Microeconomics module 12 – Third degree price discrimination practice problems

(The attached PDF file has better formatting.)

** Exercise 12.1: Third degree price discrimination

A monopolist sells to two types of consumers (Group Y and Group Z), charging the same monopoly price to everyone. For consumers in group Y, consumers' surplus is C′.

The monopolist changes its pricing strategy to use third degree price discrimination. Consumers' surplus for group Y *increases* to C′′.

A. Does the price charged for group Y increase or decrease?
B. Does the quantity sold to group Y increase or decrease?
C. Does overall producers' surplus increase or decrease with third degree price discrimination?
D. Does the price charged for group Z increase or decrease?
E. Does the quantity sold to group Z increase or decrease?
F. Which group of consumer has the greater price elasticity of demand, Y or Z?

Parts A and B: The demand curve for Group Y has not changed. If the price has increased, the quantity sold to this group has decreased; if the price has decreased, the quantity sold to this group has increased.

If consumers' surplus *increases* for group Y, they are getting more value from buying the goods, so it must be that the price has decreased and the quantity bought has increased.

Part C: Third degree price discrimination increases overall producers' surplus. If it decreased producers' surplus, the monopolist would prefer to charge the same price to both groups of consumers.

Parts D and E: If the price for group Z also decreases, the monopolist would have charged a lower price even without third degree price discrimination. We infer that the price for group Z increases.

Part E: Y has the lower price, so these consumers have the greater price elasticity of demand.

** Exercise 12.2: Price discrimination

- A monopolist practices third degree price discrimination to consumers in groups Y and Z.
- In each market, it sells a quantity of 1,000.
- The price is $100 for Group Y and $120 for Group Z.

At these equilibrium prices, which of the following is true?

A. Which group has the greater price elasticity of demand?
B. In which market does the monopolist earn the greater net profit?
C. In which market does the monopolist have the greater marginal revenue at the prices charged?

Part A: The market with the lower price has the greater price elasticity of demand (Group Y).

Part B: The cost and quantity are the same in the two markets, and net profit is Q × (P – cost), so profit is greater where price is greater (Group Z).

Part C: In each market, marginal revenue = marginal cost. The marginal cost is the same in the two markets, so the marginal revenue is the same.

Question: What if the marginal cost is not the same in the two markets? For example, insurers charge different premiums to different policyholders because their costs differ.

Answer: Price differences because of different costs are not third degree price discrimination. For most goods, the cost does not differ among consumers.

** Exercise 12.3: Third degree price discrimination

Airline travelers are adults age 21 to 64, retired persons age 65 and over, or young people below 21.

- Adults travel for work meetings; buses or trains take too long.
- Retired persons and young people travel for personal vacations; they can take buses or trains.

One airline has all the hubs at the city's airport, so it has a monopoly on flights to and from the city.

- Its marginal cost for a flight is MC = Q / 12, where Q is the number of daily passengers in thousands.
- The demand curve for adult passengers is P = 60 – Q / 3.
- The demand curve for retired and young passengers is P = 30 – 2Q / 9.

Retired passengers in this exercise refers to both retired and young passengers, to avoid excess words.

A. Express the demand curve as quantity in terms of price.
B. What is the demand curve for the market as a whole?
C. What is the marginal revenue curve for the market as a whole?
D. What is the monopoly quantity?
E. What is the monopoly price?
F. How many passengers travel from each group (adult and retired) with no price discrimination?
G. What is the price elasticity of demand for the total market at the equilibrium price?
H. How many adults and how many retired buy airline tickets at this price?
I. What is the price elasticity of demand for each group (adult and retired) at this price?
J. Which group (adult and retired) has the greater price elasticity of demand at any price?
K. What is the marginal revenue at the equilibrium quantity for each group (adult and retired)?
L. What is consumers' surplus for adults at this equilibrium price?
M. What is producers' surplus for adults at this equilibrium price?
N. What is consumers' surplus for retired passengers at this equilibrium price?
O. What is producers' surplus for retired passengers at this equilibrium price?
P. With third degree price discrimination, what are the marginal revenue curves by group?
Q. With third degree price discrimination, what are the equilibrium quantities by group?
R. With third degree price discrimination, what are the equilibrium prices by group?
S. What is consumers' surplus for adults at this equilibrium price?
T. What is producers' surplus for adults at the equilibrium price with third degree price discrimination?
U. What is consumers' surplus for retired passengers at this equilibrium price?
V. What is producers' surplus for retired passengers at this equilibrium price?
W. How does price discrimination affect consumers' surplus, producers' surplus, and social welfare?

Question: Is the demand curve P = f(Q) or Q = g(P), where g(P = f^{-1}(P)?

Answer: The demand curve is strictly decreasing: if P = f(Q), Q = f^{-1}(P). Both expressions give demand curves. Some economists call Q = f^{-1}(P) the demand curve and P = f(Q) the inverse demand curve.

- Adults: P = 60 – Q/3 ⇛ Q = 180 – 3P.
- Retired: P = 30 – 2Q/9 ⇛ Q = 135 – 9P/2.

Question: Why do we want Q as a function of P?

Answer: To add demand curves of two groups of consumers, we add the quantities at a given price.

Part B: At a given price P, the combined quantity is 180 – 3P + 135 – 9P/2 = 315 – 15P/2.

Part C: The marginal revenue curve is a function of Q, not of P. Total revenue is P × Q, and marginal revenue is ∂(total revenue)/∂(quantity). To determine the marginal revenue, we need P as a function of Q:

$15P/2 = 315 - Q \Rightarrow P = 42 - 2Q/15$.

The total revenue curve is $P \times Q = 42Q - 2\,Q^2/15$. Taking the partial derivative with respect to Q gives the marginal revenue curve as $MR = 42 - 4Q/15$.

Part D: Equate marginal cost and marginal revenue: $Q/12 = 42 - 4Q/15$

$\Rightarrow 5Q/60 + 16Q/60 = 42 \Rightarrow 21Q/60 = 42 \Rightarrow Q = 120$

Part E: The monopoly price is $42 - 2 \times 120 / 15 = 26$.

Part F: Use the demand curves for each group:

- Adults: $Q = 180 - 3P = 180 - 3 \times 26 = 102$.
- Retired: $Q = 135 - 9P/2 = 135 - 9 \times 26/2 = 18$.

Question: Is the ratio of adult people to retired people 102 to 18 (5.667 to 1) or 180 to 135?

Answer: We don't know the ratio of all adult to all retired, since some people don't fly airplanes (sick people, people afraid of flying). If the cost of flying were zero, there would be 180 adults and 135 retired passengers, or 1.333 to 1.

Question: What causes the much higher ratio of 5.667 to 1?

Answer: Retired persons have higher price elasticity of demand. As the cost of flying increases, they take buses or trains. The monopolist raises prices to get more income from adults, but loses most retired persons.

Part G: The price elasticity of demand depends on the quantity and price: $\partial Q/\partial P \times P/Q$.

For all passengers combined, this is $-15/2 \times 26/120 = -1.625$. [$120 = 315 - 15 \times 26 / 2$].

Part H: We use the demand curves for each group with the equilibrium price and quantities.

- Adults: $-3 \times 26 / 102 = -0.765$
- Retired : $-9/2 \times 26 / 18 = -6.500$

At a price of 26, adults buy tickets, but most retired and young persons do not.

Question: Isn't the price elasticity of demand in monopolistic markets always elastic (less than -1)?

Answer: Yes, so the price for adults is too low. If the monopolist raises the price for adults, the percentage decrease in quantity is less than the percentage increase in price, so it would make more money. Third degree price discrimination, with separate prices for adult and retired, allows it to do this.

Part I: We derive the price elasticity of demand as a function of price for both groups:

- Adults: $-3 \times P / (180 - 3P)$
- Retired: $-4.5 \times P / (135 - 4.5P)$

Part J: At any price, the absolute value of the numerator is greater for retired and the absolute value of the denominator is greater for adults. Retired have greater price elasticity of demand than adults at any price.

Part K: At 120 passengers (combined adults and retired), the marginal cost is $120 / 12 = 10$.

We derive the marginal revenue curves for adults and retired:

- Adults: MR = 60 – 2Q/3 = 60 – 2 × 102 / 3 = -8
- Retired: 30 – 4Q/9 = 30 – 4 × 18/9 = 22

Question: How can this be correct? This says that the marginal revenue from an adult passenger is –8. If the marginal revenue is negative, why would the airlines sell tickets to adult passengers?

Answer: Let us examine what happens if the airline reduces price $1 at the monopoly equilibrium of $26.

- Adults: Q increases 3, with marginal revenue of –8 for each one. The added revenue is 3 × –8 = –24.
- Retired: Q increases 4.5, with marginal revenue of 22 for each one. The added revenue is 4.5 × 22 = +99.

For all consumers combined, marginal revenue is –$24 + $99 = +$75. The number of additional passengers is 3 + 4.5 = 7.5. The marginal cost at a quantity of Q = 120 is $10 × 7.5 passengers = $75. Marginal revenue just equals marginal cost, so the price of $26 is optimal for the monopolist.

Question: If the marginal revenue from each adult passenger is –$8, isn't something wrong?

Answer: This is what third degree price discrimination seeks to correct. If the airline can not set separate prices for adult and retired passengers, it reduces the common price so that it gets enough retired passengers. At this common price (of $26 in this example), it is losing money on each additional adult passenger.

Question: Adult and retired passengers pay the same price and have the same marginal cost. How can the airline make money on retired passengers and lose money on adult passengers?

Answer: The airline is not losing money on adult passengers. At a price of $26 and 102 adult passengers, its revenue is 102 × $26 = $2,652. Its marginal cost at 120 total passengers is $10 and its average cost is $5 per passenger, so its total cost for adult passengers is 102 × $5 = $510. It makes money on adult passengers.

Question: But you just said that is losing money on each additional adult passenger?

Answer: Consider what happens if the airline writes more passengers. Suppose the airline lowers the price to $25 a passenger. The quantities of adult and retired passengers are

- Adults: Q = 180 – 3P = 180 – 3 × 25 = 105.
- Retired: Q = 135 – 9P/2 = 135 – 9 × 26/2 = 22.5.

The revenue from the adult drivers is 105 × $25 = $2,625, which is $2,652 – $2,625 = $27 less than with a price of $26 per passenger. Its marginal cost at a total quantity of 105 + 22.5 = 127.5 is $127.5 / 12 = $10.625; its average cost is $10.625 / 2 = $5.3125; and its total cost for the 105 adult passengers is 105 × $5.3125 = $557.8125. The additional cost is $557.8125 – $510 = $47.8125. Its revenue is lower and its cost is higher.

Question: If it is losing money on each additional adult passenger, is the price too low or too low?

Answer: The price is too low for adult passengers so too many are buying tickets. The airline would make more money by raising the price. A higher price means fewer adult will buy plane tickets. But the price elasticity of demand for adults at a price of $26 is –0.765. A 1% increase in price causes an 0.765% decrease in quantity, so revenue increases.

Question: Is the same true for retired drivers?

Answer: The opposite is true for retired passengers. The price elasticity of demand for retired passengers at a price of $26 is –6.5. A 1% increase in price causes an 6.5% decrease in quantity, so revenue decreases.

Part L: Consumers' surplus for adults is a right triangle, whose width is Q = 102 and height is $60 – $26 = $34. The area of this right triangle is ½ × 102 × ($60 – $26) = $1,734.

Part M: Producers' surplus for adults is a right triangle plus a rectangle.

- The right triangle has a width of Q = 102 and height of \$10 – \$0 = \$10. The area of this right triangle is ½ × 102 × (\$10 – \$0) = \$510.
- The rectangle has a width of Q = 102 and height of \$26 – \$10 = \$16. The area of this rectangle is 102 × (\$26 – \$10) = \$1,632.

Producers' surplus is \$510 + \$1,632 = \$2,142

Question: The marginal cost at 102 passengers is 102 / 12 = \$8.50. Shouldn't the right triangle have a height of \$8.50, not \$10? And shouldn't the height of the rectangle be \$26 – \$8.50, not \$26 – \$10?

The marginal cost depends on the total number of passengers, not the number of adult passengers. A price of \$26 leads to 102 adult passengers and 18 retired passengers, for a total of 120 passengers.

Part N: Consumers' surplus for retired passengers is a right triangle, whose width is Q = 18 and height is \$30 – \$26 = \$4. The area of this right triangle is ½ × 18 × (\$30 – \$26) = \$36.

Part O: Producers' surplus for retired passengers is a right triangle plus a rectangle.

- The right triangle has a width of Q = 18 and height of \$10 – \$0 = \$10. The area of this right triangle is ½ × 18 × (\$10 – \$0) = \$90.
- The rectangle has a width of Q = 18 and height of \$26 – \$10 = \$16. The area of this rectangle is 18 × (\$26 – \$10) = \$288.

Producers' surplus is \$90 + \$288 = \$378.

Question: The same prices are paid by adult and retired passengers, so the relative sizes of consumers' surplus and producers' surplus should be about the same.

- For adults, the ratio of producers' surplus to consumers' surplus is \$2,142 to \$1,734 = 1.235.
- For retired passengers, the ratio of producers' surplus to consumers' surplus is \$378 to \$36 = 10.500.

Answer: The disparity reflects the price elasticity of demand and the optimal monopoly prices.

- Adults have *low* price elasticity of demand and are paying *less* than the monopoly price for their group alone, so consumers' surplus is large and producers' surplus is low.
- Retired passengers have *high* price elasticity of demand and are paying *more* than the monopoly price for their group alone, so consumers' surplus is small and producers' surplus is large.

Part P: If the monopolist practices third degree price discrimination, separate prices are charged for adult vs retired passengers. The demand curves and marginal revenue curves by group are

- Adults: $P_a = 60 – Q_a/3 \Rightarrow MR_a = 60 – 2Q_a/3$.
- Retired: $P_r = 30 – 2Q_r/9 \Rightarrow MR_r = 30 – 4Q_r/9$.

The subscripts are *a* = adult and *r* = retired. The equilibrium price, quantity, and marginal revenue differ for adults vs retired passengers.

Part Q: For both groups of passengers, MC (marginal cost) = $(Q_a + Q_r) / 12$.

Marginal revenue = marginal cost in each market, giving a pair of linear equations in two unknowns:

- Adult: $60 – 2Q_a/3 = (Q_a + Q_r) / 12$.
- Retired: $30 – 4Q_r/9 = (Q_a + Q_r) / 12$.

Solving this pair of equations give $Q_a = 75$ and $Q_r = 45$.

Part R: The prices for adult and retired passengers are determined from the demand curves for each group.

- Adult: $P = 60 - Q_a/3 = 60 - 75 / 3 = \$35$.
- Retired: $P = 30 - 2Q_r/9 = 30 - 2 \times 45 / 9 = \$20$.

Part S: For adults, the price is higher than with no third degree price discrimination and the quantity is lower, so consumers' surplus is lower. Consumers' surplus for adults is a right triangle, whose width is Q = 75 and height is \$60 – \$35 = \$25. The area of this right triangle is ½ × 75 × (\$60 – \$35) = \$938.

Part T: Producers' surplus for adults is a right triangle plus a rectangle.

- The right triangle has a width of Q = 75 and height of \$10 – \$0 = \$10. The area of this right triangle is ½ × 75 × (\$10 – \$0) = \$375.
- The rectangle has a width of Q = 75 and height of \$35 – \$10 = \$25. The area of this rectangle is 75 × (\$35 – \$10) = \$1,875.

Producers' surplus is \$375 + \$1,875 = \$2,250.

Part U: For retired passengers, the price is lower than with no third degree price discrimination and the quantity is higher, so consumers' surplus is greater. Consumers' surplus is a right triangle, whose width is Q = 45 and height is \$60 – \$20 = \$40. The area of this right triangle is ½ × 45 × (\$60 – \$20) = \$900.

Part V: Producers' surplus for retired passengers is a right triangle plus a rectangle.

- The right triangle has a width of Q = 45 and height of \$10 – \$0 = \$10. The area of this right triangle is ½ × 45 × (\$10 – \$0) = \$225.
- The rectangle has a width of Q = 45 and height of \$20 – \$10 = \$10. The area of this rectangle is 45 × (\$20 – \$10) = \$450.

Producers' surplus is \$225 + \$450 = \$675.

Part W: Total producers' surplus increases with third degree price discrimination.

- For adults, the social gain from trade decreases, with a large decrease of consumers' surplus and a small gain of producers' surplus.
- For retired passengers, the social gain from trade increases, with a large increase of consumers' surplus and a small increase of producers' surplus.
- Total consumers' surplus decreases much, and total producers' surplus increases. The monopolist is concerned only with producers' surplus.

The table below shows consumers' surplus, producers' surplus, and net social gain with and without third degree price discrimination.

w/ or w/o third degree price discrimination	*Consumers' Surplus*		*Producers' Surplus*		*Social Gain*
	Adults	*Retired*	*Adults*	*Retired*	*Total*
Without	\$1,734	\$36	\$2,142	\$378	\$4,290
With	\$938	\$900	\$2,250	\$675	\$4,763

Question: Does third degree price discrimination increase or decrease net social gain?

Answer: It may increase or decrease the net social gain, depending on the scenario. The three principles are

1. Consumers’ surplus increases for the group with the greater price elasticity of demand.
2. Consumers’ surplus decreases for the group with the smaller price elasticity of demand.
3. Producers’ surplus increases for both groups combined.

Question: Is net social gain greater for competition or for third degree price discrimination?

Answer: In a perfectly competitive market, each person pays a price that just covers marginal cost. That price is the marginal revenue, and the marginal consumers’ surplus at that price is zero. The airlines makes zero economic profit, and marginal producers’ surplus is zero.

Question: If marginal producers’ surplus is zero, doesn’t the airline lose money? Producers’ surplus includes only variable costs, not fixed costs. Doesn’t the airline lose the amount of fixed costs?

Answer: the marginal producers’ surplus at the competitive quantity is zero; the overall producers’ surplus is positive and just equals fixed costs.

Exercise 12.4: Airline flights (explanations; questions in subsequent exercises)

An airline charges \$100 for a morning flight from New York to Los Angeles, \$300 for an evening flight from New York to Los Angeles, and \$150 if the passenger will accept either a morning flight or an evening flight, but will not be told until the day of the flight when the plane leaves. Assume the airline owns all the terminals at Los Angeles airport, so it is a monopoly, and none of the planes are full: the airline could offer all passengers a definite departure time without additional costs.

We might explain this pricing system by third degree price discrimination. It might seem that giving passengers definite departure times raises the value of the flight, so the airline could charge more and earn higher profits. In fact, by reducing the value of some flights, the airline can charge more to customers who need definite departure times and can raise its overall profits.

To explain the logic, let us assume simple supply and demand curves. Assume that passengers who must fly at a specific time have a demand curve $Q = 600 - P$; passengers willing to fly either in the morning or the evening have a demand curve $Q = 600 - 2P$. For simplicity, assume that all costs are fixed costs; the variable costs of operating an airline are zero. (Variable costs of zero make the mathematics simple; adding positive variable costs doesn't change the logic of the pricing system.)

If the airline sets a single price for both groups of passengers, the combined demand curve is

$$Q = 600 - P + 600 - 2P = 1{,}200 - 3P.$$

Total revenue = $P \times Q = 1{,}200P - 3P^2$. Marginal revenue is the partial derivative of total revenue with respect to quantity = $1{,}200 - 6P$. Setting marginal revenue equal to marginal cost (0) gives the monopoly price :

$$1{,}200 - 6P = 0 \Rrightarrow P = 200 \text{ and } Q = 600.$$

Consumers' surplus = ½ × 600 × (600 – 200) = 120,000 and producers' surplus = 600 × 200 = 120,000.

If the airline sets different prices for the two groups of passengers, the monopoly prices for passengers with definite departure times (Group #1) and without definite departure times (Group #2) are

- Group #1: Marginal revenue = 600 – 2P = 0
  - ⇛ P = 300 and Q = 300
  - ⇛ consumers' surplus = ½ × 300 × (600 – 300) = 45,000
  - ⇛ producers' surplus = 300 × 300 = 90,000.

- Group #2: Marginal revenue = 600 – 4P = 0
  - ⇛ P = 150 and Q = 450
  - ⇛ consumers' surplus = ½ × 450 × (300 – 150) = 33,750
  - ⇛ producers' surplus = 450 × 150 = 67,500.

Passengers in Group #1 (with definite departure times) pay 300 for a ticket instead of 200. They buy fewer tickets and have lower consumers' surplus.

Passengers in Group #2 (without definite departure times) pay 150 for a ticket instead of 200. They buy more tickets and have higher consumers' surplus.

Total consumers' surplus is now 45,000 + 33,750 = 78,750, or a decrease of 41,250. But producers' surplus has increased from 120,000 to 90,000 + 67,500 = 157,500. The airline wants to maximize producers' surplus; it doesn't care about consumers' surplus.

** Exercise 12.5: Third Degree Price Discrimination

A monopolist sells train tickets to two types of passengers (consumers).

- Group Y are business travelers who must arrive on time and are less concerned with price (the employer pays for the ticket).
- Group Z are retired persons, students, and vacationers who are more concerned with price and may travel by bus or by another carrier if the price is lower.

The demand for tickets by these two groups is

- Group Y: $Q_Y = 20 - 1 \times P_Y$
- Group Z: $Q_Z = 16 - 2 \times P_Z$

The marginal cost for a train ticket is constant at MC = 2. We explain also more complex (but realistic) scenarios where the marginal cost for a train ticket varies with the quantity.

A. What is meant by a demand curve vs an inverse demand curve?
B. When do we use the demand curve and when do we use the inverse demand curve?
C. What is the combined demand curve for the entire population?
D. What is the combined inverse demand curve for the entire population?
E. What is the monopoly price and quantity if the same price is charged to all consumers?
F. What is consumers' surplus for each type of passenger (consumer)?
G. What is producers' surplus for each type of passenger (consumer)?
H. Which each type of passenger (consumer) has more elastic demand?
I. With third degree price discrimination (separate prices for each type of passenger (consumer), what are the prices and quantities for each group?
J. What is consumers' surplus for each type of passenger (consumer) with third degree price discrimination?
K. What is producers' surplus for each type of passenger (consumer) with third degree price discrimination?

Part A: The demand curve expresses quantity as a function of price: Q = f(P). The inverse demand curve expresses price as a function of quantity: P = g(Q). The demand curve is monotonic: a higher price leads to lower quantity demanded, so the demand curve can be transformed into the inverse demand curve.

Part B: To combine the demand curves of two sub-populations, use the demand curves. If $Q_1 = f_1(P)$ and $Q_2 = f_2(P)$ then total demand = $Q_1 + Q_2 = f_1(P) + f_2(P)$.

To compute the monopoly price and quantity, use the inverse demand curves. Marginal cost is a function of quantity (not of price), so to set marginal revenue equal to marginal cost, we must express marginal revenue as a function of quantity. To do this, we express total revenue as a function of quantity and take the partial derivative with respect to quantity.

- The demand curve gives total revenue = P × Q = P × f(P), which is a function of price, not quantity.
- The inverse demand curve gives total revenue = P × Q = Q × g(Q), a function of quantity, not price.

Part C: The combined demand curve for the entire population is $Q_Y + Q_Z = 20 - P + 16 - 2P = 36 - 3P$.

Part D: The combined *inverse* demand curve for the entire population is $3P = 36 - Q_T \Rightarrow P = 12 - ⅓\, Q_T$, where Q_T is the quantity demanded by the entire population.

Part E: The marginal cost is constant at MC = 2. We use the inverse demand curve to solve for the monopoly price and quantity.

- The total revenue curve is $P \times Q = (12 - ⅓\, Q) \times Q = 12Q - ⅓\, Q^2$.
- The marginal revenue curve is partial derivative of the total revenue curve = $12 - ⅔\, Q$.

- Setting marginal revenue equal to marginal cost gives $12 - \frac{2}{3} Q = 2 \Rightarrow Q = 3/2 \times 10 = 15$.
- From the inverse demand curve, we solve for $P = 12 - \frac{1}{3} \times 15 = 7$.

Question: Why is the marginal revenue curve per unit of quantity? Why not use a marginal revenue curve per unit of price?

Answer: The marginal cost curve is 2 per unit of quantity, not per unit of price. If we write the marginal revenue curve per unit of price, we can not compare marginal cost and marginal revenue.

Part F: From the price of 7 and the demand curve for each type of passenger (consumer), we compute the quantities for each type of passenger (consumer):

- Group Y: $Q_Y = 20 - 1 \times P_Y = 20 - 7 = 13$
- Group Z: $Q_Z = 16 - 2 \times P_Z = 16 - 2 \times 7 = 2$

We also compute the price at a quantity of zero for each type of passenger (consumer):

- Group Y: $0 = 20 - 1 \times P_Y \Rightarrow P_Y = 20$
- Group Z: $0 = 16 - 2 \times P_Z \Rightarrow P_Z = 8$

We now compute consumers' surplus for each type of passenger (consumer):

- Group Y: $\frac{1}{2} \times (20 - 7) \times 13 = 84.5$
- Group Z: $\frac{1}{2} \times (8 - 7) \times 2 = 1$

Question: Why is consumers' surplus so much higher for Group Y than for Group Z?

Answer: Group Y has less elastic demand. They are willing to pay 20 for the first ticket, declining slowly to the equilibrium price of 7. The value of these first tickets is consumers' surplus. Group Z has elastic demand. They are willing to pay only 16 for the first ticket, and this price declines rapidly to the equilibrium price of 7. They receive less value in their tickets.

Question: Is the price elasticity of demand explained by other consumer attributes?

Answer: People who are busy are more concerned with other qualities, such as schedules and locations. Retired people are generally less busy, so their demand is more elastic. Some older people are less wealthy; others are more wealthy. Less wealth people generally have more elastic demand.

Part G: Producers' surplus is the area between the equilibrium price and the marginal cost curve:

- Group Y: $(7 - 2) \times 13 = 65$
- Group Z: $(7 - 2) \times 2 = 10$

Part H: The price elasticity of demand is $\partial Q/\partial P \times P/Q$:

- Group Y: $\partial Q_Y/\partial P_Y = -1$, so $\eta = -1 \times P/(20 - P)$
- Group Z: $\partial Q_Z/\partial P_Z = -2$, so $\eta = -2 \times P/(16 - 2P)$

In absolute value, Group Z's elasticity has the larger numerator and smaller denominator for any price, so it has the greater elasticity.

Part I: We compute the inverse demand curve for each type of passenger (consumer):

- Group Y: $Q_Y = 20 - 1 \times P_Y \Rightarrow P_Y = 20 - Q_Y$
- Group Z: $Q_Z = 16 - 2 \times P_Z \Rightarrow P_Z = 8 - \frac{1}{2} Q_Z$

We compute the total revenue curve as price × quantity:

- Group Y: $Q_Y \times P_Y = Q_Y \times (20 - Q_Y) = 20\ Q_Y - Q_{Y2}$
- Group Z: $Q_Z \times P_Z = Q_Z \times (8 - ½\ Q_Z) = 8\ Q_Z - ½\ Q_Z^2$

The marginal revenue curve is the partial derivative of the total revenue curve with respect to quantity:

- Group Y: $\partial(20\ Q_Y - Q_{Y2})/\partial Q_Y = 20 - 2Q_Y$
- Group Z: $\partial(5\ Q_Z - ½\ Q_Z^2)/\partial Q_Z = 8 - Q_Z$

Setting marginal revenue equal to marginal cost gives

- Group Y: $20 - 2Q_Y = 2 \Rrightarrow Q_Y = ½ \times (20 - 2) = 9$ and $P_Y = 20 - 9 = 11$
- Group Z: $8 - Q_Z = 2 \Rrightarrow Q_Z = 6$ and $P_Z = 8 - ½\ Q_Z = 5$

With third degree price discrimination, the consumers with greater price elasticity of demand get a lower price and the consumers with smaller price elasticity of demand get a higher price.

Part J: We compute consumers' surplus for each type of passenger (consumer):

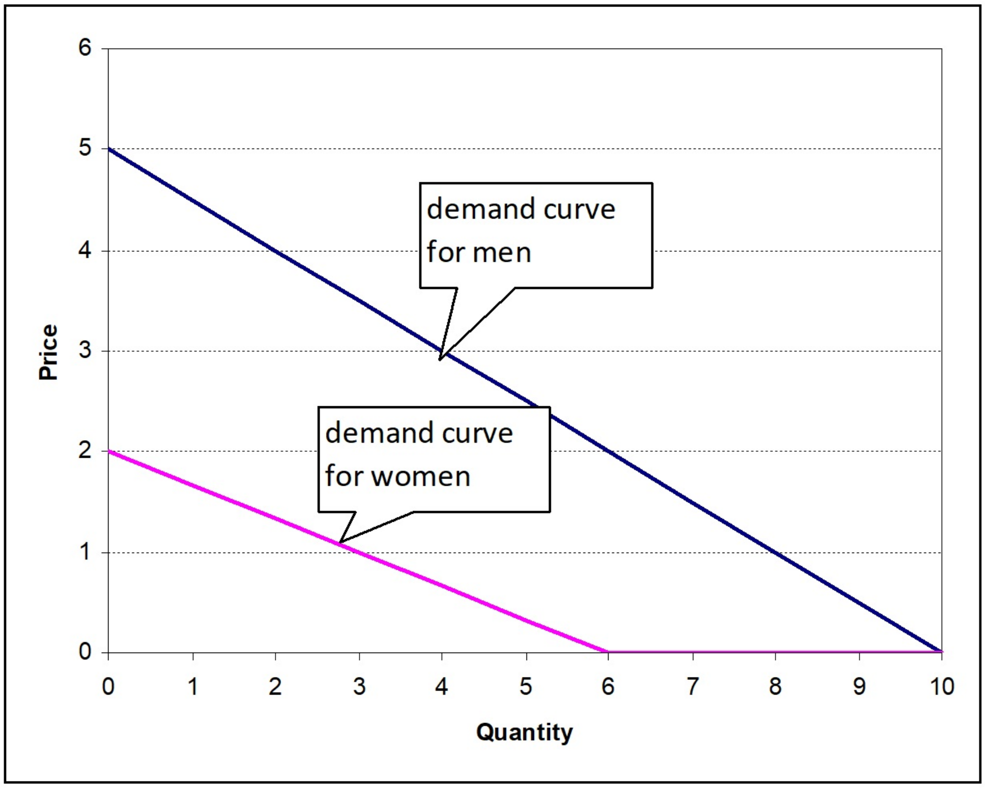
- Group Y: $½ \times (20 - 11) \times 9 = 40.5$
- Group Z: $½ \times (8 - 5) \times 6 = 9.00$

Part K: We compute producers' surplus for each type of passenger (consumer):

- Group Y: $(11 - 2) \times 9 = 81$
- Group Z: $(5 - 2) \times 6 = 18.0$

Consumers' surplus decreases for Group Y (whose price increased) and increased for Group Z (whose price decreased). Producers' surplus increased for both groups: third degree price discrimination sets the price that optimizes producers' surplus for each group.

** Exercise 12.6: Football parties, beer, and price discrimination



Beer is sold at the Harvard-Yale football party. A monopoly sells beer and sets a price to maximize profits. Beer is provided free by the colleges, so the marginal cost of the beer to the monopoly is zero. The demand curves per person for beer are Q = 10 – 2P for men and Q = 6 – 3P for women. 3,000 men and 1,000 women attend the Harvard Yale party.

A. What is the total demand curve for beer at the party?
B. If price discrimination is not allowed (everyone pays the same price), what is the equilibrium price?
C. What is the consumers' surplus for men?
D. What is the consumers' surplus for women?
E. What is producers' surplus if a single price is set for men and women?
F. If the organizers sell beer at different prices to men and women, how much beer is sold at the party?
G. What is consumers' surplus for men with price discrimination?
H. What is consumers' surplus for women with price discrimination?
I. What is producers' surplus with price discrimination?

Part A: The quantity of beer demanded by men at a price P is 3,000 × (10 – 2P) = 30,000 – 6,000P. The quantity demanded by women at a price P is 1,000 × (6 – 3P) = 6,000 – 3,000P. The total demand curve for beer is Q = 30,000 – 6,000P + 6,000 – 3,000P = 36,000 – 9,000P.

Take heed: To solve for the monopoly price and quantity, we use the inverse demand curve, P = g(Q), so that we can express marginal revenue as a function of quantity. To combined deman curves, we use Q = f(P).

Part B: If price discrimination is not allowed (everyone pays the same price), what is the equilibrium price?

We solve for the inverse demand curve. The demand curve is 9,000P = 36,000 – Q, so the inverse demand curve is P = 4 – Q / 9,000.

- The total revenue curve = quantity × price = (4 – Q / 9,000) × Q = 4Q – Q^2 / 9,000.
- The marginal revenue curve = ∂(total revenue curve)/∂Q = 4 – Q/4,500.
- The monopoly quantity is the quantity where marginal revenue equals marginal cost, which is zero:

$$4 - Q/4{,}500 = 0 \Rightarrow Q = 18{,}000.$$

- The monopoly price may be read from the inverse demand curve: P = 4 – Q / 9,000 = 4 – 2 = 2.

Question: Can we solve this problem using the direct demand curve? Can we say the following:

- The total revenue curve = quantity × price = (36,000 – 9,000P) × P = 36,000P – 9,000P^2.
- The marginal revenue curve = ∂/∂P (total revenue curve) = 36,000 – 18,000P.
- The monopoly price is the price where marginal revenue is zero: 36,000 – 18,000P = 0 ⇒ P = 2.

Answer: Your marginal revenue curve is not consistent with the marginal cost curve. A marginal cost curve is the added cost for one more unit of quantity; your marginal revenue curve is the added revenue for one more unit of price. Your method works in this scenario because marginal cost is constant at zero. Since the added cost is zero everywhere, it is zero for one more unit of price.

Part C: What is the consumers' surplus for men?

Consumers' surplus is the area of a right triangle, whose base is the equilibrium quantity and whose height is the price at Q = 0 minus the equilibrium price.

Each man buys 10 – 2P = 10 – 2 × 2 = 6 units of beer at the equilibrium price of 2. To demand zero units of beer (Q = 0), the price for men must be 0 = 10 – 2P ⇒ P = 5. The price at Q = 0 minus the equilibrium price is 5 – 2 = 3. Consumers' surplus for each man is a right triangle with a base of 6 and a height of 3, giving an area of ½ × 6 × 3 = 9. For 3,000 men, total consumers' surplus is 3,000 × 9 = 27,000.

Part D: What is the consumers' surplus for women?

The demand curve for women is Q = 6 – 3P. At a price of P = 2, Q = 6 – 3 × 2 = 0, so consumers' surplus for women is zero.

Question: Why should consumers' surplus differ for men vs women?

Answer: Men and women get different benefits from beer, since they have different demand curves. We examine consumers' surplus separately for men and women.

Question: Why do women get no benefit from beer? They do drink beer, and they have a demand curve for beer.

Answer: Men drink more beer than women, and they have less elastic demand than women (in the scenario in this exercise). Women have more elastic demand: as the price rises, they buy less beer. Men buy more beer and buy it even at higher prices. In the first half of this exercise, beer is sold at the same price to men and women. The party organizers seek to maximize their profits, so they set the monopoly price. The party has three times as many men as women, and the price that maximizes total profit is \$2. At this price, no women buy beer.

Question: Won't some women buy beer even at a price of \$2?

Answer: The solution method in this exercise assumes all women are identical, so none buys beer at a price of \$2. In truth, the demand curves for men and women are averages; persons differ within each group.

Take heed: If the same price is charged to all consumers and some consumers have very low and inelastic demand, the equations used here may show a negative quantity for that group of consumers. If the computed quantity is negative, replace it with zero.

Part E: What is producers' surplus if a single price is set for men and women?

Producers' surplus is the area above the marginal cost curve up to the equilibrium price and out to the equilibrium quantity. The marginal cost is zero in this exercise, so the area is a rectangle. The equilibrium price is 2, and the quantity is 6 units per man. With 3,000 men, producers' surplus is 3,000 × 2 × 6 = 36,000.

Part F: If the organizers sell beer at different prices to men and women, how much beer is sold at the party?

We use the separate demand curves for the two groups to solve for prices and quantity for men vs women.

For men, the aggregate demand curve is Q = 30,000 – 6,000P.

- The inverse demand curve is P = 5 – Q / 6,000.
- The total revenue curve is Total Revenue = Q × P = 5 Q – Q^2 / 6,000.
- The marginal revenue curve is Marginal Revenue = ∂(Q × P)/∂Q = 5 – P / 3,000.
- Setting marginal revenue to zero gives 5 – Q / 3,000 = 0 $\Rrightarrow$ Q = 15,000.
- Solving for P from the inverse demand curve give P = 5 – 15,000 / 6,000 = 5 – 2.5 = 2.5.

To maximize the net revenue from men, charge them 2.50 per unit of beer and sell 15,000 units.

We could also solve this problem with the single man's demand curve, which gives P = 2.50 and Q = 5. With 3,000 men, aggregate quantity is 5 × 3,000 = 15,000.

For women, the aggregate demand curve is Q = 6,000 – 3,000P.

- The inverse demand curve is P = 2 – Q / 3,000.
- The total revenue curve is Total Revenue = Q × P = 2 Q – Q^2 / 3,000.
- The marginal revenue curve is Marginal Revenue = ∂(Q × P)/∂Q = 2 – P / 1,500.
- Setting marginal revenue to zero gives 2 – Q / 1,500 = 0 $\Rrightarrow$ Q = 3,000.
- Solving for P from the inverse demand curve give P = 2 – 3,000 / 3,000 = 2 – 1 = 1.

To maximize the net revenue from women, charge them 1.00 per unit of beer and sell 3,000 units.

We could also solve this problem with the single woman's demand curve, which gives P = 1.00 and Q = 3. With 1,000 women, the aggregate quantity is 1,000 × 3 = 3,000.

The total quantity of beer sold is 15,000 + 3,000 = 18,000 units.

Part G: What is consumers' surplus for men with price discrimination?

With an equilibrium price of 2.50, an equilibrium quantity of 5 units, and 3,000 men, consumers' surplus is ½ × 2.50 × 5 × 3,000 = 18,750.

The price for men increased from 2.00 to 2.50, so consumers' surplus declined from 27,000 to 18,750.

Part H: What is consumers' surplus for women with price discrimination?

With an equilibrium price of 1.00, an equilibrium quantity of 3 units, and 1,000 men, consumers' surplus is ½ × 1.00 × 3 × 1,000 = 1,500.

The price for women decreased from 2 to 1, so consumers' surplus increased from zero to 1,500.

Part I: What is producers' surplus with price discrimination?

Producers' surplus depends on the price received, so we compute producers' surplus separately for men and women.

3,000 men buy 5 units apiece at 2.50 per unit. Marginal cost is zero for the sellers, so producers' surplus is

$$3{,}000 \times 2.50 \times 5 = 37{,}500.$$

1,000 men buy 3 units apiece at 1.00 per unit. Marginal cost is zero for the sellers, so producers' surplus is

$$1{,}000 \times 1.00 \times 3 = 3{,}000.$$

Producers' surplus is 37,500 + 3,000 = 40,500.

Producers' surplus increased from 36,000 to 40,500. Producers' surplus is always at least as great with price discrimination as without price discrimination. (If producers' surplus were smaller with price discrimination, the monopolist would charge the same price to everyone.)

Question: If the marginal cost is not zero, how would the solution differ?

Answer: We set marginal revenue equal to marginal cost to solve for the monopoly price. Producers' surplus is the area above the marginal cost curve (which was zero in this exercise). Solving monopoly problems with a marginal cost curve is covered in a previous module. For simplicity, this exercise assumes marginal cost is zero.