Microeconomics, monopoly, mathematical final exam multi-question practice problem sets

(The attached PDF file has better formatting.)

** Exercise 11.1: Monopoly Pricing (algebra)

This exercise shows the algebra for linear demand curves and marginal cost curves. The next exercise is a numerical example. The final exam uses numerical examples and multiple choice questions.

- An industry has one firm which sets prices to maximize profits.
- The market demand curve is $P = \alpha - \beta \times Q$.
- The firm's marginal cost curve is $MC = \gamma + \delta \times Q$.

[*Note:* Final exam problems may use either form of the demand curve.]

A. What is the firm's total revenue curve?
B. What is the firm's marginal revenue curve?
C. What is the monopoly quantity?
D. What is the monopoly price?
E. Whe price elasticity of demand at the monopoly price and quantity?
F. What is consumers' surplus?
G. What is producers' surplus?
H. What is the dead weight loss?

Part A: Total revenue is price times quantity = $P \times Q = Q \times (\alpha - \beta \times Q) = \alpha Q - \beta \times Q^2$.

Question: This exercise gives the demand curve as $P = \alpha - \beta \times Q$. Some exam problems gives the demand curve as $Q = \alpha - \beta \times P$. Which form should we use?

Answer: If an exam problem asks to derive the total population demand curve from the demand curves of two sub-populations, $Q = \alpha - \beta \times P$. The two demand curves from the sub-populations are $Q_1 = \alpha - \beta \times P$ and $Q_2 = \alpha - \beta \times P$. The total population demand curve is $Q_1 + Q_2 = \alpha - \beta \times P$.

For competitive pricing, we use the relation that price = marginal cost. Marginal cost is expressed as a function of quantity, so we express price as a function quantity. The marginal cost depends on the production function – how production varies with the number of items produced (quantity). It does not depend on the price.

For monopoly pricing, we use the relation that marginal revenue = marginal cost. Marginal cost is expressed as a function of quantity, so we express price as a function quantity. Marginal revenue is price × quantity, so we express marginal revenue as f(Q) × Q, not as P × f(P).

Question: What if the exam problem gives the reverse form of the demand curve? For example, what if the exam problem gives a demand curve $Q = \alpha - \beta \times P$ and asks to derive the monopoly price?

Answer: A market demand curve $P = \alpha - \beta \times Q$ is the same as $Q = \alpha / \beta - 1/\beta \times P$. Similarly, A market demand curve $Q = \alpha - \beta \times P$ is the same as $P = \alpha / \beta - 1/\beta \times Q$. Many exam problems give the reverse form and want you to convert one form to the other.

Part B: Marginal revenue is the partial derivative of total revenue with respect to quantity =

$$\partial(\alpha Q - \beta \times Q^2) / \partial Q = \alpha - 2\beta Q.$$

Question: Why not express marginal revenue as the partial derivative of total revenue with respect to quantity:

$$\partial(\alpha P - \beta \times P^2) / \partial P = \alpha - 2\beta P.$$

Answer: To solve for the monopoly price and quantity, we set marginal revenue equal to marginal cost. Since marginal cost is a function of quantity, not of price, we express marginal revenue as a function quantity, not of price.

Exception: If marginal cost is constant, we can express marginal revenue as a function of either price or quantity. Both methods give the same solution.

Part C: Let Q* = equilibrium quantity and equate marginal cost and marginal revenue:

$$\alpha - 2\beta Q^* = \gamma + \delta \times Q^* \Rightarrow Q^* = (\alpha - \gamma) / (\delta + 2\beta).$$

The next exercise shows a numerical example that solves for the equilibrium quantity and price.

Part D: Derive the equilibrium price from the demand curve, *not* the marginal cost curve. Let P* = equilibrium price, so

$$P^* = \alpha - \beta \times Q^* = \alpha - \beta \times (\alpha - \gamma) / (\delta + 2\beta)$$

Part E: The price elasticity of demand at the monopoly price is $(\partial Q/Q) / (\partial P/P) = \partial Q/\partial P \times P/Q$.

At the monopoly price and quantity, this is $P^*/Q^* \times -1/\beta$.

The elasticity is negative: as P increases, Q decreases.

Part F: Consumers' surplus is a right triangle with base Q* and height = the price at Q = zero minus the equilibrium price: $\alpha - P^*$.

Use the monopoly price and quantity, the demand curve, and the values for P* and Q* derived above.

Note: If the market demand curve is perfectly elastic, β is zero and consumers' surplus is zero.

Part G: Producers' surplus is the sum of

- a right triangle with base Q* and height = (the marginal cost at Q*) – the marginal cost at Q = 0 (γ)
- a rectangle with base Q* and height = (P* – the marginal cost at Q*)

Part H: The dead weight loss can be derived two ways.

Method #1: The dead weight loss = the decrease in net social gain from monopoly = the social gain from competition minus the social gain in monoply = (consumers' surplus + producers' surplus) in competition – (consumers' surplus + producers' surplus) in monopoly.

Method #2 (for linear demand curve and marginal cost curve): The dead weight loss = ½ × (competitive quantity – monopoly quantity) × (monopoly price – marginal cost at the monopoly price)

Method #2 follows from the geometry of consumers' surplus and producers' surplus with linear demand and marginal cost curves.

** Exercise 11.2: Monopoly pricing (numerical example)

- An industry has one firm which sets prices to maximize profits.
- The market demand curve is P = 120 – ½ Q (equivalent to Q = 240 – 2 P)
- The firm's marginal cost curve is MC = 10 + Q.

A. What is the firm's total revenue curve?
B. What is the firm's marginal revenue curve?
C. What is the monopoly quantity?
D. What is the monopoly price?
E. Whe price elasticity of demand at the monopoly price and quantity?
F. What is consumers' surplus?
G. What is producers' surplus?
H. What is the dead weight loss?

Part A: Total revenue is $P \times Q = (120 - ½ Q) \times Q = 120Q - ½ Q^2$

Part B: Marginal revenue $MR = \partial TR/\partial Q = 120 - Q$

Part C: The monopoly quantity is at the intersection of the marginal revenue and marginal cost curves.

$MR = MC \Rightarrow 120 - Q = 10 + Q \Rightarrow 2Q = 110 \Rightarrow Q = 55$

Part D: The monopoly price is derived from the monopoly quantity and the demand curve:

120 – ½ Q = 120 – ½ × 55 = $92.50

Part E: The price elasticity of demand at the monopoly price is $(\partial Q/Q) / (\partial P/P) = \partial Q/\partial P \times P/Q$.

At the monopoly price and quantity, this is $1/\beta \times P^*/Q^* = -2 \times \$92.50 / 55 = -3.364$

Alternative method: marginal revenue = price × (1 – 1/|η|)

At Q = 55, P = 92.5 and marginal revenue = 120 – 55 = 65.
Marginal revenue / price = 0.70270 = 1 – 1 / | η | ⇒ | η | = 1 / (1 – 0.7027) ⇒ η = –3.364

Question: A 1% reduction in price causes a 3.364% increase in quantity sold. Shouldn't the monopolist reduce the price to gain the higher quantity?

Answer: The monopolist loses not just the 1% on the last item sold but 1% on every item sold. The revenue lost minus the profits from the 3.364% increase in quantity offsets the marginal cost of the last item sold.

Part F: Consumers' surplus is the area under the demand curve, down to the equilibrium price, and out to the equilibrium quantity. Evaluate the area of this right triangle.

- P* = 92.5; Q* = 55.
- From the demand curve, if Q = 0, P = 120.
- The vertices of the right triangle are (0, 120), (0, 92.5), (55, 92.5).
- The area of the right triangle = ½ × 55 × (120 – 92.5) = $756.25.

Part G: Producers' surplus is the area under the equilibrium price, down to the marginal cost curve, and out to the equilibrium quantity. This is a rectangle on top of a right triangle. The base of both the rectangle and the right triangle is Q* = 55. The marginal cost at a quantity of 55 is $55 + $10 = $65.

- The height of the right triangle is the marginal cost at Q* – the marginal cost at 0: $10 + $55 – $10 = $55.

- The height of the rectangle is P* minus the marginal cost at Q*: \$92.5 – \$65 = \$27.50.

Producers' surplus is (92.5 – 65) × 55 + ½ × (65 – 10) × 55 = \$3,025.00.

Alternative: Producers' surplus is a taller rectangle minus the right triangle.

- The taller rectangle has a height of \$92.5 – \$10 = \$82.5 and a base of 55 ⇛ area = \$4,537.50
- The right triangle has a base of 55 and a height of \$65 – \$10 = \$55 ⇛ area = ½ × 55 × 55 = \$1,512.50

Producers' surplus is \$4,537.50 – \$1,512.50 = \$3,025.00

Part H: If the market were competitive, the equilibrium price and quantity are at the intersection of the supply and demand curves:

- P* = 120 – ½ Q = 10 + Q ⇛ Q* = 110 × ⅔ = 73.333
- P* = 10 + 73.333 = 83.333 or P* = 120 – ½ Q* = 83.333

- Consumers' surplus is ½ × 73.333 × (120 – 83.333) = \$1,344.44.
- Producers' surplus is ½ × 73.333 × (83.333 – 10) = \$2,688.89.
- Total social gain from competition is \$1,344.44 + \$2,688.89 = \$4,033.33.
- Dead weight loss = \$4,033.33 – \$3,025.00 – \$756.25 = \$252.08.
  - Alternative derivation: Dead weight loss = ½ × (73.333 – 55) × (\$92.50 – \$65) = \$252.08.

Take heed: Some exam problems give the market demand curve as quantity in terms of price. Do not use this form to derive total revenue in terms of price.

Question: How do we solve problems if we are given the demand curve as Q = f(P)?

Answer: Convert the demand curve to the form $P = f^{-1}(Q)$. The exercise gives both forms of the demand curve. Final exam problems give only one form.

Question: The marginal revenue curve here has twice the slope of the demand curve and the same intercept (on the vertical axis); is this always true?

Answer: It is true for linear demand curves. The Y-intercept is always the same, but the relative slopes depend on the form of the demand curve.

Question: The marginal revenue curve lies below the demand curve here; is that always true?

Answer: It is true as long as the marginal revenue curve is upward sloping.

Question: How do fixed costs affect the equilibrium price?

Answer: In the short run, fixed costs do not affect the equilibrium price. In the long-run, if fixed costs exceed producers' surplus, the net profit is negative, and the monopolist exits the industry.

Question: Is consumers' surplus lower with monopoly than in a competitive market?

Answer: In general yes, but with caveats:

- If the industry is a natural monopoly, there is no competitive equilibrium.
- A monopoly may produce economies of scale; if they are large enough, consumers' surplus may increase.
- In some industries, market power is negligible even for the largest firms, since barriers to entry are small, but economies of scale are large. In these industries, mergers are socially beneficial, since they do not create monopoly pricing but raise consumers' surplus. Examples are supermarkets, department stores (and most retail stores), groceries, ratio stations, and insurance.

Question: Is producers' surplus greater for a competitive market or a monopolistic market?

Answer: Producers' surplus is greater for a monopoly. If producers' surplus were greater at the competitive price, the monopolist would charge the competitive price, not the monopoly price.

Question: Is producers' surplus + producers' surplus greater for a competitive market or a monopolistic market?

Answer: The sum of both surpluses is smaller for a monopoly. The decrease is the dead weight loss.

** Exercise 11.3: Demand curve, marginal cost curve, consumers' surplus, and producers' surplus

- ABC is the only supplier in its market. Barriers to entry are not material, so the market is contestable and ABC acts as a competitive frm. The competitive price is \$95 and the competitive quantity is 500.
- The government grants ABC the right to supply the whole market, so ABC becomes a monopoly. The monopoly price is \$100, and the monopoly quantity is 400.

The market demand curve is linear: $P = \alpha - \beta \times Q$. The marginal cost curve is linear: $MC = \gamma + \delta \times Q$. The marginal cost curve is the same for the monopoly as for a competitive firm.

A. What is the demand curve (what are α and β)?
B. What is the marginal cost curve (what are γ and δ)?
C. What is consumers' surplus in the competitive market?
D. What is producers' surplus in the competitive market?
E. What is net social gain in the competitive market?
F. What is consumers' surplus in the monopolized market?
G. What is producers' surplus in the monopolized market?
H. What is net social gain in the monopolized market?

Part A: The demand curve is $P = \alpha - \beta \times Q$. The two markets show that

- Competitive market: $\$95 = \alpha - \beta \times 500$
- Monopoly market: $\$100 = \alpha - \beta \times 400$

This pair of linear equations gives

- $\$100 - \$95 = \$5 = -\beta \times (400 - 500) \Rrightarrow \$5 = \$100 \times \beta \Rrightarrow \beta = 0.05$
- $\$100 = \alpha - 0.05 \times 400 \Rrightarrow \alpha = \$100 + \$20 = \$120$

The demand curve is $P = \$120 - 0.05 \times Q$.

Part B: The marginal cost curve is $MC = \gamma + \delta \times Q$.

In the competitive market, price = marginal cost, so $\$95 = \gamma + \delta \times 500$.

For a monopoly market, we set marginal cost = marginal revenue. For a linear demand curve $P = \alpha - \beta \times Q$, marginal revenue = $\alpha - 2\beta \times Q \Rrightarrow$ marginal revenue = $\$120 - 2 \times 0.05 \times 400 = \$80$, so $\$80 = \gamma + \delta \times 400$.

This pair of linear equations gives

- $\$95 - \$80 = \$15 = \delta \times (500 - 400) \Rrightarrow \$15 = \$100 \times \delta \Rrightarrow \delta = 0.15$
- $\$80 = \gamma + 0.15 \times 400 \Rrightarrow \gamma = \$80 - \$60 = \$20$

The marginal cost curve is $MC = \$20 + 0.15 \times Q$.

Part C: Consumers' surplus is a right triangle whose

- base is the equilibrium quantity and
- height is the price at Q = 0 minus the price for the equilibrium quantity.

For the competitive market, consumers' surplus = $\frac{1}{2} \times 500 \times (\$120 - \$95) = \$6,250$.

Consumers' surplus considers only the demand curve, not the marginal cost curve (supply curve). It is the area above the equilibrium price and below the demand curve.

Part D: Producers' surplus is the area above the marginal cost curve and below the equilibrium price. For a competitive market with a linear marginal cost curve, it is a right triangle whose

- base is the competitive quantity, and
- height is the marginal cost at the competitive quantity minus the marginal cost at Q = 0.

Producers' surplus = ½ × 500 × ($95 – $20) = $18,750.

Question: Is producers' surplus always less than consumers' surplus in competitive markets?

Answer: The size of consumers' surplus and producers' surplus depend on the elasticities of the demand curve and the marginal cost curve. In this exercise, both curves are linear and the slope of the demand curve is 0.05 / 0.15 = one third the slope of the marginal cost curve, so consumers' surplus is one third of producers' surplus. The more elastic the curve, the less surplus is created. A horizontal demand curve (perfectly elastic) gives no consumers' surplus; a horizontal marginal cost curve gives no producers' surplus.

Part E: Social gain from trade = consumers' surplus + producers' surplus = $6,250 + $18,750 = $25,000. We can also compute the social gain from trade directly, as the area between the demand curve and the marginal cost curve. With linear curves, the area is a triangle whose

- base is the equilibrium quantity, and
- height is the difference between price and marginal cost at a quantity of zero:

½ × 500 × ($120 – $20) = $25,000.

Part F: Consumers' surplus is computed the same way in monopolies as in competitive markets: the area of a right triangle whose

- base is the equilibrium quantity and
- height is the price at Q = 0 minus the price for the equilibrium quantity.

For the monopoly, consumers' surplus = ½ × 400 × ($120 – $100) = $4,000.

Part G: Producers' surplus for a monopoly with a linear marginal cost curve is a right triangle plus a rectangle. For the right triangle:

- The base is the equilibrium quantity in the monopolized market.
- The height is the marginal cost at the equilibrium quantity minus the marginal cost at Q = 0.

The area of this triangle = ½ × 400 × ($80 – $20) = $12,000.

For the rectangle:

- The base is the equilibrium quantity in the monopolized market.
- The height is the monopoly price minus the marginal cost at the equilibrium quantity.

The area of this triangle = 400 × ($100 – $80) = $8,000.

Producers' surplus = $12,000 + $8,000 = $20,000.

Part H: Social gain from trade = consumers' surplus + producers' surplus = $4,000 + $20,000 = $24,000.

We verify by computing the dead weight loss, which is a triangle with

- height equal to the difference between the monopoly price and the marginal cost at the monopoly quantity ($100 – $80)

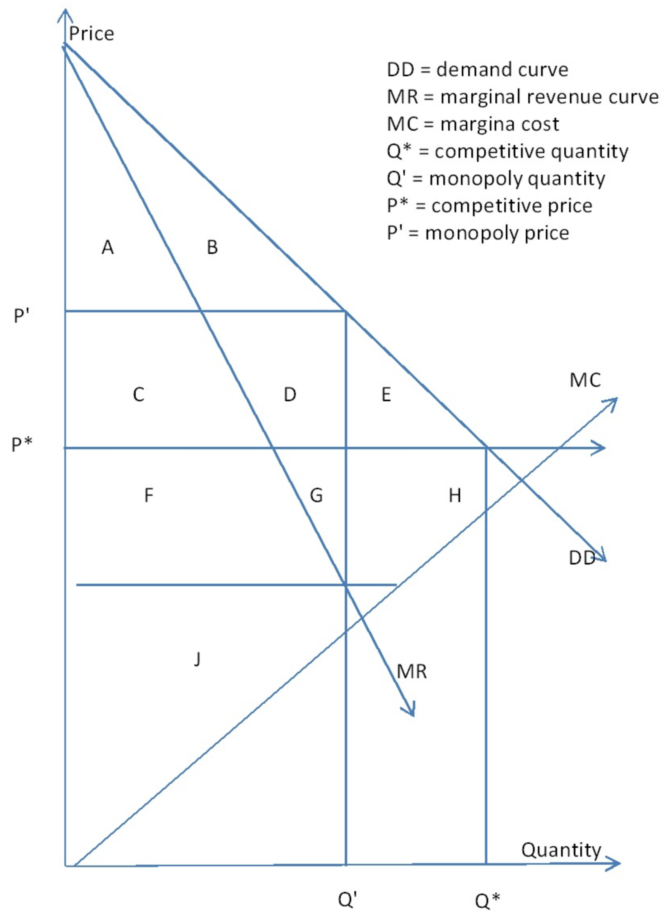
- and width equal to the difference between the competitive quantity and the monopoly quantity (500 – 400).

The area of this triangle is ½ × ($100 – $80) × 100 = $1,000. The dead weight loss is the difference between the social gains in the competitive market and the monopoly market: $1,000 = $25,000 – $24,000.

Graphic: consumers' surplus and producers' surplus for competitive and monopolistic markets.

The graphic shows consumers' surplus and producers' surplus for competitive and monopolistic markets.

- A + B + C + D + E: consumers' surplus in competitive market
- F + G + H + J: producers' surplus in competitive market
- A + B: consumers' surplus in monopolized market
- C + D+ F + G + J: producers' surplus in monopolized market
- E + H: dead weight loss from monopoly



** Exercise 11.4: Contestable market

- Firm ABC is the only supplier in its market. Barriers to entry are insignificant, so the market is contestable and ABC acts as a competitive frm. The competitive price is \$100 and the competitive quantity is 1,000.
- The government grants ABC the right to supply the whole market, so ABC becomes a monopoly. The monopoly price is \$120, and the monopoly quantity is 800.

A. If the demand curve is $P = \alpha - \beta \times Q$, what are α and β?
B. If the marginal cost curve is $MC = 0 + \gamma \times Q$, what is γ?
C. In the contestable market, what is consumers' surplus?
D. In the contestable market, what is producers' surplus?
E. In the contestable market, what is the social gain from trade?
F. In the monopolistic market, what is consumers' surplus?
G. In the monopolistic market, what is producers' surplus?
H. In the monopolistic market, what is the social gain from trade?

Part A: The demand curve is $P = \alpha - \beta \times Q$. The two markets show that

- $\$100 = \alpha - \beta \times 1{,}000$
- $\$120 = \alpha - \beta \times 800$

This gives

- $\$20 = \beta \times 200 \Rightarrow \beta = 0.100$
- $\$100 = \alpha - 0.1 \times 1{,}000 \Rightarrow \alpha = \$200$

The demand curve is $P = \$200 - 0.100 \times Q$.

Part B: The marginal cost curve is $MC = 0 + \gamma \times Q$.

In the competitive market, price = marginal cost, so $\$100 = \gamma \times 1{,}000 \Rightarrow \gamma = 0.100$.

Take heed: Some problems say that marginal costs are constant. If that were the case here, the marginal cost curve would be $\$100 + 0 \times Q$. This exercise assumes the marginal cost curve passes through the origin.

Question: What is the constant \$100?

Answer: In a competitive market, price equals marginal cost.

Part C: Consumers' surplus is a right triangle:

- The base is the competitive quantity.
- The height is the price at Q = 0 minus the competitive price in the market.

Consumers' surplus = $\frac{1}{2} \times 1{,}000 \times (\$200 - \$100) = \$50{,}000$.

Part D: Producers' surplus is a right triangle:

- The base is the competitive quantity.
- The height is the marginal cost at the competitive quantity minus the marginal cost at Q = 0.

Producers' surplus = $\frac{1}{2} \times 1{,}000 \times (\$100 - \$0) = \$50{,}000$.

Question: Producers' surplus = consumers' surplus in this exercise; is that usual?

Answer: This occurs because the demand curve and the marginal cost curve are both linear and the marginal cost is zero at Q = 0. In practice, marginal cost rises slowly with Q (the marginal cost curve is relatively flat) and the demand curve is relatively steep. Consumers' surplus is generally larger than producers' surplus in a competitive market. If marginal cost is constant (does not vary with Q), producers' surplus is zero.

Part E: Social gain from trade = consumers' surplus + producers' surplus = $50,000 + $50,000 = $100,000.

Part F: Consumers' surplus is a right triangle:

- The base is the equilibrium quantity in the monopolized market.
- The height is the price at Q = 0 minus the equilibrium price in the monopolized market.

Consumers' surplus = ½ × 800 × ($200 – $120) = $32,000.

Part G: Producers' surplus is a right triangle plus a rectangle. For the right triangle:

- The base is the equilibrium quantity in the monopolized market.
- The height is the marginal cost in the monopolized market minus the marginal cost at Q = 0.

The area of this triangle = ½ × 800 × ($80 – $0) = $32,000.

For the rectangle:

- The base is the equilibrium quantity in the monopolized market.
- The height is the monopoly price minus the marginal cost in the monopolized market.

The area of this triangle = 800 × ($120 – $80) = $32,000.

Producers' surplus = $32,000 + $32,000 = $64,000.

Part H: Social gain from trade = consumers' surplus + producers' surplus = $32,000 + $64,000 = $96,000.

We verify by computing the dead weight loss: a triangle with height ($120 – $80) and width ($1,000 – 800):

½ × $40 × 200 = $4,000 = $100,000 – $96,000.