Corporate Finance, Module 3: Value of Common Stocks, practice problems
(The attached PDF file has better formatting.)
Brealey and Myers, Chapter 4
** Exercise 3.1: Present Value of Growth Opportunities
A stock trades for $\$ 300$ and has a 10\% dividend yield, defined as next year's dividend divided by the current price: $\mathrm{DIV}_{1} / \mathrm{P}_{0}$. The dividends grow at $10 \%$ per annum.
A. What is next year's dividend?
B. What is the capitalization rate for this stock?
C. What is the present value of growth opportunities?

Part A: Next year's dividend is $\$ 300 \times 10 \%=\$ 30$.
Part B: The stock price is next year's dividend divided by the difference between the capitalization rate ( $r$ ) and the dividend growth rate $(g)$ :

$$
P_{0}=\text { DIV }_{1} /(r-g) \Rightarrow \$ 300=\$ 300 /(r-10 \%) \Rightarrow r=20 \%
$$

Know the rule: the capitalization rate is the dividend yield plus the dividend growth rate:

$$
10 \%+10 \%=20 \%
$$

Part C: If all earnings were paid in dividends, the dividend growth rate would be zero and the stock price would be the present value of perpetual payments of $\$ 30$ a year at a capitalization rate of $20 \%$ : $\$ 30 / 20 \%=\$ 150.00$.

The rest if the stock price reflects the present value of growth opportunities: $\$ 300-\$ 150.00=\$ 150.00$.
** Exercise 3.2: Stock Price Relations
A firm has book equity of $\$ 100$ per share on December 31, 20X0, and projected earnings of

| Year | Earnings per Share | Payout Ratio |
| :---: | :---: | :---: |
| 1 | $\$ 25.00$ | $20 \%$ |
| 2 | $\$ 30.00$ | $20 \%$ |
| 3 | $\$ 23.00$ | $50 \%$ |

The dividends in these three years are paid on December 31, 20X1, 20X2, and 20X3.
After the third year, the firm will have a return on book equity of $16 \%$ and will reinvest $25 \%$ of its profits. The firm's capitalization rate is $15 \%$.
A. What are the earnings and dividends per share for the first three years?
B. What is the book equity per share, payout ratio, and dividend for the fourth year?
C. What is the dividend growth rate after the forth year?
D. What is the firm's value at the beginning of the fourth year (end of the third year)?
E. What is the firm's current value?

Use the template below for Parts $A$ and $B$ of this problem.

| Year | Equity | EPS | Payout Ratio | Dividend | Retained |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\$ 100.00$ | $\$ 25.00$ | $20 \%$ |  |  |
| 2 |  | $\$ 30.00$ | $20 \%$ |  |  |
| 3 |  | $\$ 23.00$ | $50 \%$ |  |  |
| 4 |  |  |  |  |  |

Part A: Determine the earnings and dividends per share for the first three years.

| Year | Equity | EPS | Payout Ratio | Dividend | Retained |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\$ 100.00$ | $\$ 25.00$ | $20 \%$ | $\$ 5.00$ | $\$ 20.00$ |
| 2 | $\$ 120.00$ | $\$ 30.00$ | $20 \%$ | $\$ 6.00$ | $\$ 24.00$ |
| 3 | $\$ 144.00$ | $\$ 23.00$ | $50 \%$ | $\$ 11.50$ | $\$ 11.50$ |
| 4 | $\$ 155.50$ |  | $75 \%$ |  |  |

- Year 1: We are given the stock price, earnings per share, and payout ratio. The dividend is $\$ 25 \times 20 \%$ $=\$ 5$, and the retained earnings are $\$ 25-\$ 5=\$ 20$. The book equity at the beginning of year 2 is $\$ 100$ + \$20 = \$120.
- Year 2: The earnings per share are $\$ 30$ and the payout ratio is $20 \%$, so the dividend is $\$ 30 \times 20 \%=\$ 6$. The retained earnings are $\$ 24$, and the equity at the beginning of year 3 is $\$ 120+\$ 24=\$ 144$.
- Year 3: Earnings per share are $\$ 23$ and the payout ratio is $50 \%$, so the dividend is $\$ 23 \times 50 \%=\$ 11.50$. The retained earnings are $\$ 23-\$ 11.50=\$ 11.50$, and the equity at the beginning of year 4 is $\$ 144+$ $\$ 11.50=\$ 155.50$.

Part B: At the beginning of year 4, the accumulated equity is $\$ 155.50$. The return on equity is $16 \%$, so the earnings are $\$ 155.50 \times 16 \%=\$ 24.88$. The plowback ratio is $25 \%$, so the payout ratio is $1-25 \%=75 \%$ and the dividend is $\$ 24.88 \times(1-25 \%)=\$ 18.66$.

Part C: The dividend growth rate is the ROE times the plow-back ratio, or $16 \% \times 25 \%=4 \%$.
Part D: The value of the stock in year 3 by the dividend growth model is $\$ 18.66 /(15 \%-4 \%)=\$ 169.64$.
Part D: The present value of the dividends paid at the end of years 1,2 , and $3+$ the present value of the stock price at the end of year 3 (= beginning of year 4 ) is

$$
\$ 5.00 / 1.15^{1}+\$ 6.00 / 1.15^{2}+\$ 11.50 / 1.15^{3}+\$ 169.64 / 1.15^{3}=\$ 127.99
$$

Question: The exponents of 1.15 are $1,2,3$, and 3 in the formula above. Should the last exponent be 4 ?
Answer: In the dividend growth model, the stock price is one year before the dividend date.
** Exercise 3.3: Risk and Return
In one year, Stock A will pay a $\$ 5.50$ dividend and will be priced at $\$ 121$, and Stock $B$ will pay a $\$ 30$ dividend and will be priced at $\$ 100$.

The two stocks have the same systematic risk. If Stock B now trades for $\$ 120$, what is the price of Stock $A$ ?
Solution 3.3: The two stocks have the same systematic risk, so they have the same market capitalization rate - that is, the same return. The return on Stock B is $(\$ 30+\$ 100-\$ 120) / \$ 120=8.33 \%$. If Stock A now trades for $Z$, its return is

$$
(\$ 5.50+\$ 121-Z) / Z=8.33 \%
$$

$\Rightarrow Z=\$ 126.50 / 1.0833=\$ 116.77$.

## ** Exercise 3.4: Price-Earnings Ratio

A firm has the following figures:
Market capitalization rate ..... 15\%
Expected dividend one year from now ..... \$5
Projected long-term constant growth rate of dividends ..... 10\%
Payout ratio ..... 40\%
A. What is the firm's stock price?
B. What are the earnings next year?
C. What are the earnings this year?
D. What is the price-earnings ratio?

Part A: We determine the stock price from the dividend growth model as

$$
P_{0}=D_{1} /(r-g)=\$ 5 /(15 \%-10 \%)=\$ 100 .
$$

Part B: The dividend is the earnings times the payout ratio, so next year's earnings are $\$ 5 / 40 \%=\$ 12.50$.
Part C: In a steady state, the dividend growth rate is also the earnings growth rate, so the current year's earnings are $\$ 12.50 / 1.10=\$ 11.36$.

Part D: If the price-earnings ratio is the current stock price divided by next year's earnings, the price-earnings ratio is $\$ 100 / \$ 12.50=8.000$.

If the price-earnings ratio is the current stock price divided by current earnings, the price-earnings ratio is $\$ 100$ / \$11.36 = 8.803.

Note: In the Brealey and Myers textbook, the dividend yield is next year's dividend divided by the current stock price, and the price-earnings ratio is the current stock price divided by current earnings. Financial analysts differ in these definitions; some analysts speak of the dividend yield as the current dividends divided by the current stock price, and the price-earnings ratio is the current stock price divided by current earnings. Unless the exam problem specifies otherwise, use the definitions in the textbook.

Question: If next year's dividend is greater, is the price-earnings ratio higher or lower?
Answer: if the payout ratio is given, the price-earnings ratio does not change.

- The current stock price is next year's dividend divided by $(\mathrm{r}-\mathrm{g})$, the excess of the capitalization rate over the dividend growth rate.
- Next year's earning are the dividend divided by the payout ratio.
- The price-earnings ratio ratio is the payout ratio divided by $(r-g)$.

In this exercise, the price-earnings ratio is $40 \% /(15 \%-10 \%)=8.000$.
** Exercise 3.5: Payout Ratios, Dividends, and Stock Prices
Two firms have market capitalization rates of $12 \%$, earn $10 \%$ on book equity, and pay a dividend next year of $\$ 10$ per share. Firm A has a higher payout ratio than Firm B. Which firm should have a higher stock price?

Solution 3.5: We solve this problem first intuitively and then by formulas.
Intuition 1: The stock price is the discounted value of future dividends. The dividend growth rate is the return on equity times the plow-back ratio. The return on equity is the same for both firms. The greater the plowback ratio, the higher the dividend growth rate and the greater is the discounted vale of future dividends, since both firms have the same capitalization rate. Firm A has the greater payout ratio so Firm B has the greater plow-back ratio, the higher dividend growth rate, and the higher stock price.

Intuition 2: The two firms have the same dividend. Firm A has the greater payout ratio, so Firm B has the greater earnings. If the firms have the same capitalization rate, Firm B has the higher stock price.

Formulas: We express the firm's stock price as a function of its payout ratio. If the payout ratio is $p$, the plowback ratio is $(1-p)$. The dividend growth rate is $10 \% \times(1-p)$. The stock price is $\$ 10 /[12 \%-10 \% \times(1-$ $p)]=\$ 10 /(2 \%+0.1 p)$. The denominator increases with $p$, so the ratio decreases with $p$. Firm A has the higher value of $p$, so Firm $B$ has the higher stock price.
** Exercise 3.6: Payout Ratios, Earnings, and Stock Prices
Two firms have market capitalization rates of $12 \%$, earn $10 \%$ on book equity, and have earnings (not dividends) next year of $\$ 10$ per share. Firm A has a higher payout ratio than Firm B. Which firm should have a higher stock price? (This question differs from the preceding one only by replacing dividends with earnings.)

Solution 3.6: We solve this problem first intuitively and then by formulas.
Intuition: The return on equity is less than the market capitalization rate, so money plowed back into the firm is being invested in negative NPV projects. The greater the plow-back ratio, the more money is being lost. Firm A has the greater payout ratio, so Firm B has the greater plow-back ratio, it is losing more money, and it should have the lower stock price.

Formulas: We express the firm's stock price as a function of its payout ratio. If the payout ratio is $p$, and the earnings are $\$ 10$ per share, the dividend next year is $\$ 10 \times p$. The plow-back ratio is $(1-p)$, so the dividend growth rate is $10 \% \times(1-p)$. The stock price is

$$
\$ 10 p /[12 \%-10 \% \times(1-p)]=10 p /(2 \%+0.1 p)
$$

The reciprocal of this ratio is $0.01+0.002 / p$, which decreases with $p$, so this ratio increases with $p$. Firm $A$ has the higher value of $p$ (the payout ratio), so it has the higher stock price.

Question: In a later module, we learn the Modigliani and Miller proposition that dividend policy does not matter in perfect capital markets with no taxes or other friction costs. Yet this example shows that dividend policy is relevant.

Answer: In perfect capital markets, all firms invest in projects with zero NPV's. These firms are investing in negative NPV projects, so dividend policy matters.
** Exercise 3.7: Net Present Value
A potential acquisition has the following expected cash flows at the end of each year:

| Year | Cash Flows |
| :---: | :---: |
| 0 | $-\$ 1,750,000$ |
| 1 | $\$ 75,000$ |
| 2 | $\$ 75,000$ |
| $3+$ | grow at $9 \%$ per annum |

The firm's market capitalization rate is $12.5 \%$.
What is the value of the acquisition?
Solution 3.7: Treat the cash flows as dividends and value the company after payment of the $\$ 75,000$ cash flow in year 1 .

- The cash flow in the next year (year 2) is $\$ 75,000$, the growth rate is $9 \%$, and the capitalization rate is $12.5 \%$, so the value of the company (after payment of the year 1 cash flow) is $\$ 75,000 /(12.5 \%-9 \%)$ $=\$ 2,142,857$.
- The present value of the acquisition is

$$
-\$ 1,750,000+\$ 75,000 / 1.125^{1}+\$ 2,142,857 / 1.125^{1}=\$ 221,428 .
$$

## Exercise 3.8: Present Value of Growth Opportunities

Three publicly traded companies have the following characteristics:

| Firm | $S_{0}$ | EPS $_{1}$ | r |
| :---: | :---: | :---: | :---: |
| W | $\$ 50$ | $\$ 4$ | $10 \%$ |
| Y | $\$ 100$ | $\$ 7$ | $20 \%$ |
| Z | $\$ 75$ | $\$ 2$ | $20 \%$ |

$\mathrm{S}_{0}=$ stock price at time $0 ; \mathrm{EPS}_{1}=$ earnings per share at time $1 ; r=$ market capitalization rate
A. For each firm, what is the present value of growth opportunities (PVGO)?
B. Of the three stocks, which one is best described as an income stock?

Part A: The stock price is the capitalized value of earnings plus the present value of growth opportunities. For the three firms, we have

- Firm W: $\$ 50=\$ 4 / 10 \%+$ PVGO $\Rightarrow$ PVGO $=\$ 10$
- Firm Y: $\$ 100=\$ 7 / 20 \%+$ PVGO $\Rightarrow$ PVGO $=\$ 65$
- Firm Z: $\$ 75=\$ 2 / 20 \%+$ PVGO $\Rightarrow$ PVGO $=\$ 65$

Part B: The present value of growth opportunities is a lower percentage of the stock price for income stocks than for growth stocks. The lowest percentage is for Firm W.

Question: Are there other characteristics of income stocks vs growth stocks?
Answer: The ratio of the present value of growth opportunities to the stock price is the item that Brealey and Myers emphasize. In common parlance, income stocks have lower capitalization rates and higher priceearnings ratios than growth stocks have. Firm W has the lowest capitalization rate and highest price-earnings ratio. Brealey and Myers do not use these attributes; follow Brealey and Myers for final exam problems.

## ** Exercise 3.9: Dividends and Stock Prices

A firm will pay an end of year dividend of $\$ 10$ per share, after which its stock will sell at $\$ 110$. If the firm's market capitalization rate is $10 \%$, what is its current stock price?

Answer 3.9: \$109.09

- The value of the stock plus its dividend next year is $\$ 110+\$ 10=\$ 120$.
- This is $10 \%$ more than the current value, which is $\$ 120 / 1.1=\$ 109.09$.
*Question 3.10: Stock Trading
Which of the following is true regarding the trading of common stocks?
A. Sales of new shares to raise new capital occur in the secondary market.
B. All stock companies have shares traded on an exchange.
C. The New York Stock Exchange is also known as the over the counter market.
D. NASDAQ is a system of computer terminals linking a network of dealers.
E. None of $A, B, C$, or $D$ is true.

Answer 3.10: D
Statement A: Sales of new shares, whether in an initial public offering or a subsequent offering, occur in the primary market.

Statement B: Most stock companies are not publicly traded. The U.S. has hundreds of thousands of firms, but only about twenty thousand are publicly traded.

Statement C: NASDAQ, not the New York Stock Exchange, is the over the counter market.
Statement D: The New York Stock Exchange is a physical place, located on Wall Street in downtown New York. NASDAQ is a system of terminals, not a physical location.

