Microeconomics module 11: marginal costs, value, and revenue (practice problems)

** Exercise 11.1: Shutdown Price

A firm in a competitive market has fixed costs of 20 and the following marginal costs for item #N:

Item #	1	2	3	4	5	6	7	8	9	10
Marginal cost	9	5	4	3	4	7	9	10	11	12

A. What is the firm's shut-down price?
B. What is the firm's total profit at its short-run shutdown price?

Part A: The firm's shut-down price is the minimum average variable cost. The variable cost is the sum of the marginal costs of the items produced. For example, if three items are produced, variable costs are 9 + 5 + 4 = 18. Average variable costs are variable costs divided by the number of items produced.

Item #	1	2	3	4	5	6	7	8	9	10
Marginal cost	9	5	4	3	4	8	9	10	11	12
Variable cost	9	14	18	21	25	33	42	52	63	75
Avg variable cost	9	7	6	5.2	5	5.5	6	6.5	7	7.5

The minimum average variable cost is 5 when 5 items are produced, so the shut down price is 5.

Part B: At the shut down price, average variable costs equal the price of the good, so the firm's total profit is the negative of its fixed costs, or –20.

**Exercise 11.2: Shutdown Price

A firm in a competitive market has fixed costs of 20 and the following marginal costs for item #N:

Item #	1	2	3	4	5	6	7	8	9	10
Marginal cost	9	4	3	3	4	4	5	6	7	8

A. What is the firm's shut-down price?
B. What is the firm's total profit at its short-run shutdown price?

Part A: The firm's shut-down price is the minimum average variable cost. The variable cost is the sum of the marginal costs of the items produced. For example, if three items are produced, variable costs are 9 + 4 + 3 = 16. Average variable costs are variable costs divided by the number of items produced.

Item #	1	2	3	4	5	6	7	8	9	10
Marginal cost	9	4	3	3	4	4	5	6	7	8
Variable cost	9	13	16	19	23	27	32	38	45	53
Avg variable cost	9.00	6.50	5.33	4.75	4.60	4.50	4.57	4.75	5.00	5.30

The minimum average variable cost is 4.50 when 6 items are produced, so the shut down price is 4.50.

Part B: At the shut down price, average variable costs equal the price of the good, so the firm's total profit is the negative of its fixed costs, or –20.

** Exercise 11.3: Slope of Marginal Cost Curve

In the short run, as more output is produced, how does a manufacturing firm's marginal cost change?

Solution 11.3: At very low output, marginal cost may be high, since there is too little output for the optimal ratio of labor to capital. In the short run, as quantity increases, the firm runs its factories at greater than capacity, using overtime to produce more goods. The higher cost of labor (overtime pay) and the increased cost from using a ratio of labor to capital that is higher than optimal causes marginal cost to rise. This pattern is true for many firms, though it is clearest for manufacturers.

** Exercise 11.4: Study Time

The value of passing an exam is 30,000. The probability of passing (P) the exam depends on the hours the candidate studies (S) as $P = S^{0.5} / 30$, with a maximum of 100%. For example, if the candidate studies 400 hours, the chance of passing is 20 / 30 = 66.7%.

The cost of S hours of study is $2 \times S^{1.5}$. For example, the cost of 400 hours of study is $2 \times 400^{1.5} = 16{,}000$.

How many hours does the candidate study?

Solution 11.4: We derive marginal cost and benefit curves and we equate marginal cost to marginal benefit.

The marginal cost is $2 \times 1.5 \times S^{0.5} = 3 \times S^{0.5}$. The marginal benefit of another hour of study is $0.5 \times (30{,}000 / 30) \times S^{-0.5} = 500 \times S^{-0.5}$. Equating marginal cost and marginal benefit gives S = 500 / 3 = 166.7 hours.

** Exercise 11.5: Marginal cost, revenue, and profit

The demand curve is P = 40 – 5Q, and the cost curve is Total Cost = $3Q^2 + Q$. The units are not divisible: the firm must produce an integral number of units (1, 2, 3, …).

A. What is the marginal cost of producing the third unit?
B. What is the marginal revenue from selling the fifth unit?
C. How many units should the firm produce to maximize its profits?
D. What is the maximum profit this firm can earn?

Part A: The marginal cost of the third unit is the cost of three units minus the cost of two units:

- Two units total cost: $3 \times 2^2 + 2 = 14$
- Three units total cost: $3 \times 3^2 + 3 = 30$

The marginal cost of the third unit = 30 – 14 = 16.

Part B: The marginal revenue from the N^{th} unit is the revenue from N units minus the revenue from N-1 units:

- Total revenue from 5-1 units: Q × P = 4 × (40 – 5 × 4) = 4 × 20 = 80
- Total revenue from 5 units: Q × P = 5 × (40 – 5 × 5) = 5 × 15 = 75

The marginal revenue from the fifth unit is 75 – 80 = –$5.

Parts C and D: We form a table with marginal cost, marginal revenue, and profit for each level of output:

Units	*Total Revenue*	*Total Cost*	*Marginal Revenue*	*Marginal Cost*	*Profit*
1	35	5	35	5	30
2	60	14	25	9	46
3	75	30	15	16	45
4	80	52	5	22	32
5	75	80	-5	28	-5

To maximize its profits, the firm should produce two units, earning 46.

**Exercise 11.6: Marginal cost, marginal revenue, and profit for continuous functions

The demand curve is $P = 37 – 5Q$, and the cost curve is Total Cost = $3Q^2 + Q$. The units are in millions: the firm may produce 1 unit, 1.000001 units, and so forth. Solve the problem by integration, assuming the units are perfectly divisible.

A. What is the marginal cost of producing the three million^th unit?
B. What is the marginal revenue from selling the five million^th unit?
C. How many units should the firm produce to maximize its profits?
D. What is the maximum profit this firm can earn?

Part A: The marginal cost of the N^{th} unit is the partial derivative of the total cost with respect to quantity:

$$\partial TC/\partial Q = \partial(3Q^2 + Q)/\partial Q = 6Q + 1 = 19.$$

Part B: The marginal revenue is the partial derivative of the total revenue with respect to quantity:

- Total revenue = $P \times Q = (37 – 5Q) \times Q = 37Q – 5Q^2$
- Marginal revenue = $\partial(37Q – 5Q^2)/\partial Q = 37 – 10Q$

The marginal revenue at the five million^th unit is $37 – 10 \times 5 = –\$13$.

Part C: Marginal cost is $6Q + 1$, and marginal revenue is $37 – 10Q$. Setting them equal gives

$$6Q + 1 = 37 – 10Q \Rrightarrow 16Q = 36 \Rrightarrow Q = 36/16 = 2.25 \text{ million}$$

Part D: Profit is total revenue minus total cost:

- Total revenue is $2.25 \times (37 – 5 \times 2.25) = \$57.94$ million.
- Total cost is $3 \times 2.25^2 + 2.25 = \$17.44$ million

Profit is \$57.94 million – \$17.44 million = \$40.50 million