Micro module 16: Game theory: practice problems

Practice problems and illustrative test questions for the final exam

(The attached PDF file has better formatting.)

This posting gives sample final exam problems. Other topics from the textbook are asked as well; these problems are just examples. All final exam problems are multiple choice; some practice problems are not multiple choice so that the solutions can be better explained.

Exercise 16.1: Prisoner's dilemma

		Jacob's Strategy	
		Confess	*Not Confess*
Rachel's Strategy	*Confess*	Jacob gets 5 years Rachel gets 5 years	Jacob gets 8 years Rachel gets 1 year
	Not Confess	Jacob gets 1 year Rachel gets 8 years	Jacob gets 2 years Rachel gets 2 years

A prisoner's dilemma is a game in which the players have a Pareto optimal outcome which is the better outcome for both players, but each player has a dominant strategy leading to an outcome that is not Pareto optimal.

A. Does Jacob have a dominant strategy?
B. Does Rachel have a dominant strategy?
C. How many Nash equilibria does the game have?
D. How many Pareto optimal outcomes does the game have?

Part A: Know the procedure to determine if a player has a dominant strategy.

Each player has two options. Games need not be symmetric, so one player may have a dominant strategy and the other player may not. We label Jacob's strategies *right* and *left* and Rachel's strategies *up* and *down*. In the prisoner's dilemma, the two strategies are labeled *confess* and *non-confess*.

To see if Jacob has a dominant strategy, ask:

- If Rachel chooses up, should Jacob choose right or left?
- If Rachel chooses down, should Jacob choose right or left?

If Jacob chooses the same option regardless of what Rachel chooses, Jacob has a dominant strategy. If Jacob's choice depends on Rachel's choice, Jacob does not have a dominant strategy.

In the prisoner's dilemma,

- If Rachel chooses *not-confess*, Jacob chooses *confess*, since it leads to fewer years of prison: 1 year vs 2 years.
- If Rachel chooses *confess*, Jacob chooses *confess*, since it leads to fewer years of prison 5 years vs 8 years.

Most games seek to maximize a variable (usually wealth). In the prisoner's dilemma, consider years of prison as negative wealth. Minimizing years of prison is like maximizing wealth.

Part B: The prisoner's dilemma is symmetric, so Rachel's dominant strategy is the same as Jacob's dominant strategy.

Part C: Know the solution method and a few general rules.

For each outcome, ask: "If Jacob and Rachel are in that outcome, and Rachel will persist in her choice, would Jacob prefer another choice? If Jacob will persist in his choice, would Rachel prefer another choice?" If both answers are *No*, the outcome is a Nash equilibrium.

- A game may have zero, one, or more than one Nash equilibria.
- If each player has a dominant strategy, the outcome of the dominant strategies is a Nash equilibrium and it is the only Nash equilibrium.

In the prisoner's dilemma, each player has a dominant strategy, so there is a unique Nash equilibrium: Jacob confesses and Rachel confesses.

Part D: Know the solution method. For each outcome, ask: "Is there any other outcome that is better for one player and at least as good for each other player?" If the answer is *No*, the outcome is Pareto optimal.

A Nash equilibrium may or may not be Pareto optimal. In the prisoner's dilemma, *confess/confess* is the only Nash equilibrium but it is not Pareto optimal. The other three outcomes are not Nash equilibria, but they are *all Pareto optimal*. (Pareto optimal is a poor term, since it implies that the outcome is the best possible, and only one outcome can be the best possible.)

Question: If an outcome is the only Nash equilibrium, how can it not be Pareto optimal?

Answer: Consider a matrix with four cells: (a,a), (a,b), (b,a), and (b,b). *Pareto optimal* is a negative condition: Y is Pareto optimal to Z if Z if not better for all players (not if Y is better for all players). It may be that (a,a) is not worse than (a,b) and (a,b) is not worse than (b,b), but (a,a) is worse than (b,b).

In the prisoner's dilemma, *Jacob confess / Rachel confess* is better for Jacob than *Jacob not confess / Rachel confess*, and *Jacob not confess / Rachel confess* is better for Rachel than *Jacob not confess / Rachel not confess*, but *Jacob confess / Rachel confess* is worse for both Jacob and Rachel than *Jacob not confess / Rachel not confess*.

STRATEGIES AND OUTCOMES

For each set of outcomes in a two person game, know whether each person has a dominant strategy, which outcomes are Nash equilibria, and which outcomes are Pareto optimal.

The exercises here are taken from the *Jack and Jill* illustrations in the textbook. The final exam problems use other games with the same logic. Real games are more complex, since they involve many players, more than just two possible actions by each player, and mixed strategies.

Exercise 16.2: Game theory

Jacob and Rachel are players in a two person game with the following outcomes. Jacob plays *left* or *right*; Rachel plays *up* or *down*.

		Jacob's Strategy	
		Left	*Right*
Rachel's Strategy	*Up*	Jacob gets 1 Rachel gets 1	Jacob gets 4 Rachel gets 2
	Down	Jacob gets 2 Rachel gets 4	Jacob gets 3 Rachel gets 3

A. Does Jacob have a dominant strategy?
B. Does Rachel have a dominant strategy?
C. How many Nash equilibria does the game have?
D. How many Pareto optimal outcomes does the game have?

Part A: Jacob's dominant strategy is to play *right*, not *left*. Compare Jacob's *right* vs *left* for each choice by Rachel.

- If Rachel plays *up*, Jacob's *right* – *left* = 4 – 1 = +3.
- If Rachel plays *down*, Jacob's *right* – *left* = 3 – 2 = +1.

Part B: Rachel's dominant strategy is to play *down*, not *up*. Compare Rachel's *down* vs *up* for each choice by Jacob.

- If Jacob plays *left*, Rachel's *down* – *up* = 4 – 1 = +3.
- If Jacob plays *right*, Rachel's *down* – *up* = 3 – 2 = +1.

Part C: If each player has a dominant strategy, one and only Nash equilibrium exists, and it is the combination of the two dominant strategies.

To verify, consider each cell of the game matrix.

- In the *left* / *up* cell, Jacob prefers to switch (and play *right*) and Rachel prefers to switch (and play *down*).
- In the *left* / *down* cell, Jacob prefers to switch (and play *right*).
- In the *right* / *up* cell, Rachel prefers to switch (and play *down*).
- In the *right* / *down* cell, neither Jacob nor Rachel wants to switch.

Part D: The game has three Pareto optimal outcomes: *right* / *up*, *left* / *down*, and *right* / *down*.

- Compared to *right* / *up*, Jacob loses in any other cell.
- Compared to *left* / *down*, Rachel loses in any other cell.

- Compared to *right* / *down*, either Jacob loses or Rachel loses in any other cell.

See Landsburg, *Price Theory*, Chapter 12, Game Theory, page 414

Exercise 16.3: Game theory

Jacob and Rachel are players in a two person game with the following outcomes. Jacob plays *left* or *right*; Rachel plays *up* or *down*.

		Jacob's Strategy	
		Left	*Right*
Rachel's Strategy	*Up*	Jacob gets 1 Rachel gets 1	Jacob gets 2 Rachel gets 4
	Down	Jacob gets 4 Rachel gets 2	Jacob gets 3 Rachel gets 3

A. Does Jacob have a dominant strategy?
B. Does Rachel have a dominant strategy?
C. How many Nash equilibria does the game have?
D. How many Pareto optimal outcomes does the game have?

Part A: Jacob does not have a dominant strategy. Compare Jacob's *right* vs *left* for each choice by Rachel.

- If Rachel plays *up*, Jacob prefers to play *right*, since *right* – *left* = 2 – 1 = +1.
- If Rachel plays *down*, Jacob prefers to play *left*, since *right* – *left* = 3 – 4 = –1.

Part B: Rachel does not have a dominant strategy. Compare Rachel's *down* vs *up* for each choice by Jacob.

- If Jacob plays *left*, Rachel prefers to play *down*, since *down* – *up* = 2 – 1 = +1.
- If Jacob plays *right*, Rachel prefers to play *up*, since *down* – *up* = 3 – 4 = –1.

Part C: If neither player has a dominant strategy, there may be zero, one, or two Nash equilibria.

Consider each cell of the game matrix.

- In the *left* / *up* cell, Jacob prefers to switch (and play *right*) and Rachel prefers to switch (and play *down*).
- In the *left* / *down* cell, neither Jacob nor Rachel wants to switch (= Nash equilibrium)
  - If Jacob switches to *right*, he loses $1.
  - If Rachel switches to *up*, she loses $1.
- In the *right* / *up* cell, neither Jacob nor Rachel wants to switch (= Nash equilibrium)
  - If Jacob switches to *left*, he loses $1.
  - If Rachel switches to *down*, she loses $1.
- In the *right* / *down* cell, Jacob prefers to switch (and play *left*) and Rachel prefers to switch (and play *up*).

Two of the outcomes are Nash equilibria.

Part D: The game has three Pareto optimal outcomes: *right* / *up*, *left* / *down*, and *right* / *down*.

- Compared to *right* / *up*, Rachel loses in any other cell.
- Compared to *left* / *down*, Jacob loses in any other cell.
- Compared to *right* / *down*, either Jacob loses or Rachel loses in any other cell.

See Landsburg, *Price Theory*, Chapter 12, Game Theory, page 414

Exercise 16.4: Game theory

Jacob and Rachel are players in a two person game with the following outcomes. Jacob plays *left* or *right*; Rachel plays *up* or *down*.

		Jacob's Strategy	
		Left	*Right*
Rachel's Strategy	*Up*	Jacob gets 1 Rachel gets 1	Jacob gets 4 Rachel gets 4
	Down	Jacob gets 2 Rachel gets 2	Jacob gets 3 Rachel gets 3

A. Does Jacob have a dominant strategy?
B. Does Rachel have a dominant strategy?
C. How many Nash equilibria does the game have?
D. How many Pareto optimal outcomes does the game have?

Part A: Jacob has a dominant strategy. Compare Jacob's *right* vs *left* for each choice by Rachel.

- If Rachel plays *up*, Jacob prefers to play *right*, since *right* – *left* = 4 – 1 = +3.
- If Rachel plays *down*, Jacob prefers to play *right*, since *right* – *left* = 3 – 2 = +1.

Part B: Rachel does not have a dominant strategy. Compare Rachel's *down* vs *up* for each choice by Jacob.

- If Jacob plays *left*, Rachel prefers to play *down*, since *down* – *up* = 2 – 1 = +1.
- If Jacob plays *right*, Rachel prefers to play *down*, since *down* – *up* = 3 – 4 = –1.

Part C: If neither player has a dominant strategy, there may be zero, one, or two Nash equilibria.

Consider each cell of the game matrix.

- In the *left* / *up* cell, Jacob prefers to switch (and play *right*) and Rachel prefers to switch (and play *down*).
- In the *left* / *down* cell, Jacob prefers to switch (and play *right*).
- In the *right* / *up* cell, neither Jacob nor Rachel wants to switch (= Nash equilibrium)
    - If Jacob switches to *left*, he loses $3.
    - If Rachel switches to *down*, she loses $1.
- In the *right* / *down* cell, Rachel prefers to switch (and play *up*).

One outcome is a Nash equilibria.

Part D: The game has one Pareto optimal outcomes: *right* / *up*.

- Compared to *right* / *up*, both Rachel and Jacob lose in any other cell.

See Landsburg, *Price Theory*, Chapter 12, Game Theory, page 414

Exercise 16.5: Game theory

Jacob and Rachel are players in a two person game with the following outcomes. Jacob plays *left* or *right*; Rachel plays *up* or *down*.

		Jacob's Strategy	
		Left	*Right*
Rachel's Strategy	*Up*	Jacob gets 2 Rachel gets 2	Jacob gets 4 Rachel gets 1
	Down	Jacob gets 1 Rachel gets 4	Jacob gets 3 Rachel gets 3

A. Does Jacob have a dominant strategy?
B. Does Rachel have a dominant strategy?
C. How many Nash equilibria does the game have?
D. How many Pareto optimal outcomes does the game have?

Part A: Jacob's dominant strategy is to play *right*, not *left*. Compare Jacob's *right* vs *left* for each choice by Rachel.

- If Rachel plays *up*, Jacob's *right* – *left* = 4 – 2 = +2.
- If Rachel plays *down*, Jacob's *right* – *left* = 3 – 1 = +2.

Part B: Rachel's dominant strategy is to play *down*, not *up*. Compare Rachel's *down* vs *up* for each choice by Jacob.

- If Jacob plays *left*, Rachel's *down* – *up* = 4 – 2 = +2.
- If Jacob plays *right*, Rachel's *down* – *up* = 3 – 1 = +2.

Part C: If each player has a dominant strategy, one and only Nash equilibrium exists, and it is the combination of the two dominant strategies.

To verify, consider each cell of the game matrix.

- In the *left* / *up* cell, Jacob prefers to switch (and play *right*) and Rachel prefers to switch (and play *down*).
- In the *left* / *down* cell, Jacob prefers to switch (and play *right*).
- In the *right* / *up* cell, Rachel prefers to switch (and play *down*).
- In the *right* / *down* cell, neither Jacob nor Rachel wants to switch.

Part D: The game has three Pareto optimal outcomes: *right* / *up*, *left* / *down*, and *right* / *down*.

- Compared to *right* / *up*, Jacob loses in any other cell.
- Compared to *left* / *down*, Rachel loses in any other cell.
- Compared to *right* / *down*, either Jacob loses or Rachel loses in any other cell.

See Landsburg, *Price Theory*, Chapter 12, Game Theory, page 414

Exercise 16.6: Game theory

Jacob and Rachel are players in a two person game with the following outcomes. Jacob plays *left* or *right*; Rachel plays *up* or *down*.

		Jacob's Strategy	
		Left	*Right*
Rachel's Strategy	*Up*	Jacob gets 1 Rachel gets 3	Jacob gets 3 Rachel gets 1
	Down	Jacob gets 4 Rachel gets 2	Jacob gets 2 Rachel gets 4

A. Does Jacob have a dominant strategy?
B. Does Rachel have a dominant strategy?
C. How many Nash equilibria does the game have?
D. How many Pareto optimal outcomes does the game have?

Part A: Jacob does not have a dominant strategy. Compare Jacob's *right* vs *left* for each choice by Rachel.

- If Rachel plays *up*, Jacob's *right* – *left* = 3 – 1 = +2.
- If Rachel plays *down*, Jacob's *right* – *left* = 2 – 4 = –2.

Part B: Rachel does not have a dominant strategy. Compare Rachel's *down* vs *up* for each choice by Jacob.

- If Jacob plays *left*, Rachel's *down* – *up* = 2 – 3 = –1.
- If Jacob plays *right*, Rachel's *down* – *up* = 4 – 1 = +3.

Part C: If neither player has a dominant strategy, there may be zero, one, or two Nash equilibria.

Consider each cell of the game matrix.

- In the *left* / *up* cell, Jacob prefers to switch (and play *right*).
- In the *left* / *down* cell, Rachel prefers to switch (and play *up*).
- In the *right* / *up* cell, Rachel prefers to switch (and play *down*).
- In the *right* / *down* cell, Jacob prefers to switch (and play *left*).

No outcome is a Nash equilibria.

Part D: The game has two Pareto optimal outcomes: *left* / *down* and *right* / *down*.

- Compared to *right* / *up*, both Jacob and Rachel gain in *left* / *down*.
- Compared to *left* / *up*, both Jacob and Rachel gain in *right* / *down*.
- Compared to *left* / *down*, Jacob loses in any other cell.
- Compared to *right* / *down*, Rachel loses in any other cell.

See Landsburg, *Price Theory*, Chapter 12, Game Theory, page 414

Exercise 16.7: Game theory

Jacob and Rachel are players in a two person game with the following outcomes. Jacob plays *left* or *right*; Rachel plays *up* or *down*.

		Jacob's Strategy	
		Left	*Right*
Rachel's Strategy	*Up*	Jacob gets 2 Rachel gets 2	Jacob gets 1 Rachel gets 1
	Down	Jacob gets 1 Rachel gets 1	Jacob gets 3 Rachel gets 3

A. Does Jacob have a dominant strategy?
B. Does Rachel have a dominant strategy?
C. How many Nash equilibria does the game have?
D. How many Pareto optimal outcomes does the game have?

Part A: Jacob does not have a dominant strategy. Compare Jacob's *right* vs *left* for each choice by Rachel.

- If Rachel plays *up*, Jacob's *right* – *left* = 1 – 2 = –1.
- If Rachel plays *down*, Jacob's *right* – *left* = 3 – 1 = +2.

Part B: Rachel does not have a dominant strategy. Compare Rachel's *down* vs *up* for each choice by Jacob.

- If Jacob plays *left*, Rachel's *down* – *up* = 1 – 2 = –1.
- If Jacob plays *right*, Rachel's *down* – *up* = 3 – 1 = +2.

Part C: If neither player has a dominant strategy, there may be zero, one, or two Nash equilibria.

Consider each cell of the game matrix.

- In the *left* / *up* cell, neither Jacob nor Rachel prefers to switch.
- In the *left* / *down* cell, Rachel prefers to switch (and play *up*) and Jacob prefers to switch (and play *right*).
- In the *right* / *up* cell, Rachel prefers to switch (and play *down*) and Jacob prefers to switch (and play *left*).
- In the *right* / *down* cell, neither Jacob nor Rachel prefers to switch.

Two outcomes are Nash equilibria.

Part D: The game has one Pareto optimal outcome: *right* / *down*, which is best for both Jacob and Rachel.

See Landsburg, *Price Theory*, Chapter 12, Game Theory, page 414

Exercise 16.8: Game theory

Jacob and Rachel are players in a two person game with the following outcomes. Jacob plays *left* or *right*; Rachel plays *up* or *down*.

		Jacob's Strategy	
		Left	*Right*
Rachel's Strategy	*Up*	Jacob gets 2 Rachel gets 3	Jacob gets 1 Rachel gets 1
	Down	Jacob gets 1 Rachel gets 1	Jacob gets 3 Rachel gets 2

A. Does Jacob have a dominant strategy?
B. Does Rachel have a dominant strategy?
C. How many Nash equilibria does the game have?
D. How many Pareto optimal outcomes does the game have?

Part A: Jacob does not have a dominant strategy. Compare Jacob's *right* vs *left* for each choice by Rachel.

- If Rachel plays *up*, Jacob's *right* – *left* = 1 – 2 = –1.
- If Rachel plays *down*, Jacob's *right* – *left* = 3 – 1 = +2.

Part B: Rachel does not have a dominant strategy. Compare Rachel's *down* vs *up* for each choice by Jacob.

- If Jacob plays *left*, Rachel's *down* – *up* = 1 – 3 = –2.
- If Jacob plays *right*, Rachel's *down* – *up* = 2 – 1 = +1.

Part C: If neither player has a dominant strategy, there may be zero, one, or two Nash equilibria.

Consider each cell of the game matrix.

- In the *left* / *up* cell, neither Jacob nor Rachel prefers to switch.
- In the *left* / *down* cell, Rachel prefers to switch (and play *up*) and Jacob prefers to switch (and play *right*).
- In the *right* / *up* cell, Rachel prefers to switch (and play *down*) and Jacob prefers to switch (and play *left*).
- In the *right* / *down* cell, neither Jacob nor Rachel prefers to switch.

Two outcomes are Nash equilibria.

Part D: The game has two Pareto optimal outcomes: *left* / *up* and *right* / *down*.

- Compared to *left* / *up*, Rachel loses in any other cell.
- Compared to *right* / *down*, Jacob loses in any other cell.

See Landsburg, *Price Theory*, Chapter 12, Game Theory, page 414

Exercise 16.9: Game theory

Jacob and Rachel are players in a two person game with the following outcomes. Jacob plays *left* or *right*; Rachel plays *up* or *down*.

		Jacob's Strategy	
		Left	*Right*
Rachel's Strategy	*Up*	Jacob gets 12 Rachel gets 8	Jacob gets 9 Rachel gets 8
	Down	Jacob gets 15 Rachel gets 7	Jacob gets 14 Rachel gets 10

A. Does Jacob have a dominant strategy?
B. Does Rachel have a dominant strategy?
C. How many Nash equilibria does the game have?
D. How many Pareto optimal outcomes does the game have?

Part A: Jacob's dominant strategy is to play *left*, not *right*. Compare Jacob's *right* vs *left* for each choice by Rachel.

- If Rachel plays *up*, Jacob's *right* – *left* = 9 – 12 = –3.
- If Rachel plays *down*, Jacob's *right* – *left* = 14 – 15 = –1.

Part B: Rachel does not have a dominant strategy. Compare Rachel's *down* vs *up* for each choice by Jacob.

- If Jacob plays *left*, Rachel's *down* – *up* = 7 – 8 = –1.
- If Jacob plays *right*, Rachel's *down* – *up* = 10 – 8 = +2.

Part C: If only one player has a dominant strategy, there may be zero, one, or two Nash equilibria.

Consider each cell of the game matrix.

- In the *left* / *up* cell, neither Jacob nor Rachel prefers to switch; this is a Nash equilibrium.
- In the *left* / *down* cell, Rachel prefers to switch (and play *up*).
- In the *right* / *up* cell, Jacob prefers to switch (and play *left*).
- In the *right* / *down* cell, Jacob prefers to switch (and play *left*).

One outcome is a Nash equilibrium.

Part D: The game has two Pareto optimal outcomes: *left* / *down* and *right* / *down*.

- Compared to *left* / *down*, Jacob loses in any other cell.
- Compared to *right* / *down*, Rachel loses in any other cell.
- *left* / *up* and *right* / *up* are both worse than *right* / *down* for both Jacob and Rachel.

See Landsburg, *Price Theory*, Chapter 12, Game Theory, page 414