Micro Module 19, “Common property,” practice problems

(The attached PDF file has better formatting.)

** Exercise 19.1: Walden Pond

A town’s residents like to go fishing at Walden Pond, but only if few others are there. Each person values fishing at $\$300 - N^2$, where N is the number of persons at the pond.

- If the person is the only one at the pond, the fishing is worth $\$300 - \$1^2 = \$299$.
- If nine other persons are at the pond, the fishing is worth $\$300 - \$(9+1)^2 = \$200$.

If the pond has no entrance fee and people have no other uses for their time:

A. How many people fish at the pond?
B. What is the social value of fishing at the pond?

Part A: Solve $300 - N^2 = 0 \Rightarrow N = \sqrt{300} \approx 17.32$, so 17 people fish at the pond.

Part B: Each person’s utility is $300 - 17^2 = 11$. The social value of the pond is $11 \times 17 = 187$.

Question: If a common property has no entrance fee, its value is zero. Why is the social value here 187?

Answer: This exercise has discrete values: the number of visitors to the pond is an integer. If visitors could be cut into pieces, the number of people fishing at the pond would be $N = \sqrt{300} = 17.321$, each person’s utility would be zero, and the social value of the pond would be zero.

** Exercise 19.2: Walden Pond

A town's residents like to go fishing at Walden Pond, but only if few others are there. Each person values fishing at $\$300 - N^2$, where N is the number of persons at the pond.

- If the person is the only one at the pond, the fishing is worth $\$300 - \$1^2 = \$299$.
- If nine other persons are at the pond, the fishing is worth $\$300 - \$(9+1)^2 = \$200$.

People have no other uses for their time and the expenses of going fishing are zero.

The town sets an entrance fee for fishing at Walden Pond.

A. How many people fishing at Walden Pond maximizes social welfare?
B. What is the entrance fee that maximizes social welfare?
C. What is the social value of fishing at the pond with this entrance fee?

Part A: With N people, total social welfare is $N \times (300 - N^2) = 300N - N^3$.

To maximize social welfare, set the partial derivative with respect to N equal to zero:

$$300 - 3N^2 = 0 \Rrightarrow N = 10$$

Part B: If only 10 people are to fish at the pond, the entrance fee should be \$200.

Part C: The total social welfare is $\$200 \times 10 = \$2,000$.

Question: Each person pays an entrance fee of \$200 and gets $\$300 - 10^2 = \$200$ of benefit. Shouldn't the net social welfare be $\$200 - \$200$ = zero?

Answer: The entrance fee is a transfer of wealth from persons fishing at Walden Pond to recipients of municipal services funded by the entrance fee to Walden Pond. The total social welfare is \$2,000, but it is not received by the persons fishing at Walden Pond. It is received by the town that charges the entrance fee.

The total social welfare is the utility from fishing – the entrance fees paid by the persons fishing + the entrance fees received by the town. The last two terms cancel out, and the total social welfare is the utility from fishing.

Question: What should we focus on in these problems?

Answer: Economists seek to maximize social welfare. Common property tends to decrease social welfare because too many people seek to use the free goods. Many final exam problems ask for the entrance fee that maximizes social welfare and the amount of social welfare at the optimal entrance fee.

** Exercise 19.3: Strawberry Fields

Wild strawberries grow in a field outside a town.

- The time spent for a day picking strawberries is worth \$100.
- Picking strawberries is most efficient if no one else is also picking.
- With N people in the field, each person can pick (\$300 – N) worth of strawberries.

A. If there is no entrance fee to the Strawberry Fields, how many people pick strawberries?
B. To maximize social welfare, how many people should pick strawberries?
C. What entrance fee maximizes social welfare?
D. What is this maximum social welfare?

Part A: If there is no entrance fee, and N people pick strawberries, each person's utility is \$300 – N. Setting the utility equal to the marginal cost of \$100 gives $\$300 - N = 100 \Rrightarrow N = 200$.

Part B: With N people picking strawberries, total social welfare is $N \times (300 - N - 100) = 200N - N^2$.

To maximize social welfare, set the partial derivative with respect to N equal to zero:

$$200 - 2N = 0 \Rrightarrow N = 100.$$

Part C: The entrance fee E to get 100 people to pick strawberries is

$$\$300 - 100 - 100 - E = 0 \Rrightarrow E = \$100$$

Part D: Total social welfare is $100 \times \$100 = \$10{,}000$

** Exercise 19.4: Strawberry Fields

Wild strawberries grow in a field outside a town.

- The time spent for a day picking strawberries is worth $200.
- Picking strawberries is most efficient if no one else is also picking.
- With N people in the field, each person can pick ($500 – 2N) worth of strawberries.

A. If there is no entrance fee to the Strawberry Fields, how many people pick strawberries?
B. To maximize social welfare, how many people should pick strawberries?
C. What entrance fee maximizes social welfare?
D. What is this maximum social welfare?

Part A: If there is no entrance fee, and N people pick strawberries, each person's utility is \$300 – N. Setting the utility equal to the marginal cost of \$100 gives $\$300 - N = 100 \Rrightarrow N = 200$.

Part B: With N people picking strawberries, total social welfare is $N \times (300 - N - 100) = 200N - N^2$.

To maximize social welfare, set the partial derivative with respect to N equal to zero:

$$200 - 2N = 0 \Rrightarrow N = 100.$$

Part C: The entrance fee E to get 100 people to pick strawberries is

$$\$300 - 100 - 100 - E = 0 \Rrightarrow E = \$100$$

Part D: Total social welfare is 100 × \$100 = \$10,000

** Exercise 19.5: Concerts and study time

Students at a large university could study in their dorm rooms on Sunday afternoon or go to a free rock concert at a park near the campus.

- Study on Sunday afternoon is worth \$2 to each student.
- The park is small, and too many students attending ruins the fun.
- The value of the rock concerts is \$11 – N/1,000, where N is the number of students attending.
  - If only one student attends, the value of the rock concert is 11 – 1/1,000 = 10.999 to that student.
  - If ten thousand students attend, the value of the concert is 11 – 10,000/1,000 = 1 to each student.

A. If the admission fee to the rock concert is zero, how many students attend?
B. To maximize social welfare, how many students should attend the rock concert?
C. What admission fee to the rock concert maximizes social welfare?
D. What is the maximum social welfare from the rock concert?

Part A: If the admission fee is zero, students attend as long as the value of the rock concert is more than \$2.

$\$11 - N/1{,}000 = \$2 \Rrightarrow N = 9{,}000$.

Part B: Total social welfare is $N \times (\$11 - N / 1{,}000) = 11N - N^2 / 1{,}000$

Set the partial derivative with respect to N equal to zero:

$11 - 2N / 1{,}000 = 0 \Rrightarrow N = 4{,}500$

Part C: For 4,500 students to come, the admission fee *E* must be

$\$11 - 4{,}500 / 1{,}000 - E = \$2 \Rrightarrow E = \$4.50$

Part D: The social welfare with an admission fee of \$4.50 is 4,500 × \$4.50 = \$20,250.

** Exercise 19.6: Concerts and study time

Principles: An uneven distribution of benefit and an uneven distribution of costs raise the number of visitors to the common property and raise its social welfare. Benefits and costs depend on tastes. Students have different tastes for study and rock concerts. This exercise combines different tastes for the common property and different costs. The cost here is the value of study time given up.

1,000 students could study in their dorm rooms on Sunday afternoon or go to a free rock concert at a park near the campus. The students range from dull nerd to cool dude along a liner scale.

- Study on Sunday afternoon is worth \$10 to the dullest nerd and \$0.01 to the coolest dude.
- The park is small, and too many students attending ruins the fun.
- The value of the rock concerts is \$10.00 – N/100, where N is the number of students attending.

Assume all students perfectly predict who will attend the rock concert and who will study in the dorms.

A. If the admission fee to the rock concert is zero, how many students attend?
B. To maximize social welfare, how many students should attend the rock concert?
C. What admission fee to the rock concert maximizes social welfare?
D. What is the maximum social welfare from the rock concert?

Because the problem has a discrete number of students, the value of the rock concert ranges from \$0.01 to \$10.00 and the value of study time ranges from \$0.01 to \$10.00. Summing a series of discrete values adds complexity. To make the problem easier to solve, assume N is a continuous variable ranging from 0 to 1,000, the value of study time is a continuous variable ranging from 0 to 10, and the value of the rock concert is a continuous variable ranging from 0 to 10.

Part A: Students attend the rock concert if the value of the rock concert for them minus the admission fee is more than the value of study for them. The coolest dude is the first to attend (least cost of not studying), and the dullest nerd is the last to attend (greatest cost of not studying).

Suppose the admission fee is E and N students attend the rock concert. For the N^{th} student (in order from coolest dude to dullest nerd), study time is worth \$0.01 × N. The value of the rock concert minus the admission fee is \$10 – N/100 – E.

Illustration: If two students attend the rock concert and the admission fee is zero, the second (= the *marginal*) student is the second coolest dude. Study time *for him* is worth \$0.01 × 2 = \$0.02 and the value of the rock concert *for him* is \$10 – 2/100 = \$9.98.

If the marginal student has a net benefit of zero from attending the rock concert, all the cooler dudes have a positive net benefit and all the duller nerds have a negative net benefit.

If the admission fee is zero, the number of students at the rock concert is

$$\$10 - N/100 - 0 = \$0.01 \times N$$
$$\$10 = 2N / 100 \Rightarrow$$
$$N = 100 \times 10 / 2 = 500$$

The 500 coolest dudes attend the rock concert. The remaining students (500 dullest nerds) study in dorms.

Question: The principle says that different tastes raise the social welfare from the common property. How do we see that in this exercise?

Answer: The average value of study time for all students is \$10 / 2 = \$5.

The value of the rock concert is \$10 – N / 100. If these average values were true for all students, the value when 500 students attend is zero for all students, so the social welfare is zero.

Is this exercise, students' tastes differ. The value of study time ranges from zero for the coolest dude to \$5 for the 500th student. The total social value is $\int y/100\ \partial y = y^2/200 = 500^2 / 200 = 1{,}250$.

Using discrete values, as done in the textbook, gives an approximate answer.

- If the value of study time ranges from \$0 to \$4.99 for the first 500 students, the total social welfare of the rock concert is 500 × 501 / (2 × 100) = 1,252.50.
- If the value of study time ranges from \$1 to \$5.00 for the first 500 students, the total social welfare of the rock concert is 499 × 500 / (2 × 100) = 1,247.50.

The table at the bottom of page 462 gets a value of N between 4 and 5. Landsburg uses N = 5, since people don't come in halves. A continuous function gives N = 4.5.

Part B: At the optimal number of students attending the rock concert, the marginal social cost equals the marginal social benefit.

If N students attend the concert, the marginal social cost = the private cost for the N^{th} student = \$0.01 × N.

- The total social benefit from the concert is $(\$10 – N/100) \times N = 10N – N^2/100$.
- The marginal social benefit is $\partial(10N – N^2/100)/\partial N = 10 – N/50$.

Equating marginal social cost and marginal social benefit gives $10 – N/50 = N/100 \Rightarrow N = 333$. The coolest 333 students should attend the concert. The next student who attends decreases nets social welfare.

Question: Is there an intuitive explanation of this?

Consider student #500, where 500 students attend the rock concert.

- The value of the rock concert to this student is \$10 – 500 / 100 = \$5.
- The value of study time to this student is 500 / 100 = \$5.
- The student considers only private gains and costs and is ambivalent about attending the concert.

For social gains and costs, we subtract the cost of this student *to each previous student*, which is 1 / 100.

- All students cooler than the N^{th} student are already attending the concert.
- The gain from the concert decreases 1/100 × N (assuming continuous functions).

With 500 students attending the rock concert, the marginal social gain from the last student is negative.

The 333th coolest student gains \$10 – 333 / 100 = \$6.67 by attending the concert and loses 333/100 = \$3.33 in the value of study time, for a net gain of \$3.33. All cooler students lose 1/100 by the extra student attending the concert, for a total social loss of \$3.33. The 333th student is the last student whose attendance at the rock concert raises total social welfare. The next student attending causes net social welfare to decrease.

Part C: For 333 students to come, the admission fee *E* must equate the cost of studying with the value of attending the rock concert for the *last* student to attend (the marginal student):

$$\$10 – 333 / 100 – E = \$3.33 \Rightarrow E = \$3.33$$

Part D: The social welfare is the consumers' surplus plus the producers' surplus.

The consumers' surplus is the gain to the students from the rock concert minus the cost of lost study time:

With an admission fee of \$3.33333, the gain to students is 333.333 × (\$10 – 333.333/100 – 333.333/100) = $333.333^2/100$ = \$1,111.11.

The cost of lost study time is

$$\int N/100 \, \partial N = \tfrac{1}{2} \times 333.333^2 / 100 = \$555.56$$

The producers' surplus is the value of the admission tickets: \$3.33333 × 333.333 = \$1,111.11.

Total social welfare is \$1,111.11 + \$1,111.11 – \$555.56 =\$1,666.67.

The computations above use 333.3333 students and an admission fee of \$3.33333. If one uses 333 students and an admission fee of \$3.33, the figures differ slightly.

Question: Can you summarize the principles in these practice problems?

Answer: People use common property as long as the private gain exceeds the private cost.

- If all people are alike (same value from the common property and same costs of using it), and the admission fee for the common property is zero, the net social gain is zero.
- If not all people are alike, the net social gain is more than zero even if the admission fee is zero.

Final exam problems may ask to work out the admission fee that maximizes net social gain.