## Practice Problem

(The attached PDF file has better formatting.)

## Municipal Fair and Consumer Welfare

A municipality with $\$ 100,000$ taxpayers organizes a fair, for which it charges no admissions fee. The fixed costs of the fair are $\$ 100,000$, and the marginal costs are $\$ 2$ for each visitor. The benefit received by each visitor is $\$ 10-\mathrm{V} / 10,000$, where V is the number of visitors. The opportunity cost of the time spent by each visitor to the fair is $\$ 3$. The visitors all have the same tastes.
A. What is the number of visitors at the fair?
B. What is the cost per taxpayer of the fair?
C. What is the social gain or loss from the fair?
D. To maximize social welfare, what admission fee should be charged for the fair?

Part $A$ : The cost to each visitor is the opportunity cost of $\$ 3$ plus the admission fee of zero. The benefit is $\$ 10$ - V / 10,000. Equating marginal cost to benefit gives

$$
\$ 10-V / 10,000=\$ 3 \Rightarrow V=10,000 \times 7=70,000 \text { visitors. }
$$

Part B: With 70,000 visitors, the marginal cost is $70,000 \times \$ 2=\$ 140,000$. The fixed cost is $\$ 100,000$, so the total cost is $\$ 240,000$. The cost per taxpayer with 100,000 taxpayers is $\$ 2.40$.

Part C: The gain from the fair is zero for each visitor, since the benefit from the fair equals the opportunity cost. The marginal cost is $\$ 2.40$ per taxpayer. The net social gain or loss is a $\$ 240,000$ loss.

Part D: The total benefit to the municipality from the fair is the benefit from the fair minus the opportunity cost of the visitors' time $=\mathrm{V} \times(10-\mathrm{V} / 10,000-3)=7 \mathrm{~V}-\mathrm{V}^{2} / 10,000$. The marginal benefit is the derivative of the total benefit with respect to the number of visitors:

$$
\partial\left(7 \mathrm{~V}-\mathrm{V}^{2} / 10,000\right) / \partial \mathrm{V}=7-\mathrm{V} / 5,000
$$

Equating marginal benefit and marginal cost gives

$$
7-\mathrm{V} / 5,000=2 \Rightarrow \mathrm{~V}=5,000 \times(7-2)=25,000
$$

With 25,000 visitors, the benefit to each visitor is $10-25,000 / 10,000=\$ 7.50$. The opportunity cost is $\$ 3$, so the admissions fee is $\$ 4.50$, making the marginal cost equal to the marginal benefit.

The gain to each visitor is $\$ 4.50$, so the total social benefit is $\$ 4.50 \times 25,000=\$ 112,500$. The cost is $\$ 100,000+25,000 \times \$ 2=\$ 150,000$. The net social loss is $\$ 150,000-\$ 112,500=\$ 37,500$.
\{Note: This practice problem should help you with the homework assignment.\}

