Micro Module 11, Monopoly (shorter) practice problems
(The attached PDF file has better formatting.)
** Exercise 11.1: Marginal cost, price, and price elasticity of demand

- At the equilibrium monopoly price of 100 , the marginal cost for a good is 80 .
- What is the price elasticity of demand for this good at a price of 100 ?

Solution 11.1: The Lerner index says that $(P-M C) / P=1 /|\eta|$
In this exercise: $(100-80) / 100=20 / 100=0.2 \Rightarrow \eta=-5$.
** Exercise 11.2: Price, marginal revenue, and price elasticity of demand

- At the equilibrium monopoly price of 100 , the price elasticity of demand is -2 .
- At this price, what is the marginal revenue?

Solution 11.2: Marginal revenue $=P \times(1-1 /|\eta|)$.
For this exercise, marginal revenue $=100 \times(1-1 / 2)=50$.
** Exercise 11.3: Price, marginal revenue, and marginal cost
A. What is the relation between marginal cost and marginal revenue?
B. How do we use the relation between marginal cost and marginal revenue to price goods?
C. What is the relation between price and marginal revenue?

Part A: Firms produce a quantity so that marginal revenue equals marginal cost.

- If the firm produces a quantity such that marginal revenue is greater than marginal cost for the last item sold, it can increase its profits by reducing the price and selling more.
- If the firm produces a quantity such that marginal revenue is less than marginal cost for the last item sold, it can increase its profits by raising the price and selling less.

Part B: The marginal cost curve gives marginal cost as a function of quantity supplied. The demand curve gives price as a function of quantity or quantity as a function of price. Marginal revenue is the partial derivative of total revenue with respect to quantity, and total revenue is price times quantity.

1. Form price as a function of quantity, not quantity as a function of price. If the exam problem gives $Q=f(P)$, form the relation $P=f^{-1}(Q)$. If the exam problem gives $P=g(Q)$, use this relation.
2. Form total revenue as a function of quantity: total revenue $=Q \times P=Q \times f^{-1}(P)$ or $Q \times g(P)$.
3. Marginal revenue is the derivative of total revenue with respect to quantity.
4. Set marginal revenue equal to marginal cost and solve for the equilibrium quantity.
5. Determine the equilibrium price from the market demand curve.

We assume firms maximize profits, so they set a price such that marginal revenue = marginal cost.
Question: Do firms choose the price or the quantity?
Answer: Competitive firms are price takers; they sell at the market price. They choose the quantity so that the marginal cost equals the marginal revenue, which is the price (see Part B). Monopolies vary the quantity to optimize profits by setting marginal revenue equal to marginal cost. Choosing the quantity is like choosing the price; the two are related by the market demand curve.

Part C: If the demand curve facing the firm is horizontal (perfectly elastic), price = marginal revenue.

- A horizontal demand curve is $P=k$.
- Total revenue is $Q \times P=k Q$.
- Marginal revenue is $\partial(k Q) / \partial Q=k$.

In perfect competition, price $=$ marginal revenue $=$ marginal cost.
If the demand curve facing the firm is downward sloping (less than perfectly elastic), marginal revenue is less than price. If the same price is charged for all items, marginal revenue for the $q^{\text {th }}$ item sold is the price for item $q$ minus the decrease in price on all previous items. For monopolies, price is more than marginal revenue.

Question: Is there a simple relation between price and marginal revenue?
Answer: Many final exam problems use linear demand curves, which have a simple relation.

- $P=\alpha-\beta \times Q$
- Total revenue $=P \times Q=\alpha \times Q-\beta \times Q^{2}$
- Marginal revenue $\partial$ (total revenue) $/ \partial \mathrm{Q}=\alpha-2 \beta \times \mathrm{Q}$

Even if the demand curve is not linear, forming the marginal revenue curve is easy. Total revenue is $P \times Q$, and marginal revenue is $\partial$ (total revenue)/ $\partial \mathbf{Q}$.
** Exercise 11.4: Mark-up
Suppose one firm controls the entire market for a good. As a monopoly, it can choose the price.
A. When would raising the price increase profits?
B. What affects the price elasticity of demand?
C. What affects the price elasticity of supply?
D. How does the price elasticity of demand affect the mark-up chosen by the monopoly?

Part A: Raising the price reduces the quantity sold, with three effects:

- The firm receives a higher price for all goods it still sells.
- The firm sells fewer goods.
- The average production cost for the goods it still sells may be lower or higher.

Question: The marginal cost curve is upward sloping. Doesn't that mean average production cost is lower if it sells fewer goods?

Answer: Average variable production costs are lower. Average fixed costs are higher if the firm has economies of scale. Many firms form monopolies to reduce average production costs.

Part B: Price elasticity of demand depends on the need for the good and the availability of substitute goods.
Need is subjective. One might say: who needs diamonds? In fact, men need diamonds for wedding rings. In free markets, substitute goods become available quickly. Demand for new products starts inelastic and becomes more elastic as substitutes are invented.

Illustration: Demand for the first laptop computer, mobile phone, or iPad is inelastic. The first supplier charges a high mark-up. Consumers have no alternatives, and some consumers will pay high prices. As competitors make similar products, demand becomes more elastic.

Part C: In the short run, price elasticity of supply depends on the size of the factory. In the long run, price elasticity of supply depends on the price elasticity of demand for the factors of production.

Part D: If demand is elastic, a small increase in price causes a large decrease in quantity sold. Even a firm controlling the whole market charges a price close to the competitive price. If demand is inelastic, even a large increase in price causes only a small decrease in quantity sold. A monopoly charges a high mark-up.

## ** Exercise 11.5: Excise tax

A good has a linear demand curve with a slope of $-k$ and a linear marginal cost curve with a slope of $+k$.
The government imposes a $\$ 1$ per unit excise tax.
A. If the market is competitive, who pays the excise tax in the short run: consumers or producers?
B. If the market is competitive, who pays the excise tax in the long run: consumers or producers?
C. If the market is a monopoly, who pays the excise tax in the short run: consumers or producers?
D. If the market is a monopoly, who pays the excise tax in the long run: consumers or producers?

Part A: In the short run, consumers and producers each pay half the excise tax.

- Before the excise tax, the marginal cost curve is MC= $=\gamma+k \times \mathrm{Q}$.
- After the excise tax, the marginal cost curve is $\mathrm{MC}=\gamma+1+k \times \mathrm{Q}$.
- The market demand curve is $P=\alpha-k \times Q$.

Before the excise tax, the monopoly quantity is $\mathrm{P}=\mathrm{MC} \Rightarrow \alpha-k \times \mathrm{Q}=\gamma+k \times \mathrm{Q} \Rightarrow \mathrm{Q}=(\alpha-\gamma) / 2 k$
After the excise tax, the monopoly quantity is $P=M C \Rightarrow \alpha-k \times Q=\gamma+1+k \times Q \Rightarrow Q=(\alpha-\gamma-1) / 2 k$
The excise tax reduces the quantity by $1 / 2 k$. The competitive price increases $k \times 1 / 2 k=\$ 0.50$.
Question: Is the excise tax always split evenly between consumers and producers?
Answer: No; the split depends on the elasticities of the demand curve and the supply curve. This exercise is simple, with linear demand curves having $+k$ and $-k$ as the slope coefficients.

Part B: In the long run, the equilibrium price equals the average cost. The excise tax raises the average cost $\$ 1$, which is paid by consumers.

Question: Does this result depends on the elasticities of the supply and demand curves?
Answer: No; in the long run, consumers pay the entire sales tax and excise tax.
Question: Does that mean that producers do not lose (in the long run) from a sales tax or excise tax?
Answer: No. The equilibrium quantity is lower, and producers lose part of the dead weight loss.
Part C: For a monopoly, the rise in the equilibrium price is less than for competitive firms, so the monopolist pays more of the excise tax.

Before the excise tax, the marginal revenue curve is $M R=\alpha-2 k \times Q$. The monopoly quantity is
$\mathrm{MR}=\mathrm{MC} \Rightarrow \alpha-2 k \times \mathrm{Q}=\gamma+k \times \mathrm{Q} \Rightarrow \mathrm{Q}=(\alpha-\gamma) / 3 \mathrm{k}$
After the excise tax, the monopoly quantity is
$\mathrm{MR}=\mathrm{MC} \Rightarrow \alpha-2 k \times \mathrm{Q}=\gamma+1+k \times \mathrm{Q} \Rightarrow \mathrm{Q}=(\alpha-\gamma-1) / 3 \mathrm{k}$
The excise tax reduces the quantity by $1 / 3 k$. The competitive price increases $k \times 1 / 3 k=\$ 0.33$.
Question: Does this imply that monopoly is better than competition?

Answer: Before the excise tax, the monopoly price is higher than the competitive price. If the excise tax were so large that the marginal cost at $Q=0$ equals the competitive price at $Q=0$, the monopoly price would be the same as the competitive price. The excise tax reduces the spread between the competitive price and the monopoly price.

Question: Is an excise tax socially beneficial (for consumers) in a monopolized market?
Answer: No; a monopoly creates one source of dead weight loss; an excise tax adds a second source of dead weight loss. A monopoly has a lower equilibrium quantity than competition; an excise tax further reduces the equilibrium quantity.

Part D: The same monopoly is the only producer in both the short and long runs. If the marginal cost curve does not differ for the short and long runs, the division of the excise tax does not differ in the short and long runs. In theory, the monopolist might use a smaller factory with an excise tax, since the equilibrium quantity is smaller, but the effect of the factory size for realistic excise taxes is negligible.

## ** Exercise 11.6: Price elasticity of demand

- Firm $A B C$ is in a competitive market. At the competitive price $P^{*}$, the price elasticity of demand is $\eta^{*}$.
- Firm ABC acquires its competitors and becomes a monopoly. At the monopoly price $P^{\prime}$, the price elasticity of demand is $\eta^{\prime}$.
$\eta^{*}$ and $\eta^{\prime}$ are the price elasticity of demand for the overall market, not the price elasticity of demand facing the individual firm.

The market demand curve is linear for this product.
A. Is the price elasticity of demand positive or negative?
B. Is the price elasticity of demand greater at the competitive price or the monopoly price?
C. Is the price elasticity of demand greater at the competitive price more or less than -1 ?
D. Is the price elasticity of demand greater at the monopoly price more or less than -1 ?

Part A: The price elasticity of demand is the percentage change in quantity for a percentage change in price. A higher price leads to lower quantity, so the price elasticity of demand is negative.

Part $B$ : The price elasticity of demand is $\partial Q / \partial P \times P / Q$.

- For a linear demand curve, $\partial \mathrm{Q} / \partial \mathrm{P}$ is constant. The monopoly price is higher than the competitive price, and the monopoly quantity is lower than the competitive question.
- $\Rightarrow$ The absolute value of the price elasticity of demand is greater at the monopoly price than at the competitive price.

Question: What if the market demand curve is not linear?
Answer: The result is true for most realistic demand curves. However, one can form demand curves where this is not true, so this exercise specifies a linear demand curve.

Part C: Suppose the market demand curve is $Q=\alpha-\beta \times P$. The price elasticity of demand ranges from 0 to $-\infty$ as $P$ ranges from zero to $\alpha / \beta$. The marginal cost can be anywhere in this range, so the price elasticity of demand can be anywhere from zero to $-\infty$.

Part D: If the price elasticity of demand were between -1 and 0 (an inelastic demand), the monopolist would raise the price. This would decrease $Q$ proportionally less than the increase in $P$, so overall revenue would rise and overall costs would decrease (since $Q$ is lower). The monopolist's profit would increase. Since the monopolist charges a price that maximizes its profit, the price elasticity of demand must be less than -1 ; that is, the absolute value of the elasticity is more than 1.

## ** Exercise 11.7: Social gain from trade

Firms ABC and XYZ competite in a perfectly competitive industry. The demand curve is downward sloping, and the marginal cost curve is upward sloping.

- The competitive price is $\mathrm{P}^{\mathrm{c}}$ and the competitive quantity is $\mathrm{Q}^{\mathrm{c}}$.
- Consumers' surplus is $C^{c}$, producers' surplus is $R^{c}$, and the social gain from trade is $G^{c}$.

Firms ABC and XYZ merge and become a monopoly.

- The monopoly price is $\mathrm{P}^{\mathrm{m}}$, and the monopoly quantity is $\mathrm{Q}^{m}$.
- Consumers' surplus is $\mathrm{C}^{m}$, producers' surplus is $\mathrm{R}^{m}$, and the social gain from trade is $\mathrm{G}^{m}$.
A. Which is larger: $\mathrm{P}^{\mathrm{c}}$ or $\mathrm{P}^{\mathrm{m}}$ ?
B. Which is larger: $Q^{c}$ or $Q^{m}$ ?
C. Which is larger: $\mathrm{G}^{\mathrm{c}}$ or $\mathrm{G}^{\mathrm{m}}$ ?
D. Which is larger: $\mathrm{R}^{\mathrm{c}}$ or $\mathrm{R}^{\mathrm{m}}$ ?
E. Which is larger: $\mathrm{C}^{\mathrm{c}}$ or $\mathrm{C}^{\mathrm{m}}$ ?

Part A: The monopoly price is higher than the competitive price: $\mathrm{P}^{\mathrm{c}}<\mathrm{P}^{\mathrm{m}}$.
Part B: The monopoly quantity is lower than the competitive quantity: $Q^{c}>Q^{m}$.
Part C: The social gain from monopoly is less than the social gain from competition: $\mathrm{G}^{\mathrm{c}}>\mathrm{G}^{m}$, since perfect competition maximizes the social gain from trade. Monopoly causes a dead weight loss which reduces the social gain from trade.

Part D: Producers' surplus in monopoly is more than producers' surplus in competition: $\mathrm{R}^{\mathrm{c}}<\mathrm{R}^{\mathrm{m}}$. If this were not true, the monopolist would charge the competitive price and produce the competitive quantity.

Part E: Consumers' surplus + producers' surplus = social gain from trade. From the previous two relations, we infer that consumers' surplus in monopoly is less than consumers' surplus in competition: $\mathrm{C}^{\mathrm{c}}>\mathrm{C}^{m}$.

