Micro Modules 7 and 8: Competition practice problems

(The attached PDF file has better formatting.)

** Exercise 8.1: Competitive industry

Firms in a competitive constant-cost industry have marginal costs of $12q$ and fixed costs of $1,350, where q is the quantity produced by the firm.

The market demand curve for the firms' goods is P = $450 – 30% × Q or Q = 1,500 – 3.3333 × P, where P is the market price and Q is the market quantity.

A. What is the variable cost curve for each firm as a function of quantity?
B. What is the average variable cost curve for each firm as a function of quantity?
C. What is the average fixed cost curve for each firm as a function of quantity?
D. What is the average total cost curve for each firm as a function of quantity?
E. Graph the three average cost curves. What is the slope of each curve as a function of quantity?
F. What is the quantity produced by each firm? At this point, what are the slopes of the three curves?
G. What is the long-run market price for the firms' goods?
H. At this long-run market price, what is the total quantity demanded by consumers?
I. How many firms compete in this industry?
J. What is the industry's short-run supply curve?
K. What is the industry's long-run supply curve?
L. What is the consumers' surplus?
M. What is the producers' surplus?
N. The government adds a sales tax of $15 per unit. What is the change in the long-run equilibrium price from the sales tax? This is the price charged for the item by suppliers (who do not receive the tax), not the change in the effective price paid by consumers (who pay the tax).
O. What is the change in the long-run equilibrium quantity from the sales tax? This is the change in the market quantity, not the change in the quantity produced by a single firm.

Part A: Total variable costs are the integral of marginal costs from zero to the quantity produced:

$$= \int_0^q 12x\,dx = 6q^2.$$

Intuition: This is the area of a right triangle with base q and height $12q$; area = $6q^2$.

Part B: Average variables costs are $6q^2 / q = \$6q$.

Intuition: The marginal cost varies linearly from 0 to 12q, so the average variable cost = ½ × (0 + 12q) = 6q.

Part C: Average fixed costs are $1,350 / q$.

Part D: Average total costs are $6q + \$1,350 / q$.

Part E: The slope of average variable costs is +$6. The slope of average fixed costs is –$1,350 / q^2$.

Part F: In the long run, each firm produces at its minimum average cost. The market price is equal to this average cost, at which the firm earns zero economic profit. It earns the appropriate return on capital, but no economic rent. If it produced any other quantity, it would have higher costs and negative economic profit.

At this minimum average cost, the slope of the average total cost curve is zero: $6 = \$1,350 / q^2 \Rightarrow q = 15$.

Part G: The long run equilibrium price is the minimum average total cost. At a quantity of 15, this cost is $6 × 15 + $1,350 / 15 = $90 + $90 = $180. This is also the marginal cost at q = 15: $12 × 15 = $180.

Part H: The equilibrium quantity is $180 = $450 – 30% × Q ⇒ Q = 900.

Part I: Each firm produces 15 units. For a market quantity of 900, there are 900 / 15 = 60 firms.

Part J: Each firm has a marginal cost curve of MC = $12q. For 60 firms, q = Q / 60, so MC = Q / 5.

Part K: In the long run, firms enter and leave the industry as the market expands or contracts. Firms always produce at their minimum average cost of $180, so the long-run marginal cost curve is MC = $180.

Part L: Consumers' surplus is the area under the demand curve, down to the equilibrium price, and out to the equilibrium quantity. Since the demand curve is linear, consumers' surplus is a right triangle with three vertices

- When Q = 0, P = $450 (from the demand curve).
- The equilibrium price is $180.
- The equilibrium quantity 900 units.

The vertices (Q,P) of the right triangle are (0, $450), (0, $180), (900, $180).
Consumers' surplus is ½ × 900 × ($450 – $180) = $121,500.

Part M: Producers' surplus is the area above the short-run supply curve, up to the equilibrium price, and out to the equilibrium quantity. The supply curve is linear, so producers' surplus is a right triangle with three vertices

- When Q = 0, marginal cost = $0 (from the supply curve).
- The equilibrium price is $180.
- The equilibrium quantity is 900 units.

The vertices (Q,P) of the right triangle are (0, $0), (0, $180), (900, $180).
Producers' surplus is ½ × 900 × ($180 – $0) = $81,000.

Use the short run supply curve for producers' surplus.

Part N: The sales tax does not change the supply curve. The equilibrium price is the minimum average cost, which remains $180.

Part O: The sales tax changes the demand curve to (P + tax) = $450 – 30% × Q or Q = 1,500 – 3.33333 × (P + tax). *Intuition:* Consumers pay the price plus the tax, so the effective price = P + sales tax. The equilibrium quantity with the sales tax is Q = 1,500 – 3.33333 × ($180 + $15) = 850. The change in the quantity is 900 – 850 = –50. (The change is –3.33333 × 15 = –50.)

Exercise 8.2: Competitive Equilibrium

A competitive firm is at short run and long-run equilibrium; all firms in the industry are identical. Its marginal cost curve MC = 2Q, and its fixed costs are $1,000,000. The industry demand curve is Q = 700,000 – 200P.

A. What is the firm's variable cost curve? (Integrate the marginal cost curve.)
B. What is the firm's average variable cost curve? (Divide by the quantity.)
C. What is the firm's average total cost curve? (Add fixed costs divided by quantity.)
D. At what quantity is average cost minimized? (Differentiate with respect to Q and set to 0.)
E. What is the price at this quantity? Why is this the equilibrium market price? (Price equals marginal cost equals average total cost.)
F. Given this market price, what is the industry quantity demanded? (Use the demand curve.)
G. Given the industry quantity and the firm quantity, how many firms are in this industry? (Industry quantity divided by firm quantity.)

Part A: The firm's variable cost curve is $\int_0^Q 2x\, dx = x^2 \Big|_0^Q = Q^2$

Part B: The firm's *average* variable cost curve is $Q^2 / Q = Q$.

Part C: The firm's average total cost curve is Q + $1,000,000 / Q.

Part D: We minimize the average cost curve by setting its partial derivative with respect to Q equal to zero:

$$\partial(Q + \$1{,}000{,}000/Q)/\partial Q = 1 - \$1{,}000{,}000Q^{-2} = 0 \Rightarrow Q = 1{,}000.$$

Part E: The price at this quantity is the marginal cost at Q = 1,000, or 2 × 1,000 = 2,000. The equilibrium competitive price in the long-run is the minimum average cost.

Part F: The industry quantity demanded at P = 2,000 is

$$700{,}000 - 200 \times 2{,}000 = 300{,}000.$$

Part G: Each firm produces Q = 1,000, so there are 300 firms in this industry.