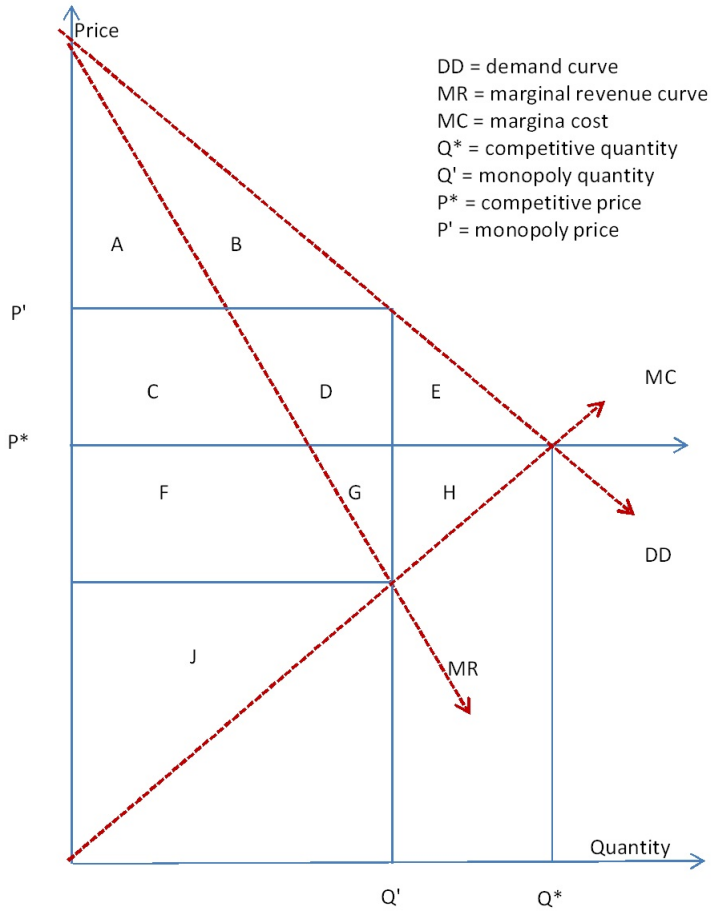
Micro Module 9: Welfare economics and the gains from trade practice problems

(The attached PDF file has better formatting.)

** Exercise 9.1: Gains from trade

The graphic below shows consumers' surplus and producers' surplus for competition and monopoly.



Note: This exercise combines material from chapter 8 (gains from trade) and chapter 10 (monopoly). Skip this exercise until you have studied the modules on monopoly, then come back to this exercise and focus on consumers' surplus, producers' surplus, and the dead weight loss.

Note: This graphic shows the marginal cost curve as a straight line passing through the origin. In practice, marginal cost curves are either flat or downward sloping at low quantities. The graphic here simplifies the mathematical computations. Final exam problems generally use either flat marginal cost curves or linear curves with a positive Y intercept (a positive marginal cost at Q = 0).

A. DD is the market demand curve. What is the demand curve facing a competitive firm in this market?
B. MR is the marginal revenue curve for a monopoly. What is the marginal revenue curve for a competitive firm in this market?
C. How are the competitive price and quantity determined?
D. How are the monopoly price and quantity determined?
E. What is consumers' surplus in a competitive market?
F. What is producers' surplus in a competitive market?
G. What is consumers' surplus in a monopolized market?
H. What is producers' surplus in a monopolized market?
I. What is the dead weight loss from monopoly? (Derive from the social gains in competition and monopoly).
J. Derive the dead weight loss directly from the areas in the graphic.

Part A: The *market* demand curve is downward sloping; it does not differ for perfect competition vs monopoly.

- The demand curve *facing a monopoly* is the market demand curve.
- The demand curve *facing a competitive firm* is a horizontal line at the competitive price P*.

Part B: Competitive firms sell all their output at the market price P*. The additional revenue from one more unit of output is P*, so the marginal revenue curve is a horizontal line at P*.

Question: Why is the marginal revenue curve for a monopoly downward sloping?

Answer: In theory, the demand curve and the marginal revenue curve can be of various shapes. Many final exam problems use linear demand curves: $P = \alpha - \beta \times Q$. Total revenue = $P \times Q = \alpha \times Q - \beta \times Q^2$ and marginal revenue = ∂(total revenue)/$\partial Q = \alpha - 2\beta \times Q$.

Part C: The competitive price and quantity are determined by the intersection of the market demand curve and the marginal cost curve. Most final exam problems are numeric:

Final exam problems may give a linear demand curve and linear marginal cost curve; derive the intersection. An exam problem may also give a cumulative (total) variable cost curve or an average variable cost curve.

- The marginal cost curve is ∂(variable cost curve)/∂Q.
- The variable cost curve is average variable cost curve) × Q.

Part D: The monopoly quantity is at the intersection of the marginal revenue curve and the marginal cost curve. The monopoly price is the price on the market demand curve corresponding to the monopoly quantity.

For final exam problems, determine

- total revenue = price × quantity
- marginal revenue = ∂(total revenue)/∂Q.

For a linear demand curve, the mathematics is easy.

Part E: Consumers' surplus is the area under the market demand curve and above the market price, or areas A + B + C + D + E.

For numerical problems with a linear market demand curve, consumers' surplus is a right triangle:

- The width is the competitive quantity.
- The height is the price on the demand curve at Q = 0 minus the competitive price.

Consumers' surplus = the area of the right triangle = ½ × height × width.

Part F: Producers' surplus is the area above the marginal cost curve and below the market price, or areas F + G + H + J.

For numerical problems with a linear marginal cost curve, producers' surplus is a right triangle:

- The width is the competitive quantity.
- The height is the competitive price minus the *variable* cost on the marginal cost curve at Q = 0.

Producers' surplus = the area of the right triangle = ½ × height × width.

Question: Is producers' surplus the same as the net profit received by producers?

Answer: Net profit is revenue – total costs = revenue – variable costs – fixed costs. The marginal cost curve considers only variable costs, not fixed costs, so producers' surplus is revenue – variable costs. In a perfectly competitive market, net economic profit is zero (in the long run).

Question: Isn't producers' surplus a right triangle plus a rectangle?

Answer: For a monopoly, producers' surplus is a right triangle plus a rectangle. The height of the rectangle is proportional to the slope of the demand curve facing the firm. A competitive firm faces a horizontal demand curve, so its rectangle has a height of zero.

Part G: Consumers' surplus is the area under the market demand curve and above the monopoly price, or areas A + B.

For numerical problems with a linear market demand curve, consumers' surplus is a right triangle:

- The width is the *monopoly* quantity.
- The height is the price on the demand curve at Q = 0 minus the *monopoly* price.

Consumers' surplus = the area of the right triangle = ½ × height × width.

- The monopoly quantity is smaller by $Q^* - Q'$.
- The monopoly price is higher by $P' - P^*$.

The reduction in consumers' surplus is areas C + D + E.

Part H: Producers' surplus is the area above the marginal cost curve and below the monopoly price, or areas C + D + F + G + J.

For numerical problems with a linear marginal cost curve, producers' surplus is a right triangle + a rectangle.

For the right triangle:

- The width is the monopoly quantity.
- The height is the marginal revenue at the monopoly quantity (= the intersection of the marginal revenue curve and the marginal cost curve) minus the variable cost on the marginal cost curve at Q = 0.

For the rectangle

- The width is the monopoly quantity.
- The height is the monopoly price minus marginal revenue at the monopoly quantity (= the intersection of the marginal revenue curve and the marginal cost curve).

Producers' surplus = the area of the right triangle = ½ × height × width + the area of the rectangle = height × width.

Part I: The dead weight loss is the social gain from competition minus the social gain from monopoly. The social gain is the consumers' surplus + the producers' surplus. In the graphic, this is area

+ (A + B + C + D + E)
+ (F + G + H + J)
– (A + B)
– (C + D + F + G + J)

= E + H.

Part J: For linear demand and marginal cost curves, the dead weight loss is the area of a triangle:

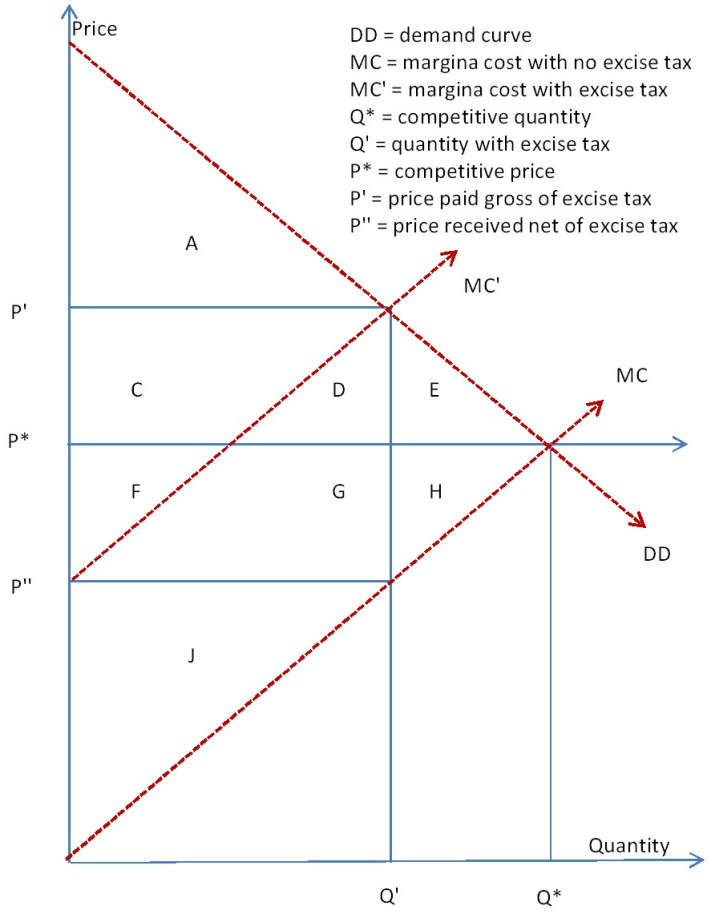
- The height is the competitive quantity – the monopoly quantity.
- The base is the market demand curve – the marginal cost curve at the monopoly quantity.

The net social gain is the area under the market demand curve minus the area under the marginal cost curve from zero to the quantity sold. The monopoly quantity Q' is less than the competitive quantity Q^*, so the dead weight loss is E + H. With a government subsidy, the equilibrium quantity is too high, and the dead weight loss appear on the right side of the competitive quantity.

** Exercise 9.2: Excise tax

The graphic below shows the effect of an excise tax as a fixed dollar amount per item.

A. Does an excise tax affect the demand curve or the supply curve?
B. What is the amount of the excise tax in this graphic?
C. What is consumers' surplus with no excise tax?
D. What is producers' surplus with no excise tax?
E. What is consumers' surplus with the excise tax?
F. What is producers' surplus with the excise tax?
G. What is the tax collected by the government?
H. What is the dead weight loss?



Part A: The excise tax is paid by the producers of the good, not by consumers. The excise tax here is a fixed dollar amount per item, so it raises the marginal cost curve a fixed amount at every point. In the graphic, MC′ is parallel to MC. If the tax depends on the price of the good, the two marginal cost curves are not parallel.

Part B: The amount paid by consumers is P′. The amount received by producers net of the excise tax is P′′. The excise tax is P′ – P′′. This is *not* the difference in price with vs without the excise tax, which is P′ – P*.

Part C: Consumers' surplus with no excise tax is A + C + D + E.

Part D: Producers' surplus with no excise tax is F + G + H + J.

Part E: Consumers' surplus with the excise tax is A. Consumers lose two ways from the tax:

- For the items bought, they pay their portion of the tax, or C + D.
- The tax causes a smaller equilibrium quantity, on which the consumers' surplus is lost (E).

Part F: Producers' surplus with the excise tax is J. Producers lose two ways from the tax:

- For the items bought, they pay their portion of the tax, or F + G.
- The tax causes a smaller equilibrium quantity, on which the producers' surplus is lost (H).

Part G: The tax collected by the government is C + D + F + G. If the government is perfectly efficient, this area is not lost; it is just shifted from consumers and producers to the recipients of government aid. In practice, much of this area is lost, for several reasons. First, tax collection costs money; for each dollar of tax, the tax authorities may pay 15¢ of collection expense. Each firm must be audited and sales must be monitored. Second, firms spend money to minimize their taxes. Free markets maximize social gain if the firms produce consumer goods. Markets serving to avoid taxes are not maximizing social gain. Third, governments often use money inefficiently, spending it to reward their supporters, not to maximize social gain. These items are not easily shown on the graphic. The textbook mentions them but does not quantify them.

Part H: We work out the dead weight loss two ways: (1) the change in social gain and (ii) the social loss from the quantity not produced.

1. The difference in social gain is A + C + D + E + F + G + H + J – (A + J + C + D + F + G) = E + H.
2. The quantity not produced is Q* – Q′, so the dead weight loss is E + H.

Question: In final exam problems, is the sales tax or excise tax always a fixed amount per item?

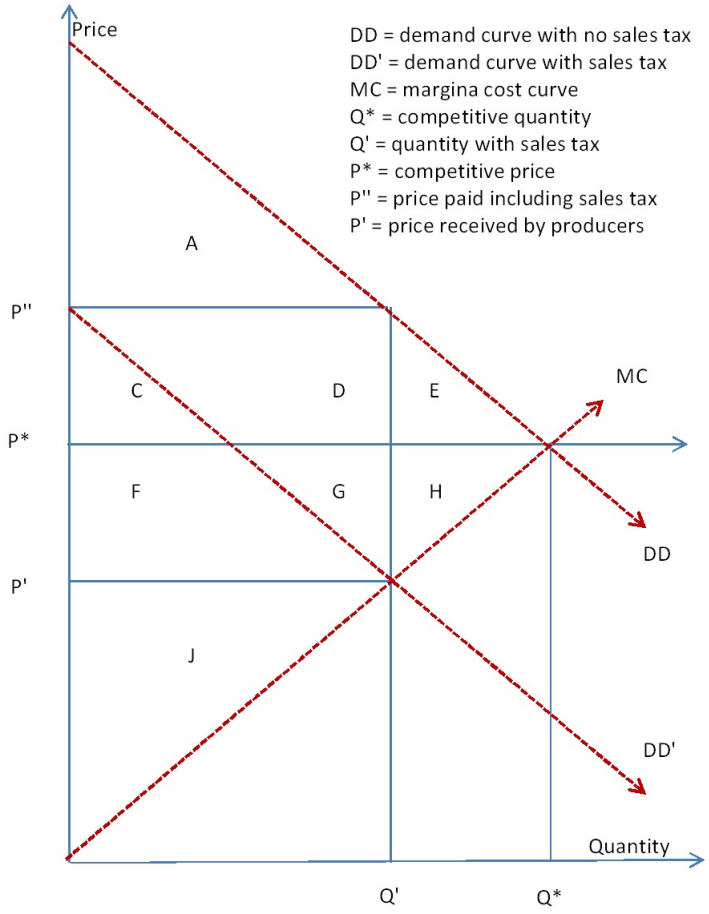
Answer: No. The sales tax or excise tax may a percentage of the pre-tax price.

If the pre-tax demand curve is $P = \alpha + \beta \times Q$, and the government imposes a sales tax of T%, the new demand curve is $P / (1 + T\%) = \alpha + \beta \times Q$.

** Exercise 9.3: Sales tax

The graphic below shows the effect of a sales tax as a fixed dollar amount per item.

A. Does an sales tax affect the demand curve or the supply curve?
B. What is the amount of the sales tax in this graphic?
C. What is consumers' surplus with no sales tax?
D. What is producers' surplus with no sales tax?
E. What is consumers' surplus with the sales tax?
F. What is producers' surplus with the sales tax?
G. What is the tax collected by the government?
H. What is the dead weight loss?



Part A: The excise tax is paid by the consumers of the good, not by producers. The sales tax here is a fixed dollar amount per item, so it lowers the demand curve a fixed amount at every point. In the graphic, DD′ is parallel to DD. If the tax depends on the price of the good, the two demand curves are not parallel.

Part B: The amount paid by consumers (including the sales tax) is P′′. The amount received by producers net of the sales tax is P′. The sales tax is P′′ – P′. This is *not* the difference in price with vs without the sales tax, which is P′′ – P*.

Part C: Consumers' surplus with no sales tax is A + C + D + E.

Part D: Producers' surplus with no sales tax is F + G + H + J.

Part E: Consumers' surplus with the sales tax is A. Consumers lose two ways from the tax:

- For the items bought, they pay their portion of the tax, or C + D.
- The tax causes a smaller equilibrium quantity, on which the consumers' surplus is lost (E).

Part F: Producers' surplus with the sales tax is J. Producers lose two ways from the tax:

- For the items bought, they pay their portion of the tax, or F + G.
- The tax causes a smaller equilibrium quantity, on which the producers' surplus is lost (H).

Part G: The tax collected by the government is C + D + F + G. If the government is perfectly efficient, this area is not lost; it is just shifted from consumers and producers to the recipients of government aid. In practice, much of this area is lost, for several reasons. First, tax collection costs money; for each dollar of tax, the tax authorities may pay 15¢ of collection expense. Each firm must be audited and sales must be monitored. Second, firms spend money to minimize their taxes. Free markets maximize social gain if the firms produce consumer goods. Markets serving to avoid taxes are not maximizing social gain. Third, governments often use money inefficiently, spending it to reward their supporters, not to maximize social gain. These items are not easily shown on the graphic. The textbook mentions them but does not quantify them.

Part H: We work out the dead weight loss two ways: (1) the change in social gain and (ii) the social loss from the quantity not produced.

1. The difference in social gain is A + C + D + E + F + G + H + J – (A + J + C + D + F + G) = E + H.
2. The quantity not produced is Q* – Q′, so the dead weight loss is E + H.

Question: The graph is different for the sales tax vs excise tax, but all the areas (consumers' surplus, producers' surplus, social gain, dead weight loss) are the same. Is this just a coincidence?

Answer: The economic incidence a sales tax is identical to the economic incidence of an excise tax. All the areas must be exactly the same. Some people think that if businesses pay the tax, they are better off. This is not true, though government often levy taxes on businesses to make voters feel better.

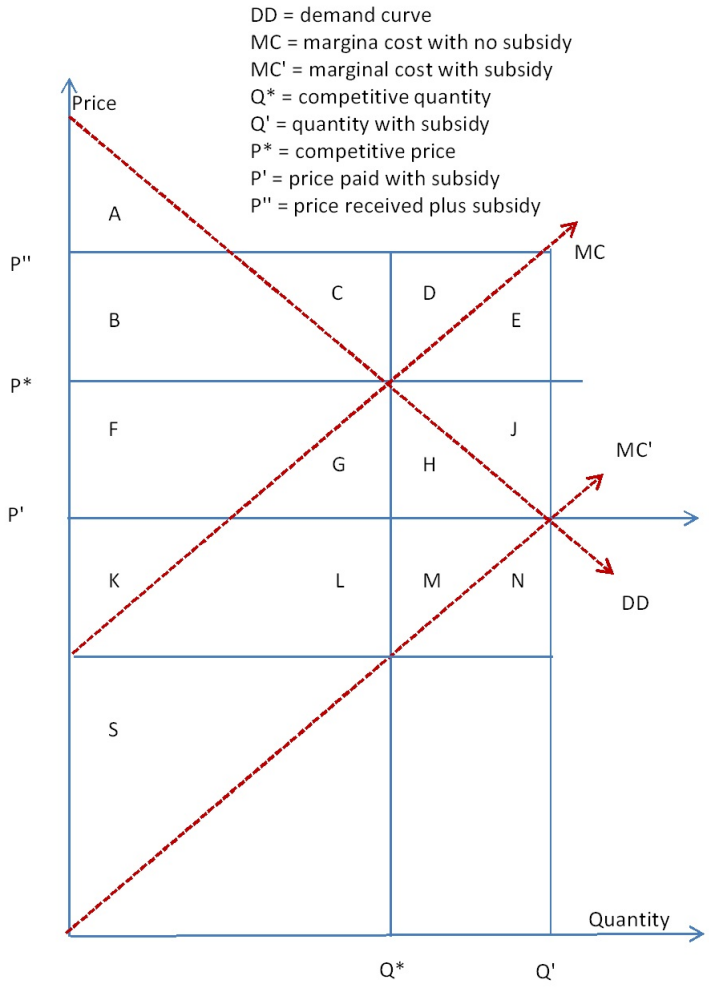
Question: The United States uses mostly sales taxes, not excise taxes. Does your comment apply to the United States as well?

Answer: The United States has many excise taxes, but consumers are not always aware of them.

- In the United States, half the social security taxes are paid by workers and half are paid by their employers. These are sales taxes and excise taxes on the market for labor. Most labor markets are competitive, so workers end up paying both taxes.
- If you buy auto insurance, you pay no taxes directly to the government. The insurer pays 3% of premium to the state and about 5% of premium to the federal government. The entire tax is shifted to consumers, who pay 8% of the premium in tax.

** Exercise 9.4: Subsidy

Electric cars are not yet economically viable. The government believes that electric cars reduce the nation's dependence on foreign sources of oil, so it pays a subsidy to producers of electric cars. The subsidy is a fixed dollar amount per electric car. The graphic below shows the effect of this subsidy.



A. Does this subsidy affect the demand curve or the supply curve?
B. What is the amount of the subsidy in this graphic?
C. What is consumers' surplus with no subsidy ?
D. What is producers' surplus with no subsidy ?
E. What is consumers' surplus with the subsidy ?
F. What is producers' surplus with the subsidy ?
G. What is the subsidy paid by the government?
H. What is the dead weight loss?

Part A: The subsidy is paid to producers, not consumers. It lowers the marginal cost by the amount of the subsidy.

- The marginal cost curve before the subsidy is MC.
- The marginal cost curve after the subsidy is MC′.

Question: Does the marginal cost curve after the subsidy pass through the origin?

Answer: No; the marginal cost curve after the subsidy can intersect the price axis above or below the origin. The graphic draws it through the origin for consistency with the other graphics in this set of practice problems.

Part B: The amount of the subsidy is P′′ – P′.

- P′′ is the amount received by producers including the subsidy.
- P′ is the amount paid by consumers. If producers receive a government subsidy, consumers pay less.

Part C: With no subsidy, the competitive quantity is Q* and the competitive price is P*. Consumers' surplus is A + B.

Part D: With no subsidy, the competitive quantity is Q* and the competitive price is P*. Producers' surplus is F + K.

Part E: With the subsidy, consumers pay P′ and the equilibrium quantity is Q′. Consumers' surplus is A + B + F + G + H.

Part F: With the subsidy, producers receive P′′ and the equilibrium quantity is Q′. Producers' surplus is B + C + D + F + K.

Part G: The subsidy paid by the government (by taxpayers) is B + C + D + E + F + G + H + J.

Part H: The dead weight loss is

$$(A + B) + (F + K) – [A + B + F + G + H + B + C + D + F + K – (B + C + D + E + F + G + H + J)] =$$
$$(A + B) + (F + K) – [A + F + K + B – (E + J)] = E + J$$

Question: With no subsidy, B is in consumers' surplus and F is in producers' surplus. With the subsidy, B and F are part of consumers' surplus, producers' surplus, and the subsidy. Is this correct?

Answer: The subsidy means consumers pay less than producers receive, but taxpayers pay for this overlap *and more*. The extra amount they pay is the dead weight loss.

Question: Is this just an exercise for the VEE course, or does this actually happen?

Answer: The federal government bailed out General Motors and provides subsidies for electric cars. Society loses two ways. First, General Motors was inefficient: high labor costs, poor product design, and self-serving management. Second, the subsidy for electric cars has a large dead weight loss.

Question: Both consumers and producers are better off; the quantity produced is more than in the competitive equilibrium. It seems everyone gains. Is this good?

Answer: There are no free lunches. If government interference in the market makes some persons better off, others are worse off. In this scenario, taxpayers fund the subsidy. On the whole, society loses; this is the dead weight loss (see below). The subsidy to General Motors to build electric cars illustrates this. Buyers of electric cars (green consumers) gain, since they pay less. Producers of electric cars (auto workers) gain, since they have jobs and earn income above the value of their labor. (They are producing cars that people don't value

as much they cost to produce.) The taxpayers who fund the subsidy lose more than the consumers and producers gain.

Question: If society loses, why have the subsidy? Who gains from the GM bailout and the subsidies?

Answer: The auto unions gained from the bailout, and they are strong supporters of the current administration. Progressive voters who want green cars at below market prices gain from the subsidy, and they are also strong supporters of the current administration. The moral, which applies to all taxes and subsidies: the dead weight loss is not shared equally by all persons. Taxes and subsidies reduce social welfare, but they improve the lot of specific groups.