Corporate finance, Module 2: “How to Calculate Present Values”

Practice Problems

(The attached PDF file has better formatting.)

Exercise 2.1: Compounding Intervals

What is the value of $200 after 5 years invested at (a) 12% per annum, (b) 3% a quarter, and (c) 1% a month?

Solution 2.1:

- Part A: At 12% per annum, value is $\$200 \times 1.12^{5} = \$352.47$
- Part B: At 3% per quarter, value is $\$200 \times 1.03^{4 \times 5} = \$361.22$
- Part C: At 1% per month, value is $\$200 \times 1.01^{12 \times 5} = \$363.34$

Exercise 2.2: Compounding Intervals

What is the equivalent annual effective yield of each of the following?

A. 6% each half year
B. 3% a quarter
C. 1% a month

Solution 2.2:

Part A: $1.06^{2} - 1 = 12.36\%$
Part B: $1.03^{4} - 1 = 12.55\%$
Part C: $1.01^{12} - 1 = 12.68\%$

Question: How important is the compounding interval? And how important is the interest rate?

Answer: If we know the capitalization rate, it is good to be accurate. If the capitalization rate is 12%, we should not use 10% or 11%. For investment analysis, accurate interest rates are essential. Capital markets are efficient, and a slight difference in yield brings large changes in supply and demand.

Illustration: If the market yield is 12% per annum compounded quarterly, a bank that offers a yield of 12% with annual compounding may face much lower demand for its products.

For financial analysis, the increased accuracy pales compared to accurate estimates of future cash flows. A project may bring in $10 million in cash next year or $20 million. Good estimates of cash flows are the *sine qua non* of financial analysis. The proper capitalization rate is useful, but it is less important than the proper cash flows.

Exercise 2.3: Doubling Investments

How long will it take $1 to double when it is invested at (a) 3%, (b) 5%, (c) 10%, (d) 12%, (e) 15%? (Use logarithms to compute the answer.)

Solution 2.3:

Part A: With an annual effective interest rate of 3%:

$\$1 \times 1.03^{z} = \$2 \Rightarrow \ln 2 = z \ln 1.03 \Rightarrow z = \ln 2 / \ln 1.03 = 23.450$ years

Part B: With an annual effective interest rate of 5%:

$1 × 1.05^z = $2 ⇛ *ln* 2 = z *ln* 1.05 ⇛ z = *ln* 2 / *ln* 1.05 = 14.207 years

Part C: With an annual effective interest rate of 10%:

$1 × 1.10^z = $2 ⇛ *ln* 2 = z *ln* 1.10 ⇛ z = *ln* 2 / *ln* 1.10 = 7.273 years

Part D: With an annual effective interest rate of 10%:

$1 × 1.12^z = $2 ⇛ *ln* 2 = z *ln* 1.12 ⇛ z = *ln* 2 / *ln* 1.12 = 6.116 years

Part E: With an annual effective interest rate of 15%:

$1 × 1.15^z = $2 ⇛ *ln* 2 = z *ln* 1.15 ⇛ z = *ln* 2 / *ln* 1.15 = 4.959 years

Exercise 2.4: Discount Factors and Annuity Formula

An investment of $1,000 will produce income of $270 a year for 5 years. Calculate its NPV at a discount rate of 10% by the following methods:

A. The conventional NPV method, using separate discount factors
B. Using the annuity formula

Solution 2.4:

Part A: Discount factors: The present value of $270 per annum for 5 years at 10% is

$270 / 1.10^1 + $270 / 1.10^2 + $270 / 1.10^3 + $270 / 1.10^4 + $270 / 1.10^5 = $1,023.51

The net present value of the project is $1,023.51 – $1,000 = $23.51

Part B: Annuity Formula: $270 × $\left\{ \frac{1}{r} - \frac{1}{r \times (1+r)^t} \right\}$ = $1,023.51

The net present value of the project is $1,023.51 – $1,000 = $23.51

Exercise 2.5: Three Year Investment

An investment of $2,000 in year 0 produces cash flows of $700 in year 1, $700 in year 2, and $900 in year 3. Calculate its net present value at (a) 0%, (b) 5%, (c) 10%, (d) 15%.

Solution 2.5:

Part A: At 0%, – $2,000 + $700 + $700 + $900 = $300

Part B: At 5%, –$2,000 + $700 / 1.05^1 + $700 / 1.05^2 + $900 / 1.05^3 = $79.04

Part C: At 10%, –$2,000 + $700 / 1.10^1 + $700 / 1.10^2 + $900 / 1.10^3 = ($108.94)

Part D: At 15%, –$2,000 + $700 / 1.15^1 + $700 / 1.15^2 + $900 / 1.15^3 = ($270.24)

Exercise 2.6: Savings and Consumption

An actuarial candidate has savings of $1,200, and she expects to save an additional $600 next year. She will use the savings to pay exam fees of $800 in 2 years' time and $900 in 3 years' time. How much can she afford to spend now on textbooks if her savings earn (a) 5%, (b) 7%, (c) 9%?

Solution 2.6:

Part A: At 5%, $\$1{,}200 + \$600 / 1.05^1 - \$800 / 1.05^2 - \$900 / 1.05^3 = \$268.35$

Part B: At 7%, $\$1{,}200 + \$600 / 1.07^1 - \$800 / 1.07^2 - \$900 / 1.07^3 = \$327.33$

Part C: At 9%, $\$1{,}200 + \$600 / 1.09^1 - \$800 / 1.09^2 - \$900 / 1.09^3 = \$382.15$

Exercise 2.7: Estate Value

An actuary will receive $40,000 from his uncle's estate in 1 year and annually thereafter in perpetuity. What is the value of this perpetuity at an interest rate of (a) 8% (b) 10%?

Solution 2.7:

Part A: At 8%, $\$40{,}000 / 0.08 = \$500{,}000$

Part B: At 10%, $\$40{,}000 / 0.10 = \$400{,}000$

Exercise 2.8: Delayed Perpetuity

How much is the previous perpetuity worth if it begins in 5 years time instead of in 1?

Solution 2.8: If it begins in 5 years time instead of 1 year, it begins 4 years later than in the previous problem:

Part A: At 8%, $\$40{,}000 / (0.08 \times 1.08^4) = \$500{,}000 / 1.08^4 = \$367{,}514.93$

Part B: At 8%, $\$40{,}000 / (0.10 \times 1.10^4) = \$400{,}000 / 1.08^4 = \$273{,}205.38$

Exercise 2.9: Increasing Perpetuity

If the uncle's will provides $40,000 in 1 year, increased annually by 6%. What is the present value of this growing stream of income at an interest rate of (a) 8% (b) 10%?

Solution 2.9:

Part A: At 8%, $\$40{,}000 / (0.08 - 0.06) = \$2{,}000{,}000$

Part B: At 10, $\$40{,}000 / (0.10 - 0.06) = \$1{,}000{,}000$

Question: These problems are not hard.

Answer: The first two modules are background; if you have dealt with these topics, the first five modules are not difficult.

Question 2.10: Yield to Maturity

All but which of the following would likely increase the yield to maturity on a corporate bond?

A. An increase in the firm's business risk
B. An increase in the firm's leverage ratio
C. An increase in the risk-free rate
D. An increase in the firm's profitability ratio.
E. All of A, B, C, and D are true.

Answer 2.10: D

Statement A and B: Riskier firms have higher debt rates.

Statement C: The yield is the risk-free rate plus the firm's risk premium.

Statement D: More profitable firms pay lower debt interest rates. Less profitable firms have higher probabilities of bankruptcy, so they pay higher debt interest rates.