## Microeconomics, Module 12, "Price Discrimination" (Chapter 10)

## Concepts and Overview

(The attached PDF file has better formatting.)

Price discrimination is charging different prices for identical items.
Incentive to price discriminate:

- The firm produces where marginal value (MV) > marginal cost (MC),
- So it can sell additional items at less than the consumer's marginal value.

Price discrimination is possible only if the monopolist can prevent resales.

## Question 12.1: Price Discrimination and Resale Prevention

For which of the following goods is third degree price discrimination least likely?
A. Bus fares
B. Movie tickets
C. Grocery sales
D. New car sales
E. Magazine subscriptions

Answer 12.1: D
Statement A: Bus fares are lower for high school students and seniors
Statement B: Movie tickets are lower for seniors
Statement C: Grocery prices are lower for shoppers with coupons
Statement D: Magazine subscriptions are lower for college students
Statement E: If new car sales differed by type of consumer, such as seniors or students, people would have these types of consumer buy their cars for them.

Question: I've heard that car dealers offer lower prices to some consumers, such as lower prices for men than for women. Is this true? And is this price discrimination?

Answer: The dealer may "size up" the consumer and charge the maximum possible price. This is first degree price discrimination, not third degree price discrimination. This occurs in any market where the price is not fixed and the buyer and seller negotiate a price. Men don't get lower prices than women (or vice versa). Whichever consumer is better at negotiating the sale gets the better price. Many consumers don't like this arrangement, since no matter what price they get, they think they are being cheated. Some sellers have fixed prices now.
(1) First degree price discrimination

- Charges each customer the maximum he or she is willing to pay.
- Captures all social gains for the monopolist; consumers' surplus is zero.
- The monopolist produces if $P>M C$ and it produces the competitive quantity, so there is no deadweight loss.

The monopolist gains by

- Appropriating consumers' surplus.
- Producing the competitive quantity, creating and keeping additional welfare gains.
(2) Second degree price discrimination
- Different prices for identical items.
- Each customer is offered the same set of prices.
- Second degree price discrimination increases output, so it increases social welfare for both the producer and the consumer.


## Exercise 12.2: Second Degree Price Discrimination

Suppose each consumer has a demand curve for ice cream cones of $Q=5-5 P$, where $Q$ is scoops of ice cream and $P$ is the price of a scoop in dollars. The marginal cost curve is flat at $\$ 0.20$ per scoop, and fixed costs are zero.

At $\$ 1.00$ per scoop, the consumer wants no scoops. At zero per scoop, the consumer wants five scoops. At $\$ 0.60$ per scoop, the consumer wants two scoops. We work this problem as if scoops were a continuous function, not a discrete function; the answer is slightly different if scoops are discrete.
A. If the market for ice cream cones is competitive, what are the equilibrium price and quantity for ice cream scoops? What are consumers' surplus and producers' surplus?
B. If the market for ice cream cones is monopolistic with no price discrimination, what are the equilibrium price and quantity for ice cream scoops? What are consumers' surplus and producers' surplus?
C. If the monopolistic practices second degree price discrimination, what are the equilibrium price and quantity for ice cream scoops? What are consumers' surplus and producers' surplus?

Solution 12.2:
Part A: In a competitive market, the marginal cost curve is the supply curve. The marginal cost is $\$ 0.20$ a scoop, so the price is $\$ 0.20$ a scoop, and the quantity is four scoops per consumer.

Consumers' surplus is a right triangle with a base of 4 scoops and a height of $\$ 0.80$. The area is $1 / 2 \times 4 \times 4 / 5$ $=\$ 1.60$. Producers' surplus is zero.

Part $B$ : We rewrite the demand curve as $P=1-Q / 5$. The mean reversion curve is $P=1-2 Q / 5$. Setting this equal to marginal cost gives $2 Q / 5=\$ 1-\$ 0.20 \Rightarrow Q=2$ scoops and $P$ (from the demand curve) is $\$ 0.60$.

Consumers' surplus is a right triangle with a base of 2 scoops and a height of $\$ 0.40$. The area is $1 / 2 \times 2 \times 2 / 5$ $=\$ 0.40$. Producers' surplus is a rectangle with a height of $\$ 0.40$ and a width of 2 scoops, or $\$ 0.80$. The dead weight loss is $\$ 1.60-\$ 0.40-\$ 1.20=\$ 0.40$.

Part C: With second degree price discrimination, the seller charges a different price for each scoop of ice cream. If the seller could charge a different price for each part of the scoop, it would charge a price of \$1 a scoop for the first drop of ice cream, with the price dropping to $\$ 0.80$ a scoop at the end of the first scoop, $\$ 0.60$ a scoop at the end of the second scoop, and so forth. The consumers' surplus is zero, and the seller gets all the surplus: $\$ 1.60$.

In practice, it is hard vary the price continuously. A seller might differentiate between the first scoop and subsequent scoops.
(3) Third degree price discrimination

- The monopolist sets different prices in markets with different elasticities.
- This is the most common form of price discrimination.
- The markets have different demand curves which the monopolist recognizes.
- The monopolist can prevent resales.
- The marginal revenue is the same in each market, since it equals the marginal cost.
- The market with the more elastic demand is charged the lower price.
\{The final exam tests the mathematics of third degree price discrimination, not that of first or second degree price discrimination. You must know the meaning of first and second degree price discrimination, but you will not be asked to work out problems like the one above. The exam problems are similar to the problem below. Additional examples are in the practice problems and the homework assignments.\}


## Exercise 12.3: Third Degree Price Discrimination

(Adapted from question 32 of the May 1994 actuarial exam)
A single insurer sells professional liability insurance for consulting actuaries. The cost of producing Q policies is $\$ 10,000 \mathrm{Q}$. Consulting actuaries can be divided into small firms with demand of $p=\$ 30,000-\$ 300 q_{s}$ and large firms with demand of $p=\$ 50,000-\$ 100 q_{L}$.
A. What is the marginal cost curve? This does not depend on the market in which the good is sold.
B. What is the aggregate demand for small and large firms combined? To aggregate the markets, re-write the demand curves as $Q$ in terms of $P$.
C. Let the price be $\mathrm{P}_{0}$ for both types of firms. Give an algebraic expression for the price elasticity of demand at a price of $P_{0}$ for each type of firm. Which firms have more elastic demand, small firms or large firms, at the price of $\mathrm{P}_{0}$ ? (Which elasticity is more negative?)
D. If the insurer uses third degree price discrimination to maximize profits, which firms will be charged the lower price, small firms or large firms? Consumers with more elastic demand are charged a lower price.
E. With third degree price discrimination, what is the price and quantity for each type of firm?
F. What is consumers' surplus and producers' surplus for small and large firms?
G. The state regulator says that varying the price by size of the insured is unfair price discrimination, and the same price must be charged to all firms. What is the price and quantity if the same price is charge to all insureds?

## Solution 12.3:

Part A: The marginal cost of producing the $Q^{\text {th }}$ policy is the cost of $Q$ policies minus the cost of $\mathrm{Q}-1$ policies: $\$ 10,000 \mathrm{Q}-\$ 10,000(\mathrm{Q}-1)=\$ 10,000$. The marginal cost is flat at $\$ 10,000$. This is expected for insurance, which does not have economies of scale.

Part B: We rewrite the demand curves as $Q$ in terms of $P$ :

- $\mathrm{q}_{\mathrm{s}}=[30,000-\mathrm{p}] / 300=100-\mathrm{p} / 300$
- $\mathrm{q}_{\mathrm{L}}=[50,000-\mathrm{p}] / 100=500-\mathrm{p} / 100$

At a given price $p$, the total demand is $q_{s}+q_{L}$

$$
=[100-p / 300]+[500-p / 100]=600-p / 75
$$

We could also write this as $p=45,000-75 q$.
\{Note for candidates: When you work this problem on the final exam, check the quantities at a price of zero. This helps catch arithmetic and other careless errors. For a price of $\$ 0$, large firms buy 500 policies and small firms buy 100 policies, for a total of 600 policies.\}

Part C: We derive the price elasticities of demand for each type of firm:

- Small firms: $\eta_{s}=-1 / 300 \times p /(100-p / 300)=-p /(30,000-p)$
- Large firms: $\eta_{L}=-1 / 100 \times p /(500-p / 100)=-p /(50,000-p)$

For any $\mathrm{P}_{0}$ :

$$
50,000-p>30,000-p \text {, so }\{p /(30,000-p)\}>\{p /(50,000-p)\} \text {, so }\left|\eta_{s}\right|>\left|\eta_{L}\right| \text {. }
$$

The small firms have more elastic demand. In practice, less wealthy persons have higher price elasticity of demand, so this result is reasonable.

Question: Can we generalize this formula?
Answer: Suppose the demand curve is $Q=\alpha-\beta P$.
The elasticity at $P$ is $-\beta P /(\alpha-\beta P)=-P /(\alpha / \beta-P)$. The consumer with the smaller $\alpha / \beta$ has the greater elasticity.

Question: That's fine if $\mathrm{P}<\alpha / \beta$; what is P is larger?
Answer: If P larger, the consumer buys zero goods.
Question: Why should elasticity depend on the wealth of the consumer?
Answer: Less wealthy consumers are more concerned about the price of the good.
Question: Do smaller consumers have more elastic demand for insurance products?
Answer: Not always. For some products, larger consumers have more insurance options and they have greater elasticities.

Part D: Since small firms have more elastic demand, they have the lower price under third degree price discrimination. We will see this as we work the problem.

Part $E$ : At the profit-maximizing price for each market, the marginal revenues in the two markets are equal, and $M R=M C$, which is 10,000 per unit for both markets.

For small firms, total revenue $=P_{s} \times Q_{s}=Q_{s} \times\left(30,000-300 Q_{s}\right)=30,000 Q_{s}-300 Q_{s 2}$, and marginal revenue $=\partial T R / \partial Q=30,000-600 Q_{s}$. Setting MR $=M C$ gives

$$
10,000=30,000-600 Q_{\mathrm{s}} \Rightarrow Q_{\mathrm{s}}=33.33
$$

We derive $P_{s}$ from the demand curve:

$$
P_{s}=30,000-300 \times 33.33=20,000
$$

For large firms, total revenue $=P_{L} \times Q_{L}=Q_{L} \times\left(50,000-100 Q_{L}\right)=50,000 Q_{L}-100 Q_{L 2}$, and marginal revenue $=\partial T R / \partial Q=50,000-200 Q_{L}$. Setting MR $=$ MC gives

$$
10,000=50,000-200 Q_{L} \Rightarrow Q_{L}=200 .
$$

We derive $P_{\mathrm{L}}$ from the demand curve:

$$
P_{L}=50,000-100 \times 200=30,000 .
$$

Question: Is there a way to check our answers for the price and quantity?

Answer: With a linear demand curve and a flat marginal cost curve, the monopolistic quantity is half the competitive quantity and the monopolistic price is halfway between the marginal cost and the price at a quantity of zero. The marginal cost is $\$ 10,000$, which is the competitive price. The price at a quantity of zero is $\$ 30,000$ for small firms and $\$ 50,000$ for large firms, so halfway between is $\$ 20,000$ for small firms and $\$ 30,000$ for large firms. The competitive quantity is 400 for large firms and 66.67 for small firms, so half of this is 200 for large firms and 33.33 for small firms.

Part F: For small firms:

- Consumers' surplus is the area of a triangle with the points $(0,20,000),(0,30,000),(33.33,20,000)$. This is $1 / 2 \times 33.33 \times 10,000=\$ 166,667$.
- Producers' surplus is the area of a rectangle with the points $(0,10,000),(0,20,000),(33.33,10,000)$, $(33.33,20,000)$. This is $33.33 \times 10,000=\$ 333,333$.

For large firms:

- Consumers' surplus is the area of a triangle with the points $(0,30,000),(0,50,000),(200,30,000)$. This is $1 / 2 \times 200 \times 20,000=\$ 2,000,000$.
- Producers' surplus is the area of a rectangle with the points $(0,10,000),(0,30,000),(200,10,000),(200$, $30,000)$. This is $200 \times \$ 20,000=\$ 4,000,000$.
- Total consumers' surplus is $\$ 166,667+\$ 2,000,000=\$ 2,166,667$.
- Total producers' surplus is $\$ 333,333+\$ 4,000,000=\$ 4,333,333$.

Part G: If the insurer can not set different prices for large and small buyers, we use the aggregate demand curve. Total revenue $=P \times Q=$

$$
Q \times(45,000-75 Q)=45,000 Q-75 Q^{2}
$$

Marginal revenue is $\partial T R / \partial Q=45,000-150 Q$. Setting $M R=M C$ gives

$$
10,000=45,000-150 Q \Rightarrow Q=233.333
$$

The equilibrium price is $45,000-75 \times 233.333=\$ 27,500$.
Conditions for profitable price discrimination:

- The producer have some monopoly power.
- Resales are controllable.
- For third degree price discrimination, the producer have a means to charge lower prices to consumers with more elastic demand.

Sometimes, what appears to be price discrimination is not discrimination, since products that appear identical may differ.

- Lower price at salad bar if also buy entree.
- Charge less for second scoop of ice cream.
- Different insureds get different insurance rates.

