoids mathematics that his readers may not know. Most final exam questions use the continuous method.

We refer to these two methods as arithmetic vs calculus, or units that are not divisible vs units that are divisible. Instead of units that are divisible, we might say: *units are in millions*.

Question 5.1: Marginal Cost (Arithmetic)

If units are not divisible, what is the marginal cost of producing the third unit?

Answer 5.1: If the units are not divisible, the marginal cost of the third unit is the total cost of three units minus the total cost of two units:

- Two units total cost: $3 \times 2^2 + 2 = 14$
- Three units total cost: $3 \times 3^2 + 3 = 30$

Marginal cost of third unit = 30 – 14 = 16.

Question 5.2: Marginal Cost (Calculus)

If units are in millions (so they are divisible), what is the marginal cost of producing the three millionth unit?

Answer 5.2: If the units are divisible (and millions are divisible into a million parts), the marginal cost of the N^{th} unit is the partial derivative of the total cost with respect to quantity:

$$\partial TC/\partial Q = \partial(3Q^2 + Q)/\partial Q = 6Q + 1 = \$19.$$

Question 5.3: Marginal Revenue (Arithmetic)

If the units are not divisible, what is the marginal revenue received from selling the fifth unit?

Answer 5.3: The marginal revenue from the N^{th} unit is the total revenue from N units minus the total revenue from N-1 units:

- Total revenue from N-1 units: $Q \times P = 4 \times (40 - 5 \times 4) = 4 \times 20 = 80$
- Total revenue from N units: $Q \times P = 5 \times (40 - 5 \times 5) = 5 \times 15 = 75$

Marginal revenue = 75 – 80 = –$5.

Question: Why can't the firm sell the first four units at $20 a unit and the fifth unit at $15 a unit, so the total revenue is $95?

Answer: Both legal and business reasons prevent this.

- It is not legal to sell units at different prices to different consumers, unless the costs of these units differ.
- If consumers expected the firm to sell the fifth unit at $15, the consumer might wait for the firm to reduce its price, thereby reducing the total revenue.

In practice, firms attempt to divide their markets and sell goods at high prices to consumers willing to pay high prices and at low prices to consumers who would not buy the goods at the higher prices. We cover this in the module on price discrimination.

Question 5.4: Marginal Revenue (Calculus)

If the units are in millions (they are divisible), what is the marginal revenue from selling the five millionth unit?

Answer 5.4: The marginal revenue is the partial derivative of the total revenue with respect to quantity:

- Total revenue as a function of quantity = $P \times Q = (40 - 5Q) \times Q = 40Q - 5Q^2$
- Marginal revenue = $\partial(40Q - 5Q^2)/\partial Q = 40 - 10Q$

The marginal revenue at the five millionth unit is $40 - 10 \times 5 = -\$10$.

Question 5.5: Profit Maximization (Arithmetic)

According to the equimarginal principle, if units are not divisible, how many units should the firm produce to maximize its profits?

Answer 5.5: 2 units.

Units	*Total Revenue*	*Total Cost*	*Marginal Revenue*	*Marginal Cost*	*Profit*
1	35	4	35	4	31
2	60	14	25	10	46
3	75	30	15	16	45
4	80	52	5	22	28
5	75	80	-5	28	-5

We select the optimal row either as

- The row where profit is maximized.
- The last row where marginal revenue exceeds marginal cost.

Question 5.6: Maximum Profit (Arithmetic)

What is the maximum profit this firm can earn (if units are not divisible)?

Answer 5.6: 46

Question 5.7: Profit Maximization (Calculus)

If units are in millions, how many units should the firm produce to maximize its profits?

Answer 5.7: Marginal cost is 6Q + 1; marginal revenue is 40 – 10Q. Setting them equal gives

$$6Q + 1 = 40 - 10Q \Rightarrow 16Q = 39 \Rightarrow Q = 39/16 = 2.438 \text{ million.}$$

Question 5.8: Maximum Profit (Calculus)

If units are in millions, what is the maximum profit this firm can earn?

Answer 5.8:

- Total revenue is 2.438 × (40 – 5 × 2.438) = \$67.80 million.
- Total cost is $3 \times 2.438^2 + 2.438$ = \$20.27 million

Profit is \$67.80 million – \$20.27 million = \$47.53 million

Exercise 5.9: Study Time

Suppose the value of passing Course M is \$20,000. The chance of passing (P) depends on the average hours of study each week (S) as P = S / 40, with a maximum of 100%. (If the candidate studies 30 hours a week, the chance of passing is 30 / 40 = 75%.)

The marginal cost of an hour of study per week is \$25 × S. The first hour of study causes a loss of utility equal to \$25; the second hour of study causes a loss of utility of \$50; etc.

A. How many hours does the candidate study each week?
B. What is the candidate's chance of passing Course M?
C. What is the total cost of study? That is, what is the dollar cost of studying these hours each week?
D. What is the net value received by the candidate from studying for Course M?
E. If the candidate's employer gives 10 hours of study time each week at work, how many additional hours does the candidate study? What is the total hours of study?
F. What is the candidate's chance of passing Course M?

Solution 5.9:

Part A: The marginal revenue of each hour of study a week is \$20,000 / 40 = \$500. The marginal cost is \$25 × S. Equating the two gives $25 \times S = 500 \Rightarrow S = 500 / 25 = 20$.

Part B: The candidate's chances of passing Course M is 20 / 40 = 50%.

Part C: The total cost of study is the integral of 25 × S from 0 to 20, or $\int_0^{20} 25S\,dS = 12.5\,S^2 \Big|_0^{20}$ = 12.5×20^2 = \$5,000.

Question: This assumes that candidates stop studying when it is no longer worthwhile. But candidates study for these exams as long as they can; they stop studying only when they are no longer absorbing the material.

Answer: The economist says the same thing. Candidates don't study as long as they can; they do other things as well, such as eat and sleep and work and play. It study were worth more to them, they would cut out more play and eat quicker and sleep less and take more vacation time. When they have studied so much that they are no longer absorbing the material, an additional hour of study has less marginal revenue.

Part D: The net value is 50% × \$20,000 – \$5,000 = \$5,000.

Part E: Since the marginal revenue curve is flat, the marginal revenue is still \$500 per hour of study (up to 40 hours of total study a week). The candidate studies an additional 20 hours a week, for a total of 30 hours a week (including study time at work).

Part F: The candidate's chance of passing Course M is 30 / 40 = 75%.

Exercise 5.10: Profit Maximization

A firm faces a demand curve of P = 100 – Q. Fixed costs are 1,000; marginal costs are 10 + Q. Assume that the firm produces a quantity and charges a price to maximize profits.

A. What is the marginal revenue curve facing the firm?
B. What is the quantity produced by the firm?
C. What is the equilibrium price charged by the firm?
D. What are the total variable costs of the firm?
E. What are the total costs of the firm?
F. What is the total revenue of the firm?
G. What is the net profit of the firm?

Solution 5.10:

Part A: The total revenue curve is $P \times Q = (100 - Q) \times Q = 100Q - Q^2$. The marginal revenue is $\partial(TR)/\partial Q = 100 - 2Q$.

Part B: We equate marginal revenue and marginal cost: $100 - 2Q = 10 + Q \Rightarrow 90 = 3Q \Rightarrow Q = 30$.

Part C: The equilibrium price is 100 – 30 = 70 (from the demand curve).

Part D: Total variable cost is the integral of the marginal costs from 0 to 30: $\int(10+Q)\,\partial Q = 10Q + \frac{1}{2}Q^2$ from 0 to 30 = $10 \times 30 + \frac{1}{2}\, 30^2 = 300 + 450 = 750$.

Part E: The total costs are variable costs plus fixed costs = 1,750.

Part F: The total revenue is P × Q = 70 × 30 = 2,100.

Part G: The net profit is 2,100 – 1,750 = 350.

Exercise 5.11: *Profit Maximization*

A firm has the following cost and demand schedules.

Quantity	*Price*	*Total Cost*

1	100	50
2	90	100
3	80	160
4	70	230
5	60	320
6	50	450

A. What is the total revenue at each quantity?
B. What is the total profit at each quantity?
C. What quantity should the firm produce to maximize profits? If two quantities have the same total profit, choose the higher quantity.
D. What is the marginal cost at each quantity?
E. What is the marginal revenue at each quantity?
F. At what quantity is marginal cost equal to marginal revenue?

Solution 5.11:

The table below shows the columns needed to answer the practice problem.

Quantity	*Price*	*Total Revenue*	*Marginal Revenue*	*Total Cost*	*Marginal Cost*	*Total Profit*
1	100	100	100	50	50	50
2	90	180	80	100	50	80
3	80	240	60	160	60	80
4	70	280	40	230	70	50
5	60	300	20	320	90	-20
6	50	300	0	450	130	-150