## Microeconomics, Module 16, "The Theory of Games" (Chapter 12)

## Homework Assignment

(The attached PDF file has better formatting.)

A town has 10,000 drivers and two auto insurers, $Y$ and $Z$. The cost of an auto insurance policy is $\$ 700$ per driver, including all expenses.

Each insurer has a choice of charging either $\$ 800$ or $\$ 850$ per policy. An $\$ 800$ policy gives a $\$ 100$ profit provision. Each insurer would be satisfied with this profit, though the insurer would prefer a higher profit. A premium above $\$ 850$ would attract new entrants into the market, which neither insurer wants.

- If $Y$ and $Z$ charge the same price, each insures $50 \%$ of the drivers.
- If $Y$ and $Z$ charge different prices, the insurer with the lower price gets $90 \%$ of the drivers and the insurer with the higher price gets $10 \%$ of the drivers.
A. Draw the game matrix for the two insurers. The format is shown below; fill in the cells with the Greek letters.
- Draw one insurer on the horizontal axis and the other insurer on the vertical axis.
- Each insurer has two possible strategies: charge $\$ 800$ or charge $\$ 850$.
- The cells of the game matrix show the profit for each insurer.
B. What is the dominant strategy for Insurer $Y$ ? Since the game is symmetric, this is also the dominant strategy for Insurer Z. Show that this is the dominant strategy as follows:
- If $Z$ charges $\$ 800$, does $Y$ make a greater profit by charging $\$ 800$ or $\$ 850$ ?
- If $Z$ charges $\$ 850$, does $Y$ make a greater profit by charging $\$ 800$ or $\$ 850$ ?

A dominant strategy is preferred in all scenarios. If the answer to the two questions above is the same, this price is the dominant strategy.
C. If both insurers have a dominant strategy, this is a Nash equilibrium. What is the profit of each insurer in this Nash equilibrium? Is it Pareto optimal? Explain.
D. What is the Pareto optimal cell for the two insurers? Is it a Nash equilibrium? Explain. (The Nash equilibrium in this exercise is not Pareto optimal, and the Pareto optimal cell is not a Nash equilibrium. This is the prisoner's dilemma.)

The format of the game matrix template is:
Z
\$800 \$850

| $Y$ | $\$ 800$ | $\alpha$ | $\beta$ |
| :--- | :--- | :--- | :--- |
|  | $\$ 850$ | $\gamma$ | $\delta$ |

Illustration: For cell $\alpha, Y$ charges $\$ 800$ per policy and $Z$ charges $\$ 800$ per policy.

- Y insures 5,000 drivers and earns $(\$ 800-\$ 700)=\$ 100 /$ driver, for a total of $\$ 5$ million.
- Z insures 5,000 drivers and earns ( $\$ 800-\$ 700$ ) $=\$ 100 /$ driver, for a total of $\$ 5$ million.

For cell $\beta$, Y charges $\$ 800$ per policy and $Z$ charges $\$ 850$ per policy.

- Y insures 9,000 drivers and earns $(\$ 800-\$ 700)=\$ 100 /$ driver, for a total of $\$ 9$ million.
- Z insures 1,000 drivers and earns $(\$ 850-\$ 700)=\$ 150 /$ driver, for a total of $\$ 1.5$ million.

For cell $\delta, Y$ charges $\$ 850$ per policy and $Z$ charges $\$ 850$ per policy.

- $Y$ insures 5,000 drivers and earns $(\$ 850-\$ 700)=\$ 150 /$ driver, for a total of $\$ 7.5$ million.
- Z insures 5,000 drivers and earns $(\$ 850-\$ 700)=\$ 150 /$ driver, for a total of $\$ 7.5$ million.

Question: The module on market power says that in a Bertrand oligopoly, the firms charge the competitive price and produce the competitive quantity; in a Cournot oligopoly, the firms charge more than the competitive price (but not as much as the monopoly price) and produce less than the competitive quantity (but not as little as the monopoly quantity). How does oligopoly differ from game theory?

Answer: They are related; game theory is used primary with oligopolies. But the module on oligopoly assumes that firms have a strict pricing or production rule. Firms do not act with foresight; they don't anticipate how their rivals will respond to their own actions. Now we assume that each firm figures out what its rivals will do if it acts in a certain way.

Question: Do insurers do this? Do they price based on the expected actions of their rivals and the anticipated reactions to their own behavior?

Answer: Insurance executives spend much time anticipating their rivals' actions. Some insurers have formal procedures: The personal auto direct writers tabulate each others' rates, market shares in each location, agents, and so forth. Other insurers analyze rival's activities informally.

But the desire to know a rival's intentions is far from the ability to do so. Insurer Y may wish to know Insurer Z's pricing strategy. But Insurer Y has only a general sense of its own pricing strategy; it doesn't know its own rates for a given insured more than a few months in advance, and it surely can't anticipate Insurer Z's potential rate actions.

Question: The SOA and CAS syllabus present cost-based pricing (based on their loss costs and expenses) or return on capital pricing (achieving a required return). If game theory is used, why don't we study it on the exams?

Answer: This topic is critical, but it is nebulous; we have no strict rules and no mathematical procedures. In addition, game theory is relevant when firms have market power; if the market is competitive, firms price at minimum average cost. Most insurance markets are highly competitive, and game theory is not always useful.

Question: For Parts C and D, what is the difference between Nash equilibrium and Pareto optimal? Aren't these two ways of expressing the same concept?

Answer: Nash equilibrium means that neither firm has an incentive to shift its strategy. If both firms shift to a different strategy (e.g., both prisoners do not confess), they both gain. But the firms cannot collude or price in unison. They presume that even if they are in a Pareto optimal long-term strategy, giving an outcome which is better for both than the equilibrium, their rival has an incentive to cheat. The other strategy is better in the short run for one firm, but it destroys the long-run equilibrium and is worse for both firms in the long-run.

Question: Insurers are run by bright people. Surely they can figure out that both gain if they don't cheat and stick to the Pareto optimal strategy.

Answer: People seek self-interest. They may agree that everyone gains if they don't cheat; they then try to cheat without being discovered. This is the magic of free markets: because each firm seeks its own selfinterest, the outcome is optimal for society as a whole.

