

Microeconomics, Module 12: "Price Discrimination"

Homework Assignment

(The attached PDF file has better formatting.)

{See the *Concepts and Overview* posting and the practice problems posting for problems to review before doing the homework assignment.}

A monopolist practices third degree price discrimination by separating its customers into two groups: consumers under 65 and senior citizens. The monopolist's marginal cost is $MC = 0.05q$, where q is the *total* output in both markets. The marginal cost does not depend on the market in which the goods are sold. The demand curves are

- Adults: $P_A = 25 - 1/6 \times Q_A = 25 - 0.1667 \times Q_A$
 - Seniors: $P_S = 15 - 1/8 \times Q_S = 15 - 0.125 \times Q_S$
- A. What is the total industry demand curve? (Rewrite each demand curve as Q in terms of P . The total industry demand curve is the total Q in terms of P , so add Q_A and Q_S .)
 - B. What is the price elasticity of demand for adults? (Use the equation of Q in terms of P to get $\partial Q/\partial P$ and multiply by P/Q . Write the elasticity as a function of P .)
 - C. What is the price elasticity of demand for seniors?
 - D. Which market is more elastic? (Elasticity depends on the price; for a given price, which market is more elastic?)
 - E. What is the total revenue (TR) curve for adults? ($TR = P \times Q$)
 - F. What is the marginal revenue (MR) curve for adults? ($MR = \partial TR/\partial Q$)
 - G. What is the total revenue curve for seniors?
 - H. What is the marginal revenue curve for seniors?
 - I. What are the equilibrium quantities Q_A and Q_S ? (Marginal cost is $0.05q$, where q is the *combined* quantity for adults and seniors. Solve the simultaneous linear equations: (i) marginal revenue for adults = marginal cost, (ii) marginal revenue for seniors = marginal cost, and (iii) adult quantity plus senior quantity = total quantity.)
 - J. What are the equilibrium prices P_A and P_S ? (Use the demand curve for each group of consumers; the prices are different.)
 - K. What are the price elasticities of demand for the markets at their equilibrium prices?
 - L. If price discrimination is prohibited, what the combined demand curve? (This is the industry demand curve worked out above; nothing new is required here.)
 - M. If price discrimination is prohibited, what is the equilibrium quantity Q_C ?
 - N. If price discrimination is prohibited, what is the equilibrium price P_C ?
 - O. What is the price elasticity of demand at the equilibrium price P_C ? (These last three parts are monopoly pricing from the previous module.)

ELASTICITIES

Question: How do we compare elasticities if it is not constant over the whole demand curve?

- ~ For a logarithmic demand curve $\ln(Q) = \alpha - \beta \ln(P)$ the price elasticity of demand is constant.
- ~ For a linear demand curve, the price elasticity of demand depends on the point.

The regression analysis on-line course discusses the mathematics of elasticities. In that course, we learn to convert logarithmic curves to linear curves. For the microeconomics on-line course, we learn the formula for the price elasticity of demand:

$$\eta = \frac{\frac{\partial Y}{Y}}{\frac{\partial X}{X}} = \frac{\partial Y}{\partial X} \times \frac{X}{Y} = \beta X / Y$$

X is price (P) and Y is quantity (Q). For a linear demand curve, β is constant, but X / Y varies.

- ~ For a very low price, where $P \approx 0$, the price elasticity of demand is low. The quantity demanded is near its maximum. Cutting the price in half or doubling the price has a small effect on the quantity demanded.
- ~ For a very high price, where $Q \approx 0$, the price elasticity of demand is high. The price is near its maximum. To double the quantity (a small additive increase), only a small price reduction is needed.

We can't speak about the price elasticity of demand for an entire curve. How do we compare two groups of consumers?

Answer: We compare the elasticities at a *given price*. We ask: "For a price P_0 , which consumers have the higher price elasticity of demand?"

DEMAND CURVE

Question: Sometimes the demand curve is written as Q in terms of P and sometimes it is written as P in terms of Q. Which way should we use?

Answer: We can write supply and demand curves as P in terms of Q or Q in terms of P.

- ~ For the intersection of the two curves, we can use either form. We solve a pair of linear equations. Both forms make sense. We ask: "For a given price, how many units do consumers demand?" or "For a given quantity, what is the maximum that consumers will pay?"
- ~ To add demand curves of two groups of consumers, we write Q in terms of P. We ask: "For a given price, what is the total demand?" We are not asking: "For a given quantity, what is the total price?" That last question doesn't make sense.
- ~ For a sales tax or excise tax, we write Q in terms of P.
- ~ For elasticities, we generally write Q in terms of P. We don't have to use this form, but it makes the computation easier. We find $\partial Q / \partial P$, which is easier to derive if we write Q in terms of P.
- ~ For the marginal revenue curve, we write P in terms of Q. We solve for the additional revenue from one more unit sold, not for an additional dollar of price.

COMPARING ELASTICITIES

Question: Can you show how we compare elasticities at a given price?

Answer: We write the demand curve as Q in terms of P:

- Adults: $Q_A = 150 - 6P_A$
- Seniors: $Q_S = 120 - 8P_S$

At $P = 10$, the elasticities are

- Adults: $-6 \times 10 / 90 = -0.667$
- Seniors: $-8 \times 10 / 40 = -2.000$

We chose $P = 10$ arbitrarily. For the homework assignment, you observe that this is true at all prices, or you observe that this is true at the equilibrium price and quantity.

ACTUAL ELASTICITIES

Question: Is this what we expect in real life?

Answer: That depends on the scenario. Some seniors are not working, so they have less current income and more free time. They are more likely to choose a lower cost product. Their higher price elasticity of demand leads to senior discounts for many products.

For some products, seniors have a lower price elasticity of demand. An older person may have less information about competing prices or may be disabled and is not able to price shop. The senior will remain with a given supplier even if a less expensive product is offered by another supplier.

Question: One candidate writes on the discussion forum that for $P=21$, the elasticity for adults is -5.25 and 3.5 for seniors, and for $P = 19$, the elasticity for adults is -3.167 and 4.75 for seniors. Is this correct?

Answer: The price elasticity of demand is never positive. At $P = 19$ or $P = 21$, seniors don't buy any of the good. At $P = 15$, the quantity demanded is zero. At $P > 15$, the quantity demanded remains zero.

Question: Is the price elasticity of demand zero for $P > 15$?

Answer: The price elasticity of demand is not defined at $P > 15$.

ELASTICITIES OF LINEAR DEMAND CURVES

Question: If the demand curve is linear, how we easily see which curve is more elastic?

Answer: See the *Concepts and Overview* posting: if $Q = \alpha - \beta P$, the market with greater price elasticity of demand has a smaller α/β .

TOTAL MARKET DEMAND CURVES

Question: What is the total market demand curve?

Answer: We add the two demand curves:

$$Q_{\text{total}} = Q_T = Q_A + Q_S = 150 - 6P + 120 - 8P = 270 - 14P$$

To add two demand curves, we must have Q in terms of P , not P in terms of Q .

Question: Do we write Q_T in terms of $P_A + P_S$ or in terms of a single P ?

Answer: We write Q_T in terms of a single P . We ask: "What is the total quantity demanded in the market if we do not discriminate by group?" We are not asking: "What is the total quantity demanded by both groups if they each have their own price."

EQUILIBRIUM PRICE AND QUANTITY

Question: How do we solve for the equilibrium price and quantity?

Answer: We set marginal cost = marginal revenue. We write the demand curve as P in terms of Q . Both marginal cost and marginal revenue are in terms of Q , not in terms of P . They are the additional cost or revenue for one more unit, not for one more dollar of price.

Question: Is there a way to write marginal cost and marginal revenue in terms of P ?

Answer: Marginal cost has nothing to do with price, so we can't write it in terms of P . For the demand curve, Q is a function of P , so we could write marginal revenue in terms of P . But marginal cost is function of Q , so we must have marginal revenue as a function of Q .

MARGINAL COST BY GROUP

Question: How do we get the marginal cost curve for adults vs seniors?

Answer: The marginal cost curve depends on the total quantity produced. The cost of a plane ticket, a train ticket, a theater seat, or a magazine is the same regardless who sits in the seat or reads the magazine.

MARGINAL REVENUE

Question: How do we get the marginal revenue curve?

Answer: We use the demand curve for that market, which differs for seniors, adults, or the total. We write P in terms of Q . We then multiply $P \times Q$, to get their product in terms of Q (not in terms of P).

The product of total revenue. The partial derivative of total revenue with respect to quantity is marginal revenue.

Question: Is there a simple formula for a linear demand curve?

Answer: If the demand curve is $P = \alpha - \beta Q$, the marginal revenue curve is $MR = \alpha - 2\beta P$.

SOLVING THE HOMEWORK ASSIGNMENT

Question: How do we solve the homework assignment for two separate groups?

Answer: We write three equations in three unknowns. The equations are simple linear equations, so the arithmetic is easy.

The three unknowns are Q_A , Q_S , and Q_{total} . One linear equation is $Q_A + Q_S = Q_{total}$.

- ~ For adults, we write the marginal revenue as a function of Q_A and the marginal cost as a function of Q_{total} .
- ~ For seniors, we write the marginal revenue as a function of Q_S and the marginal cost as a function of Q_{total} .

INSURANCE

Question: For insurance we also set different prices by group, such as men vs women or old vs young. Do the same principles apply?

Answer: Insurance is not price discrimination. Men and women have different costs, and we price each group separately.

Question: We have many pricing actuaries at my company, and no one estimates marginal revenue or marginal cost. Is Landsburg's text realistic?

Answer: Actuaries set rates for the next policy year in highly competitive markets. This is long-term competitive pricing. We set premium rates at the minimum total cost, including both variable and fixed costs.

Almost all insurance expenses are variable costs. They may vary with policy counts or exposures, not necessarily with premium, but they are still variable costs. This makes long-term pricing easy.

Short term pricing is done by underwriters, not by actuaries. (For reinsurance pricing, actuaries often work with the underwriters.) The underwriter must consider elasticities, sunk costs, and the other items that Landsburg discusses.

Good underwriters do this, but without the equations. Landsburg does not say that firms use these equations to price their products. Firms price this way by common sense, trial and error, and experience. If they price too high, they don't sell the product. If they price too low, they lose money.