Microeconomics, Module 24, “Risk and Uncertainty” (Chapter 18)

Homework

(The attached PDF file has better formatting.)

Risk Aversion and Insurance Coverage

Economists often view insurance from a *risk aversion* perspective. This is not always realistic: consumers buy permanent life insurance for the tax advantaged investment and auto insurance because it is required by the state; governments provide tax write-offs for group health insurance and mortgage lenders require Homeowners insurance.

Insurance markets depend on many things, only one of which is risk aversion, but for this homework assignment we focus on risk aversion. We show an equilibrium for insurance markets based on risk aversion. The mathematics is simplified, but the concepts are used by economists.

Suppose consumers have homes worth $300,000 apiece and no other wealth. The chance of a fire is 1% each year. We use two alternatives for the size-of-loss distribution: fire losses are (i) all $300,000 or (ii) distributed uniformly from zero to $300,000. (In truth, small losses are much more frequent than large losses: over 90% of fire losses are less than 10% of the home value. We use a skewed distribution in the last part of this posting.)

Consumers are risk averse. The consumer’s utility is the square root of his or her wealth. A consumer with no money and no home has utility of $\sqrt{0} = 0$. A consumer with a home has a utility of $\sqrt{300,000} = 547.72$. (This utility function is simple to work with for the homework assignment; it is not used in practice.)

A. A consumer has a home with a 1% chance of a fire loss. If the fire loss will be a total loss (the first assumption above), what is the consumer expected wealth at the end of the year? (99% probability of $300,000 and 1% probability of $0.)
B. What is the expected utility of this consumer at the end of the year? (99% probability of 547.72 and 1% probability of zero.) The expected utility is not the same as the utility of the expected wealth, since the consumer is risk averse.
C. Assume that fire losses are uniformly distributed over the range [0, $300,000]. What is the consumer’s expected wealth at the end of the year? (The likelihood of a loss of size L is 1/300,000. We can integrate x/300,000 from 0 to 300,000 to find the average loss. The integration gives $\frac{1}{2} x^2/300,000$ at x = 300,000 = $150,000. This exercise is simple, and we don’t need to integrate, since the average loss is $150,000. The expected wealth at the end of the year is a 99% probability of $300,000 and 1% probability of $150,000.)
D. If fire losses are uniformly distributed over the range [0, $300,000], what is the consumer’s expected utility at the end of the year? (With a loss of size L, the utility is $(300,000 – L)^{1/2}$. Given that a loss occurs, the likelihood of a loss of size L is 1/300,000. For the expected utility if there is a loss, we integrate $(300,000 – L)^{1/2}$ / 300,000 from zero to 300,000. The expected utility at the end of the year is the weighted average of this utility (1% probability) and the full 547.72 utility (99% probability). The integration gives $\frac{2}{3} \times (300,000 – L)^{3/2}$ evaluated at L = 0.
E. We return to the scenario of only a total loss. If the insurance coverage is sold at fair odds (at the actuarial pure premium), what is the premium? (The *actuarial pure premium* means no expenses or profit. The pure premium, or PPr, is 1% × $300,000.)
F. What is the utility of a consumer who buys insurance coverage? {The consumer pays the premium but is recompensed for any fire loss. The consumer’s wealth is $300,000 – PPr, and the consumer’s utility is $(\$300,000 – Ppr)^{1/2}$.} No integration is needed.
G. Would a risk averse consumer buy insurance coverage if it is offered at the actuarial pure premium? (Is the utility at the end of the year with insurance coverage greater than the expected utility at end of the year

without insurance coverage? The textbook explains intuitively why the risk-averse consumer buys insurance; this homework assignment uses a simple numerical example.)

H. If losses are uniformly distributed from 0 to 300,000, what is the actuarial pure premium? (What is the average loss, as we worked out earlier? The actuarial pure premium is 1% of the average loss. This should be half the pure premium of the scenario when all losses are total losses.)
I. What is the utility of a consumer who buys insurance coverage? {The consumer pays the premium but is recompensed for any fire loss. The consumer's wealth is $300,000 – PPr, and the consumer's utility is ($300,000 – PPr)^{½}. The procedure is the same as used above, but the pure premium is only half as large.} No integration is needed.
J. In practice, insurers don't sell coverage at the actuarial pure premium. Assume again that all losses are total losses. The insurer has expenses and profit requirements that are included in the gross premium. What is the maximum expense and profit provision at which the consumer still buys the coverage? (We solve for the gross premium, GPr, by equating the consumer's utility with wealth of $300,000 – GPr to the expected utility with no insurance coverage. We worked out the expected utility with no insurance coverage above. Square this number and subtract the result from $300,000. Given the actuarial pure premium and the maximum gross premium, the maximum expense and profit commission is GPr – PPr. The maximum expense and profit ratio is 1 – PPr/GPr.)
K. Find the maximum expense and profit provision if the losses are uniformly distributed from zero to 300,000.

Question: Is this how insurance coverage is priced? If it is not actually priced this way, would this be how insurance coverage should be priced if we knew the loss distribution and the utility function of consumers?

Answer: Insurance markets are competitive. We said before that in a competitive market, the long-term equilibrium price is based on the producers' cost functions. The price is the minimum long-run average cost. This does not depend on the utility function of the consumer. Actuarial pricing depends on long-term expenses provisions of the insurer, not the utility function of the consumer.

Question: Doesn't the utility function of the consumer affect the demand curve?

Answer: For competitive markets, the long-run equilibrium price does not depend on the demand curve of consumers; it depends on the cost curves of the firm. In a competitive insurance market, the equilibrium price does not depend on consumers' utility curves.

Question: How would we use utility functions for insurance pricing?

Answer: Sometimes there is no equilibrium for first-dollar insurance coverage. Suppose that consumers are only slightly risk averse; they buy coverage if the expense and profit provision is 20% but not if it is 40%. Insurers have a 40% expense and profit provision. We might conclude that no coverage will be sold in this market. But a good actuary may restructure the policy to make it more desirable, as shown below.

The following example is not part of the homework assignment, since it requires more mathematics than is appropriate for this course.

A. Suppose losses have an exponential size-of-loss distribution from zero to the value of the home ($300,000). Since the exponential distribution is unbounded, we must re-balance the likelihood of a loss severity between zero and $300,000 to 100%. If the probability of a loss $\leq$ $300,000 is P, we multiply the likelihood of each loss by 1/P.
B. Consumers' utility is the square root of their wealth.
C. The probability of a fire loss is 1%.
D. If insurers offered coverage at the actuarial pure premium, consumers would buy full coverage. This is true as long as consumers are risk-averse.
E. What is the maximum expense ratio at which consumers buy full coverage? This depends of the size of loss distribution. If all losses are total losses, consumers buy insurance coverage even at high expense

ratios. If all losses are very small losses, even risk averse consumers won't buy insurance cover at high expense ratios. The exponential distribution gives a mix of small and large losses.

F. Assume that insurers actually have a higher expense ratio. Let us choose an expense ratio which is 5 percentage points higher.
G. The insurer offers coverage above a *franchise* deductible D. If the loss is less then D, the insurer pays nothing. If the loss exceeds D, the insurer pays the full loss.
H. For a deductible of D, what is the actuarial pure premium? For a deductible of D, only the losses larger than D are included.
I. Given the expense ratio we are using, what is the gross premium?
J. What is the smallest deductible D such that consumers will buy coverage?
K. Since consumers are risk averse whereas insurers are not (in this exercise), this smallest deductible is also the equilibrium coverage that is best for consumers.
L. The optimal deductible depends on the consumers' utility function, and the expense ratio of the insurer. The relations are

   - A more risk averse consumer wants a lower deductible.
   - A higher expense ratio needs a higher deductible.