Corporate finance, Module 2: How to Calculate Present Values

(Practice problems covering the fundamental concepts in the textbook chapter. Review these for the homework assignments; the final exam questions will be modeled on these practice problems.)

Exercise 2.1: Compounding Intervals

What is the value of $2,000 after 5 years invested at

A. 12% per annum
B. 1% a month?

Solution 2.1:

- Part A: At 12% per annum, value is $\$2{,}000 \times 1.12^5 = \$3{,}524.68$
- Part B: At 1% per month, value is $\$2{,}000 \times 1.01^{12 \times 5} = \$3{,}633.39$

Exercise 2.2: Doubling Investments

How long will it take a dollar to double if it is invested at

A. 3%
B. 10%
C. 15%

Use logarithms to compute the answer: $\$2 = \$1 \times (1 + r)^t \Rightarrow ln\ 2 = t \times ln\ (1 + r)$.

Solution 2.2:

Part A: With an annual effective interest rate of 3%:

$\$1 \times 1.03^z = \$2 \Rightarrow$
$ln\ 2 = z\ ln\ 1.03 \Rightarrow$
$z = ln\ 2 / ln\ 1.03 = 23.450$ years

Part B: With an annual effective interest rate of 10%:

$\$1 \times 1.10^z = \$2 \Rightarrow$
$ln\ 2 = z\ ln\ 1.10 \Rightarrow$
$z = ln\ 2 / ln\ 1.10 = 7.273$ years

Part C: With an annual effective interest rate of 15%:

$\$1 \times 1.15^z = \$2 \Rightarrow$
$ln\ 2 = z\ ln\ 1.15 \Rightarrow$
$z = ln\ 2 / ln\ 1.15 = 4.959$ years

Exercise 2.3:

What is the net present value of an investment of $1,000 that produces income of $270 a year for 5 years at a discount rate of 10% per annum? Show the solution with discount factors and with the annuity formula.

Solution 2.3:

Discount factors: The present value of $270 per annum for 5 years at a 10% rate is

$$\$270 / 1.10^1 + \$270 / 1.10^2 + \$270 / 1.10^3 + \$270 / 1.10^4 + \$270 / 1.10^5 = \$1{,}023.51$$

The net present value of the project is $1,023.51 – $1,000 = $23.51

Annuity Formula: $270 × { 1/r – 1/[r × (1 + r)^t] } = $1,023.51

$$\$270 \times \left\{ \frac{1}{r} - \frac{1}{r \times (1+r)^t} \right\} = \$1{,}023.51$$

The net present value of the project is $1,023.51 – $1,000 = $23.51

Exercise 2.4:

What is the net present value of an investment costing $2,000 that produces cash flows of $700 in year 1, $700 in year 2, and $900 in year 3 if the discount rate is

A. 5%
B. 15%

Solution 2.4:

Part A: At 5%, $-\$2{,}000 + \$700 / 1.05^1 + \$700 / 1.05^2 + \$900 / 1.05^3 = \$79.04$

Part B: At 5%, $-\$2{,}000 + \$700 / 1.15^1 + \$700 / 1.15^2 + \$900 / 1.10^3 = (\$270.24)$

Exercise 2.5:

A worker now has $12,000 and expect to save an additional $6,000 next year and then pay $8,000 in 2 years' time and $9,000 in 3 years' time for a new car. How much of the present savings can the worker afford to spend now on a dining room set if savings earn

A. 5%
B. 9%

Solution 2.5:

Part A: At 5%, $\$12{,}000 + \$6{,}000 / 1.05^1 - \$8{,}000 / 1.05^2 - \$9{,}000 / 1.05^3 = \$2{,}683.51$

Part B: At 9%, $\$12{,}000 + \$6{,}000 / 1.09^1 - \$8{,}000 / 1.09^2 - \$9{,}000 / 1.09^3 = \$3{,}821.50$

Exercise 2.6:

A lottery winner receives $400 in 1 year's time and annually thereafter in perpetuity. What is the value of this perpetuity at an interest rate of 8%?

Solution 2.6:

At 8%, $400 / 0.08 = $5,000.00

Exercise 2.7:

How much is the previous perpetuity worth if it begins in 5 years instead of in 1 year?

Solution 2.7: If it begins in 5 years time instead of 1 year, it begins 4 years later than in the previous problem:

At 8%, $\$400 / (0.08 \times 1.08^4) = \$5{,}000.00 / 1.08^4 = \$3{,}675.15$

Exercise 2.8:

If the lottery winner receives $400 in 1 year's time and this amount increases 6% per annum, what is the present value of this growing income stream at an 8% interest rate?

Solution 2.8:

At 8%, $\$400 / (0.08 - 0.06) = \$20{,}000.00$